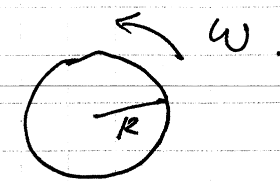
Phys. 101.

lecture March. 2.-4.

- Collect Midterm.
- Ch.10 Summary.



— rotational kinematics:

angular velocity: $\omega = \frac{d\theta}{dt}$.

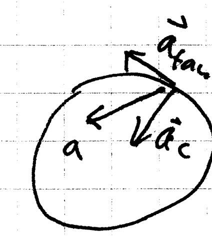
angular acceleration $\alpha = \frac{d\omega}{dt}$.

$\begin{pmatrix} ccw. + , \\ cw: - \end{pmatrix}$ — angular displacement $\Delta\theta$ $(+/-)$.

linear velocity: $v = R \cdot \omega$. — ω in rad/sec.

linear acc.: $a_{tan} = R\alpha$. — $\left. a = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = r \cdot \frac{d\omega}{dt} = r\alpha \right\}$. change of speed.

centripetal acc: $a_c = \omega^2 R = \frac{v^2}{R}$



$\vec{a} = \vec{a}_{Tan} + \vec{a}_c$.

(vector sum). $\vec{a}_{tan} \perp \vec{a}_c$.

— rotational dynamics.

$\tau = I\alpha$.

net torque; moment of inertia; angular acceleration.

— Kinetic energy:

$$K = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2 .$$

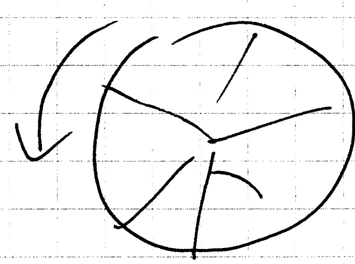
v_{CM} — velocity of C.M.

I_{CM} — Moment of Inertia about C.M.

Note: Total momentum $= M \cdot \vec{v}_{CM}$ = momentum of C.M.

but: Total K.E $\neq \frac{1}{2} M v_{CM}^2$ (K.E. of C.M).

e.g: a turning wheel.



$\vec{P}_{Total} = 0$.

but. $K_{Total} \neq 0$.

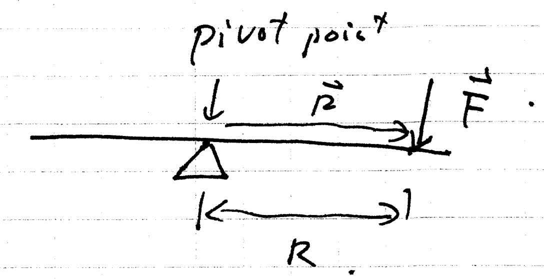
Note: Moment of Inertia. ~~p.260~~.

formulas on p.260 will be given in exams.

physical meaning: natural resistance to rotation. (like mass for linear motion).

mass distribution: { near axis, I small; harder to rotate → { away from axis, I large.

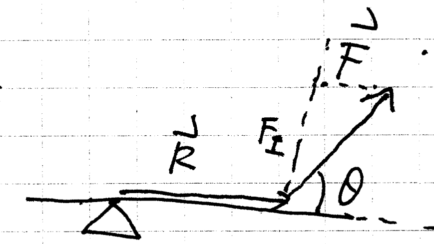
Note: Torque : τ.



Torque of $\vec{F}$:

If $\vec{F} \perp \vec{R}$.

then $\tau = FR$. (+ — ccw)
(− — cw).

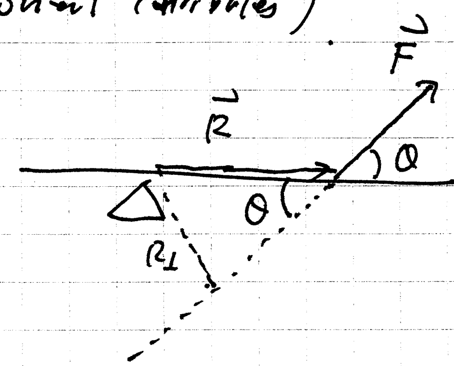


If $\vec{F}$ is not perpendicular to $\vec{R}$:

then: $\tau = F_\perp R$.

$= F \sin\theta \cdot R$.

(only perpendicular component contributes)



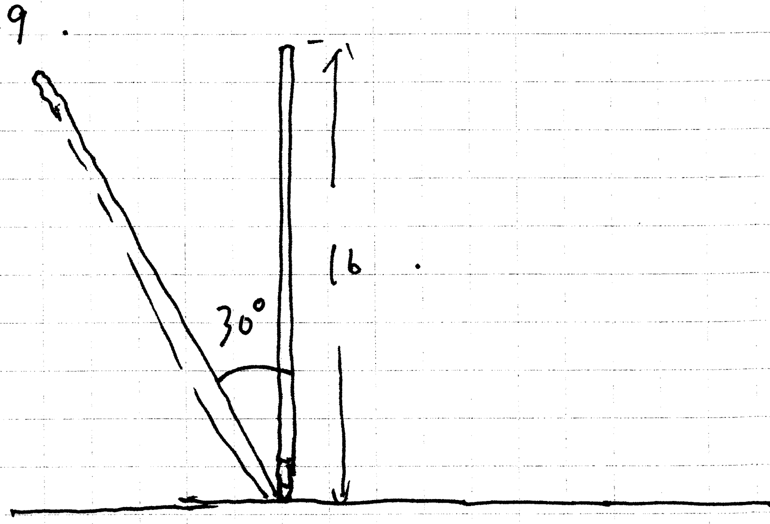
OR: $\tau = F \cdot R_\perp$

$= F R \cdot \sin\theta$.

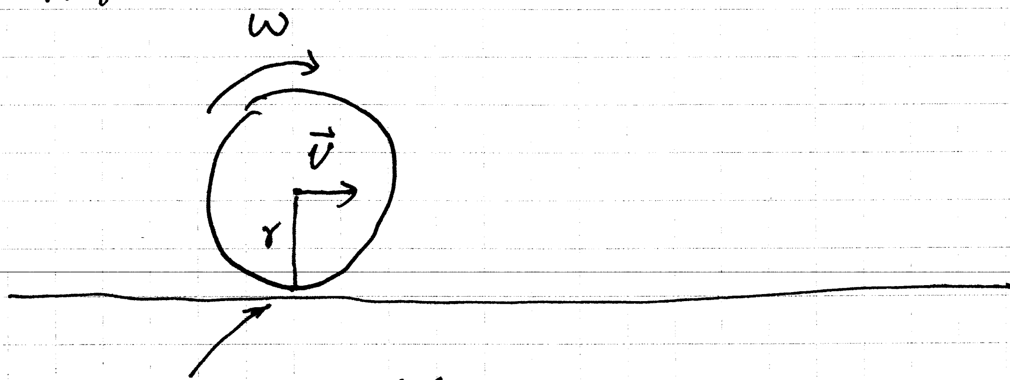
e.g: old final exam. # 19.

a). $\alpha = ?$ when $\theta = 30°$.

b). $\omega = ?$ when $\theta = 30°$.



- Rolling Motion:



No relative sliding.

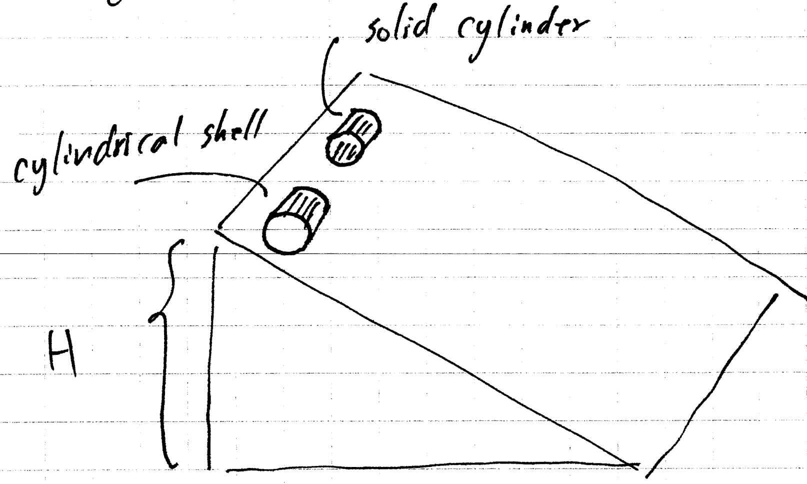
(No relative motion at the point of contact).

$\vec{v}$ — Translational velocity (of the centre of mass)

ω — Angular velocity about the centre of mass.

Important relation: $v = r\omega$.

e.g. Rolling down on an inclined plane.



which one reaches the bottom first?

No non conservative work : $W_{nc} = 0$.

(rolling friction — static friction
No heat dissipation)

For an object rolling down by height h.

$$E_i = E_f .$$

$$mgh = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

$$v_c = v = r \cdot \omega .$$

$$\therefore \ mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \cdot \frac{v^2}{r^2} .$$

$$2mgh = mv^2 \left(1 + \frac{I}{mr^2} \right) .$$

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}} .$$

($I \propto mr^2$ for all ~~shapes~~ shapes)

$\therefore \ \frac{I}{mr^2} \rightarrow$ fraction depending on shape.

The smaller the $\frac{I}{mr^2}$, the faster the v.

(the larger the I, the slower).

— solid cylinder : $I = \frac{1}{2}mr^2$, $\frac{I}{mr^2} = \frac{1}{2}$, $v = \sqrt{\frac{4gh}{3}}$

— Hollow cylinder : $I = mr^2$: $\frac{I}{mr^2} = 1$, $v = \sqrt{gh}$.

$\therefore$ Solid cylinder is faster and will reach bottom first.

- Conceptual reasoning.

$U = mgh \xrightarrow[\text{converted to}]{} K = K_L + K_R$. $=$ const.

large $I \Rightarrow$ large $K_R \Rightarrow$ small K_L
(small v)

e.g. race : solid cylinder and solid sphere.

which one wins ?

— solid sphere.

why ? sphere : mass distribution closer to axis

i.e : $I_{\text{solid sphere}} < I_{\text{solid cylinder}}$

$\therefore$ K_R is small, K_L — larger

$\therefore$ v is faster.

· Angular momentum. (ch. 11. — 11.-1 only).

(very important.

Newton's law for rotation:

$$\tau = I \cdot \alpha = I \cdot \frac{d\omega}{dt} = \frac{d(I\omega)}{dt}.$$

define: $I\omega = L$ as angular momentum.

Then: $\boxed{\tau = \frac{dL}{dt}}$ (1).

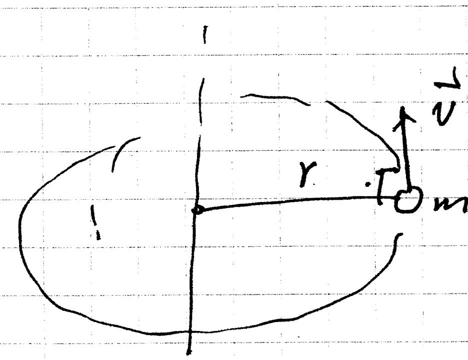
Note: more general definition of angular momentum

quantity of rotation)

$$\boxed{L = m r^2 \cdot v \quad \cancel{= m v / r}}$$

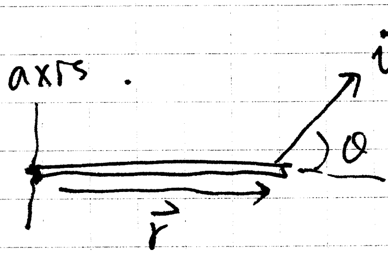
$\vec{v} \perp \vec{r}$:

$$L = mv \cdot r = p \cdot r.$$



when $\vec{v}$ is not perpendicular to $\vec{r}$:

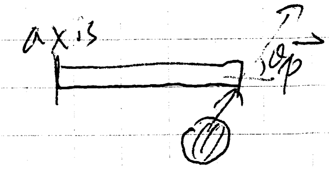
$$L = m v_\perp \cdot r = m v\, r_\perp = m v r \sin\theta$$ axis.



meaning: linear momentum

can cause rotation

e.g. use a basket ball to open a door.



$$L = r p_\perp = r p \sin\theta.$$