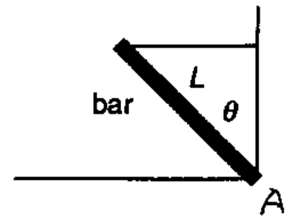
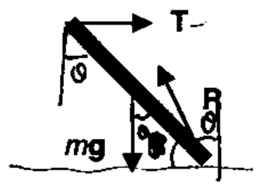
3. A bar of mass M and length L is held against a wall by a horizontal wire, as shown. The bottom of the bar is held wedged against the base of the wall, making an angle θ with respect to the wall. What is the magnitude of the force exerted on the bar at the corner? (7 marks)



Solution

A free-body diagram of the bar looks like



1

From the condition of zero torque: about A

clockwise torque = counterclockwise torque

$(mgL/2)\sin\theta = TL\cos\theta$ 2

so

$T = (mg/2)\sin\theta/\cos\theta$. 1

Thus, the components of the reaction force exerted by the hinge must be

$R_x = (mg/2)\sin\theta/\cos\theta = (mg/2)\tan\theta$ 1

$Mg\,\frac{L}{2}\sin\theta - TL\cos\theta = 0$.

and

$T - R_x = 0$

$R_y = mg,$ 1

$R_y - mg = 0$

so

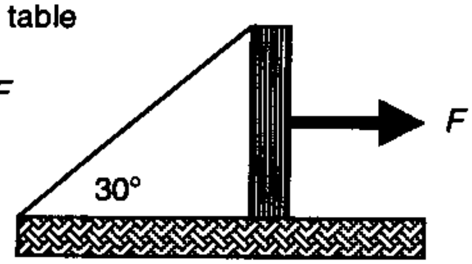
$R^2 = [(mg/2)\tan\theta]^2 + (mg)^2$

$= (mg)^2[1 + (\tan\theta/2)^2]$ 1

Lastly,

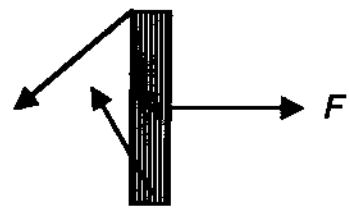
$R = mg[1 + (\tan\theta/2)^2]^{1/2}$.

3. The base of a massless bar of length L is fixed to a table so that it cannot slip. The top of the bar is attached to the table by a thin wire, as shown. A horizontal force F is applied to the mid-point of the bar by a rope. If the thin wire snaps when it experiences a tension greater than 400 N, what is the maximum force that can be applied through the rope?
(Include a free-body diagram) (7 marks)



Solution

The free-body diagram of the bar looks like



From the condition of zero torque about the bottom of the bar:

clockwise torque = counterclockwise torque

$(FL/2) = TL \sin 60°$

so

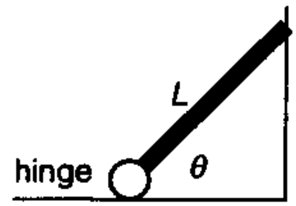
$F = 2T \sin 60°$.

If the thin wire snaps at 400 N, then the maximum force that can be applied is

$F = 2 \bullet 400 \sin 60°$

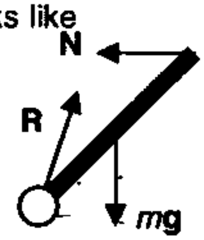
$= 693$ N.

3. A bar of mass M and length L leans up against a frictionless wall, as shown. The bottom of the bar is held against the floor by a hinge, around which the bar is free to rotate. The hinge cannot slide, so the bar makes a fixed angle θ with respect to the floor. What is the magnitude of the force that the hinge exerts on the bar? (7 marks)



Solution

A free-body diagram of the bar looks like



1

From the condition of zero torque: 2

clockwise torque = counterclockwise torque

$(mgL/2)\cos\theta = NL\sin\theta$

so

$N = (mg/2)\cos\theta/\sin\theta$. 1

Thus, the components of the reaction force exerted by the hinge must be

$R_x = (mg/2)\cos\theta/\sin\theta$ 1

and

$R_y = mg$, 1

so

$$R^2 = [(mg/2)\cos\theta/\sin\theta]^2 + (mg)^2$$
$$= (mg)^2\,[\,1 + (\cos\theta/2\sin\theta)^2]$$

Lastly, 1

$$R = mg\,[\,1 + (\cos\theta/2\sin\theta)^2]^{1/2}.$$

(v) Two thin disks are cut from the same metal sheet. Disk A has a moment of inertia I, while disk B has $I/16$. If disk A has a radius R, what is the radius of disk B?

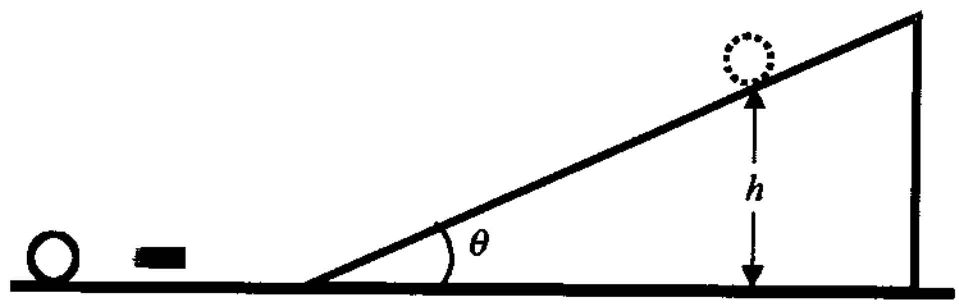
(a) $R/16$ (b) $R/4$ (c) $R/2$ (d) R (e) none of [a]-[d]

The moment of inertia of a disk is $I = MR^2/2$. If the disks are cut from the same sheet, then their mass changes with radius according to

$$mass = density \times \pi R^2.$$

In other words, the moment is proportional to R^4. Thus, if the moment is reduced by a factor of 4, the radius is reduced by $16^{1/4} = 1/2$.

2. As shown in the figure below, a small solid ball of radius R rolls without slipping on a horizontal surface at a linear velocity $v_o = 20$ m/s and then rolls up the incline. If friction losses are negligible, what is the linear speed of the ball when its height $h = 21$ m? For numerical convenience, use $g = 10$ m/s^2. (11 marks)



Solution

The kinetic energy of a rolling object has both translational and rotational contributions

$$K = mv^2/2 + I\omega^2/2.$$

The moment of inertia of a sphere is

$$I = 2mR^2/5,$$

so

$$K = mv^2/2 + mR^2\omega^2/5.$$

But $v = \omega R$ here, so

$$= mv^2/2 + mv^2/5$$
$$= mv^2\,(1/2 + 1/5)$$
$$= mv^2\,(7/10).$$

By conservation of energy, as the ball rolls up the hill

$$\Delta K = -\Delta U,$$

so

$$(7/10) \bullet (mv_o^2 - mv^2) = mgh$$

or

$$v_o^2 - v^2 = (10/7)\,gh$$

$$v^2 = v_o^2 - (10/7)\,gh.$$

Numerically, this becomes

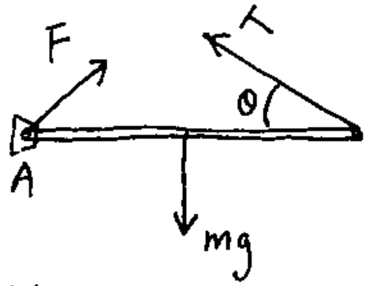
$$v^2 = 20^2 - (10/7) \bullet 10 \bullet 21$$
$$= 100$$
$$v = 10 \text{ m/s}.$$

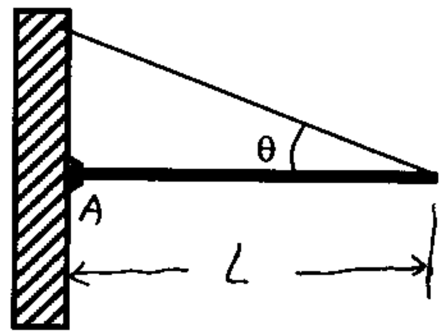
2(6marks). A uniform steel rod, with a mass of 20.0 kg and 3.00 m long, is supported by a loose bolt attached to the wall at one end and by a wire at the other end. The wire makes an angle of θ=35° with the horizontal as shown in the figure.

(a). What is the magnitude of the force exerted by the bolt on the rod?

(b). If the wire breaks, what is the angular acceleration of the rod?

[Solution].





1/6

static Equilibrium :

$\Sigma \vec{F_i} = 0$. $\quad F_x - T\cdot\cos\theta = 0$ ①

$F_y + T\sin\theta - mg = 0$ ②

1/6

$\Sigma \vec{\tau_i} = 0$ (about point A) : $\quad TL\sin\theta - mg\frac{L}{2} = 0$

$T\cdot\sin\theta = mg/2$. ③

from ③: $\quad T = \dfrac{mg}{2\sin\theta} = \dfrac{(20)(9.8)}{2\sin 35^\circ} = 171\ N$

1/6

from ①: $\quad F_x = T\cdot\cos\theta = (171)(\cos 35^\circ) = 140\ N$.

from ② and ③ : $\quad F_y = mg/2 = \dfrac{(20)(9.8)}{2} = 98\ N$

(a) : $\quad F = \sqrt{F_x^2 + F_y^2} = \sqrt{140^2 + 98^2} = 171\ N$.

2/6

(b). when the wire breaks, $T = 0$.

$\tau = -mg\frac{L}{2}$ (clockwise) .

angular acceleration of the rod about A : $\quad \alpha = \dfrac{\tau}{I}$.

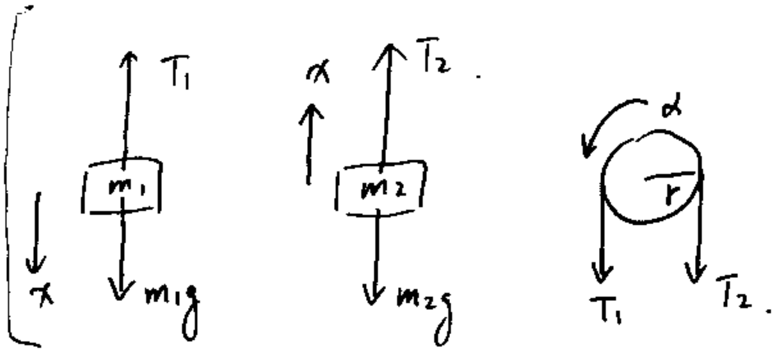
$$\alpha = \frac{-mg\frac{L}{2}}{\frac{1}{3}mL^2} = -\frac{3g}{2L} = -\frac{3\,(9.8)}{2\,(3)} = 4.9\ \text{rad}/s^2$$

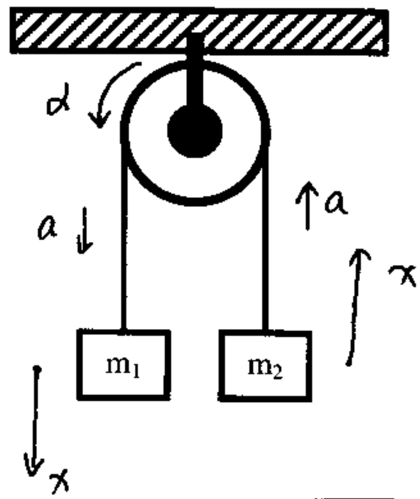
2(6 marks). A mass of 375 g hangs from one end of a string that goes over a pulley with a moment of inertia of 0.0125 kg·m^2 and a radius of 15.0 cm. A mass of 800 g hangs from the other end. When the masses are released, the larger mass accelerates downward, the lighter mass accelerates upward, and the pulley turns without the string slipping on the pulley.

(a) What is the tension in the string on the side of the 800-g mass?

(b) What is the angular acceleration of the pulley?

[solution]: Free Body diagram: 1/6





$\vec{F} = m\vec{a}$ for m_1 and m_2:

1/6
$$\begin{cases} m_1 g - T_1 = m_1 a & \text{①} \\ T_2 - m_2 g = m_2 a & \text{②} \end{cases}$$

$\vec{\tau} = I\vec{\alpha}$ for the pulley:

1/6
$$\begin{cases} T_1 r - T_2 r = I\alpha & \text{③} \\ a = \alpha r & \text{④} \end{cases}$$

①+②:

$$(m_1 - m_2)g + (T_2 - T_1) = (m_1 + m_2)a$$

④ ⇒ ③: $$T_1 - T_2 = \frac{Ia}{r^2}.$$

Then: $$(m_1 - m_2)g - \frac{Ia}{r^2} = (m_1 + m_2)a.$$

1/6
$$a = \frac{(m_1 - m_2)g}{m_1 + m_2 + \frac{I}{r^2}}$$

$$= \frac{(0.8 - 0.375)(9.8)}{0.8 + 0.375 + \frac{0.0125}{(0.15)^2}} = 2.41 \text{ m/s}^2$$

a): 1/6 from ①,

$$T_1 = m_1(g - a)$$
$$= (0.8)(9.8 - 2.41)$$
$$= 5.91 \text{ N}$$

b). 1/6
$$\alpha = \frac{a}{r} = \frac{2.41}{0.15} = 16.1 \text{ rad/s}^2$$