March . 4. 2009

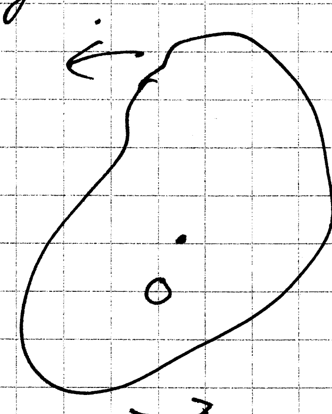
- Angular momentum .

  — A quantity that describes rotational motion .

  (just like momentum is a quantity that describes linear motion . — also called linear momentum)

  $\vec{p} = m\vec{v}$

- Angular momentum of a rotating rigid body

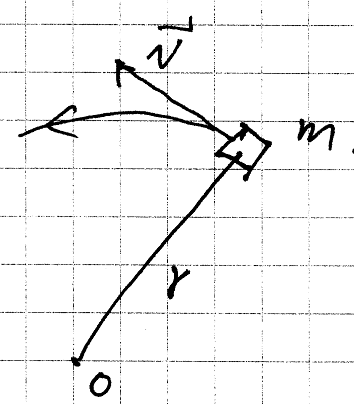


$$L = I \cdot \omega \; .$$

I — moment of inertia about O .

ω — angular velocity about O .

e.g.: A particle at the end of a massless rod in a circular motion .



$$L = I\omega \; .$$
$$= mr^2 \cdot \frac{v}{r}$$
$$= r \cdot mv \; .$$
$$= rp$$

$L = r p_t$. p_t — tangential component of $\vec{p}$

- Angular momentum of a moving particle.

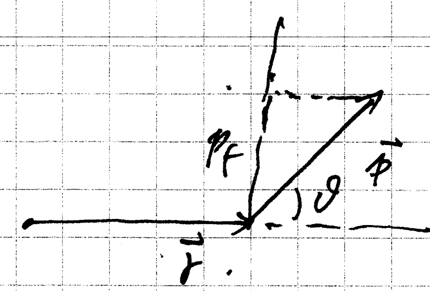
  └ can be moving in a straight line

The angular momentum of m about O:

$L = r \cdot P_t$.

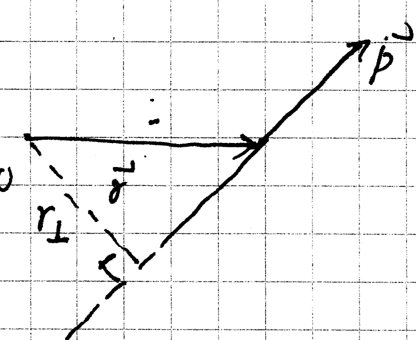
$= r \cdot p \sin\theta$

└ θ — angle between $\vec{p}$ and $\vec{r}$



or: $L = r_\perp p$.

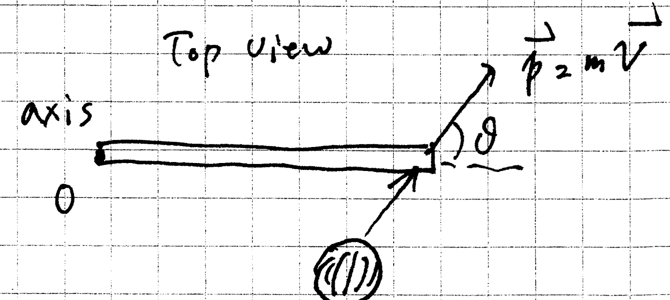
$r_\perp$ — moment arm.



- How can a particle in a linear motion have angular momentum?

e.g.: use a basketball to open a door.

The angular momentum of the ball about the axis O can be transfered to the door can causes the rotation of the door.



- conservation of angular momentum

when $\tau = 0$ (No net torque)

$$\frac{dL}{dt} = 0$$

$$L = \text{const.}$$

$$L_i = L_f \, .$$

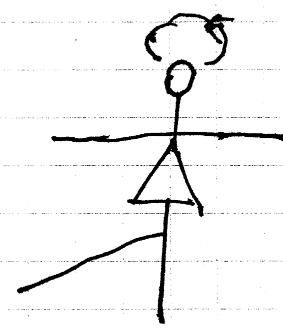
e.g. Figure skater.

$$\tau = 0 .$$

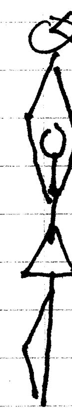
$$L_f = L_i$$

$$I_f W_f = I_i W_i$$

$$W_f = \frac{I_i}{I_f} W_i \, .$$



large I_i.

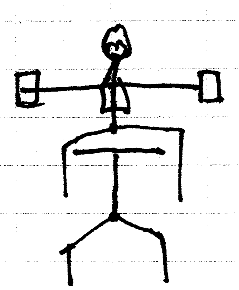


small I_f.

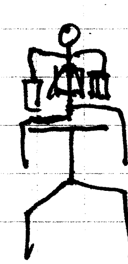
Since $I_i > I_f$,

$W_f > W_i$. — spin faster.

Demo:



$I_i W_i$



$I_f W_f$.

$$I_f < I_i , \quad W_f > W_i \, .$$

e.g. 11-4. p. 288.

Running on a circular platform.

$m = 60$ kg.

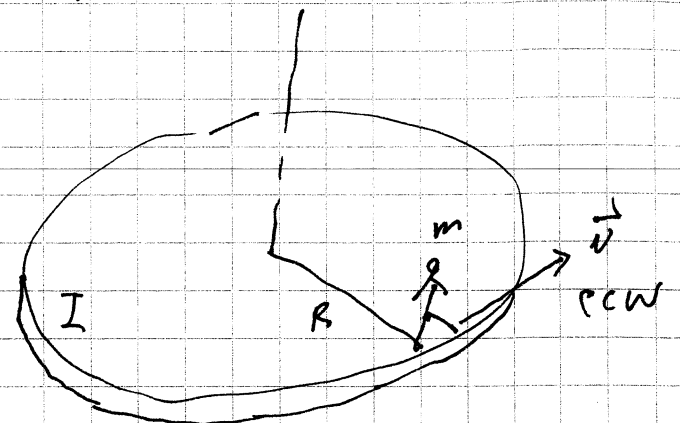
$R = 3$ m.

$I = 1800$ kg$\cdot$m^2.

$\omega_i = 0$.

$v = 4.2$ m/s.

$\omega_f = ?$



Angular momentum : $L = L_{per} + L_{plat}$.

$$L_i = L_f.$$

$$0 = rmv + I\omega.$$

$$\omega = -\frac{mrv}{I} = -\frac{60 \times 3 \times 4.2}{1800}$$

$$= -0.42 \text{ rad/s}.$$

(cw)

Q#11-19.

given: v_{mT}

need $v = v_{mT} + v_{TG}$.

$$L_i = L_f$$

$$0 = Rm(v_{mT} + R\omega) + I\omega$$

$$0 = Rmv_{mT} + mR^2\omega + I\omega$$

$$\omega = \frac{-mRv_{mT}}{I + mR^2}.$$