Chapter 7

Work and Energy

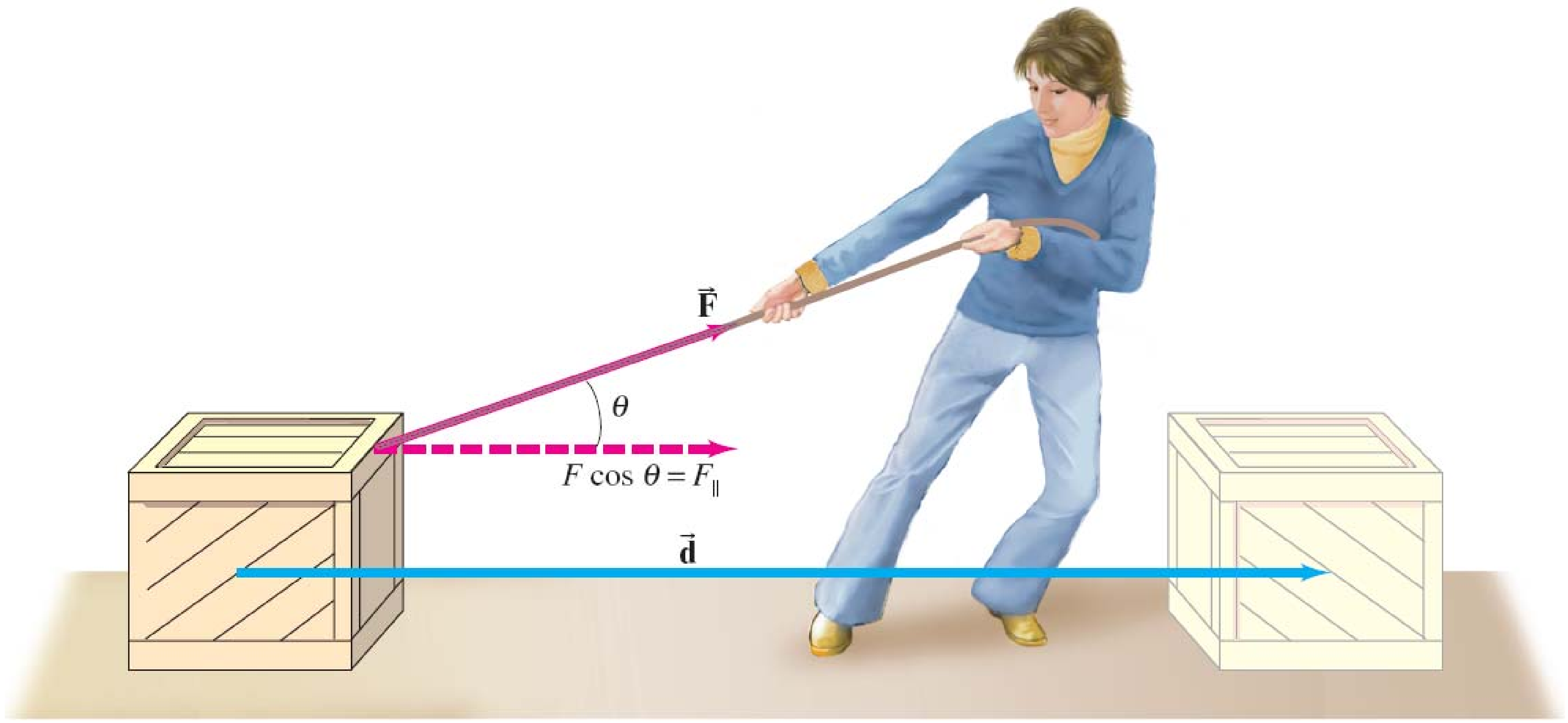
Units of Chapter 7

- Work Done by a Constant Force
- Scalar Product of Two Vectors
- Work Done by a Varying Force
- Kinetic Energy and the Work-Energy Principle

7-1 Work Done by a Constant Force

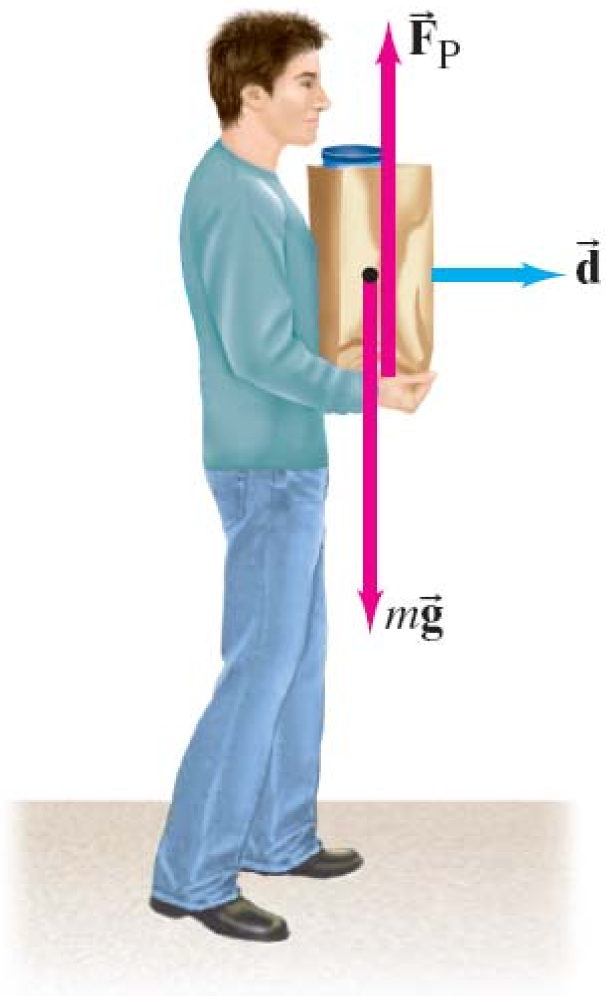
The **work** done by a **constant** force is defined as the **distance** moved multiplied by the component of the **force** in the direction of displacement:

$$W = Fd\cos\theta.$$



7-1 Work Done by a Constant Force

In the SI system, the units of work are **joules**:



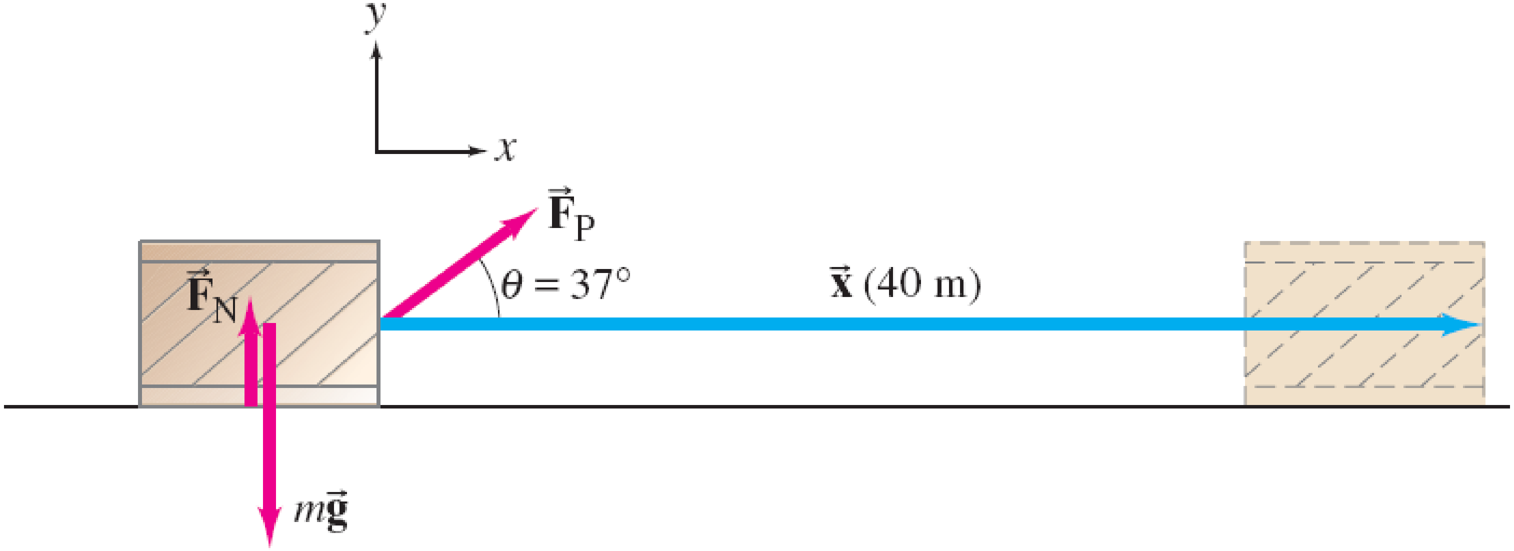
$$1\,\mathrm{J} = 1\,\mathrm{N\cdot m}.$$

As long as this person does not lift or lower the bag of groceries, he is doing **no work** on it. The force he exerts has no component in the direction of motion.

7-1 Work Done by a Constant Force

Example 7-1: Work done on a crate.

A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force F_P = 100 N, which acts at a 37° angle as shown. The floor is smooth and exerts no friction force. Determine (a) the work done by each force acting on the crate, and (b) the net work done on the crate.

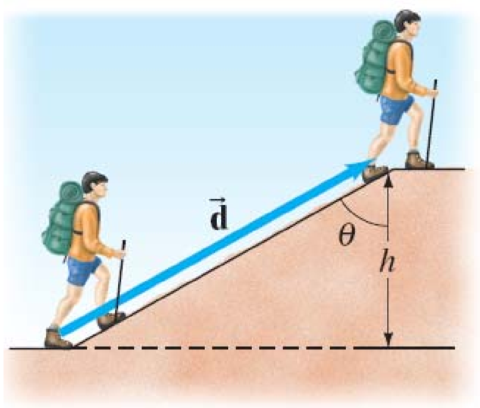


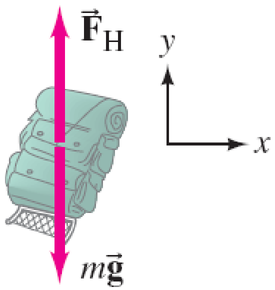
7-1 Work Done by a Constant Force

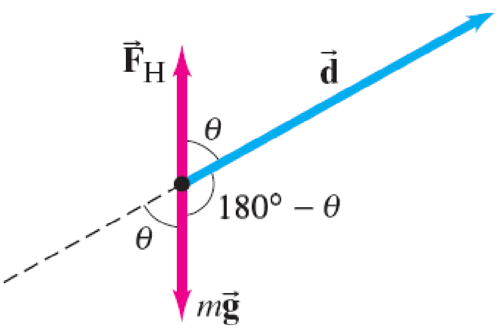
Solving work problems:

1. **Draw** a free-body diagram.
2. Choose a **coordinate** system.
3. Apply **Newton's laws** to determine any unknown forces.
4. Find the **work** done by a specific force.
5. To find the **net work**, either
   a) find the **net force** and then find the work it does, or
   b) find the work done by each force and **add**.

7-1 Work Done by a Constant Force







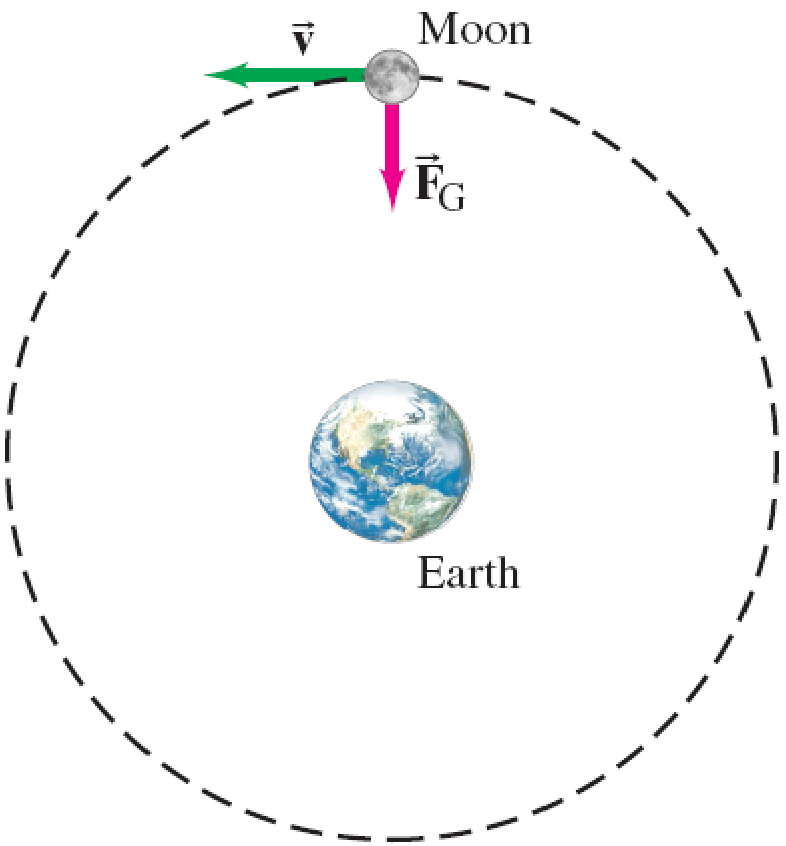
Example 7-2: Work on a backpack.

(a) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height h = 10.0 m, as shown. Determine also (b) the work done by gravity on the backpack, and (c) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity (i.e., acceleration is zero).

7-1 Work Done by a Constant Force

Conceptual Example 7-3: Does the Earth do work on the Moon?

The Moon revolves around the Earth in a nearly circular orbit, with approximately constant tangential speed, kept there by the gravitational force exerted by the Earth. Does gravity do (a) positive work, (b) negative work, or (c) no work at all on the Moon?



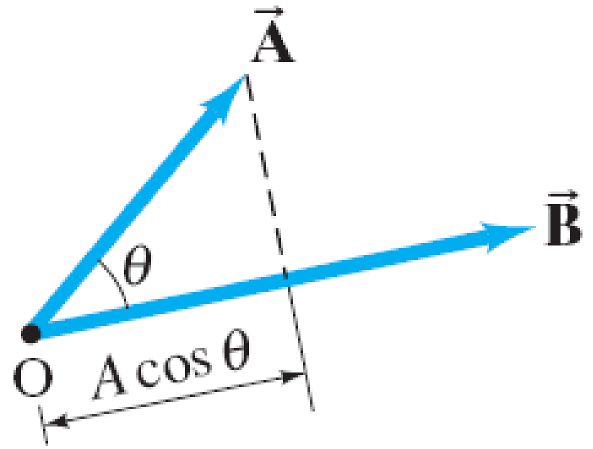
7-2 Scalar Product of Two Vectors

Definition of the scalar, or dot, product:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta$$

Therefore, we can write:

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta.$$



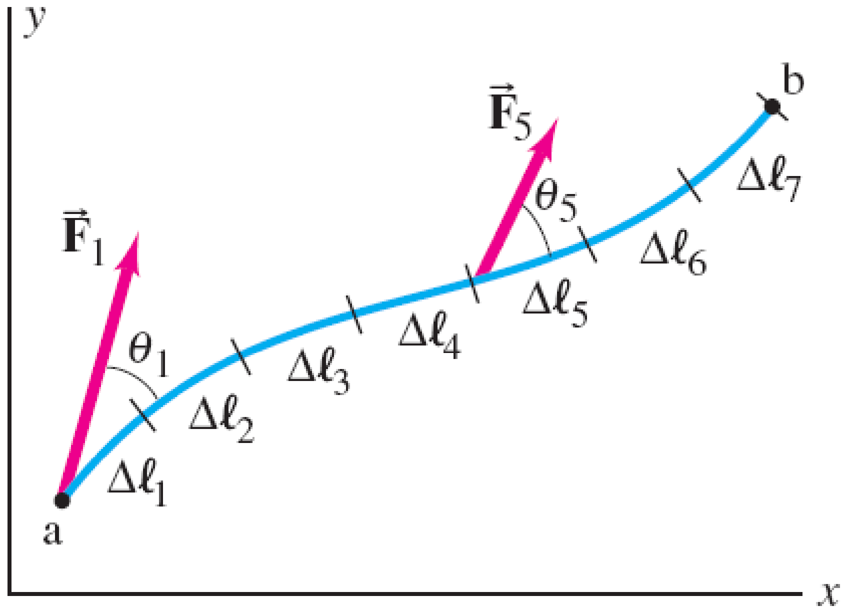
7-2 Scalar Product of Two Vectors

Example 7-4: Using the dot product.

The force shown has magnitude F_P = 20 N and makes an angle of 30° to the ground. Calculate the work done by this force, using the dot product, when the wagon is dragged 100 m along the ground.

7-3 Work Done by a Varying Force

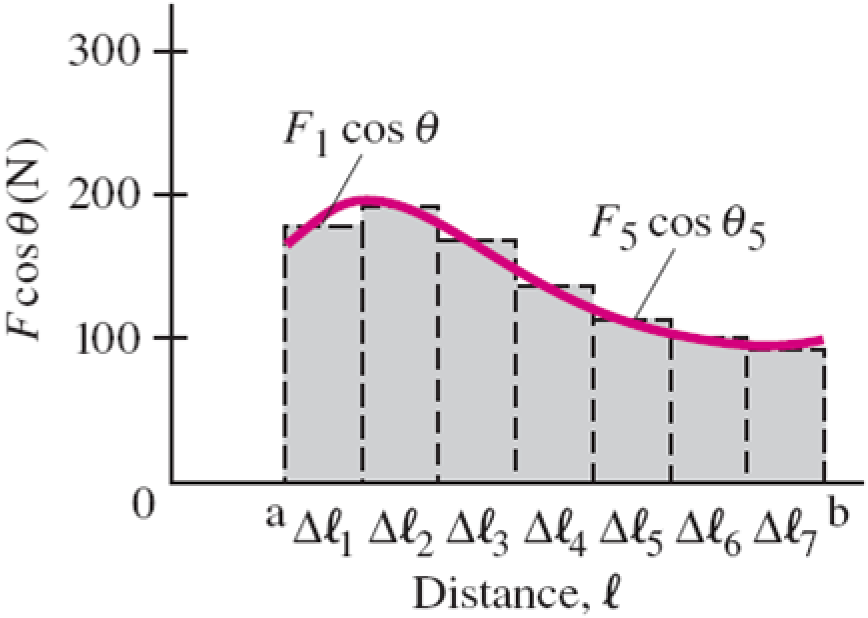
Particle acted on by a varying force. Clearly, $\vec{F}\cdot d$ is not constant!



7-3 Work Done by a Varying Force

For a force that varies, the work can be approximated by dividing the distance up into small pieces, finding the work done during each, and adding them up.

$$W \approx \sum_{i=1}^{7} F_i \cos\theta_i \, \Delta\ell_i$$



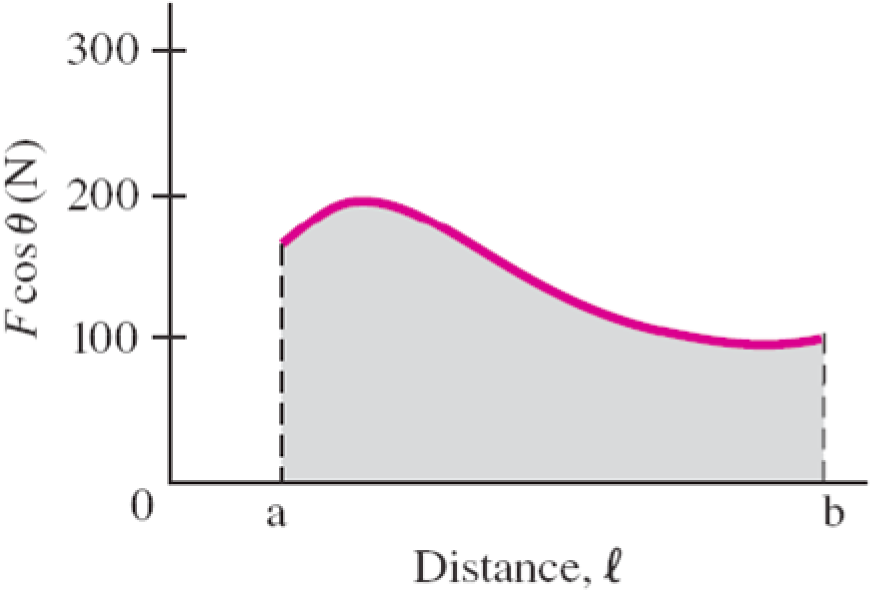
7-3 Work Done by a Varying Force

In the limit that the pieces become infinitesimally narrow, the work is the area under the curve:

$$W = \lim_{\Delta \ell_i \to 0} \sum F_i \cos\theta_i \, \Delta \ell_i = \int_a^b F \cos\theta \, d\ell.$$

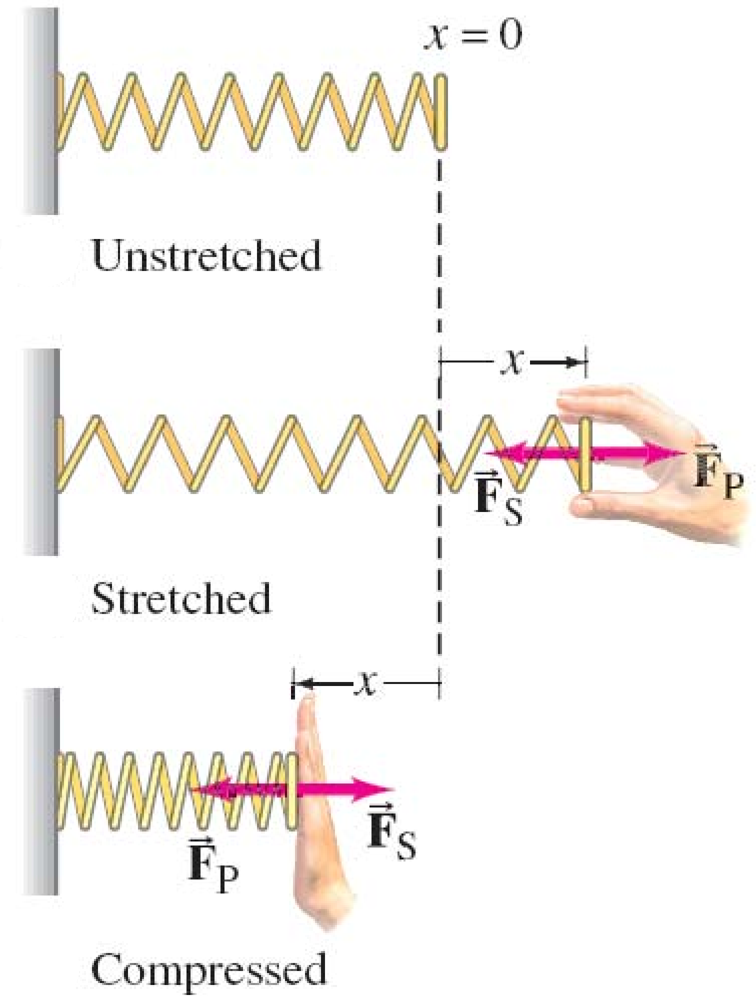
Or:

$$W = \int_a^b \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}}.$$



7-3 Work Done by a Varying Force

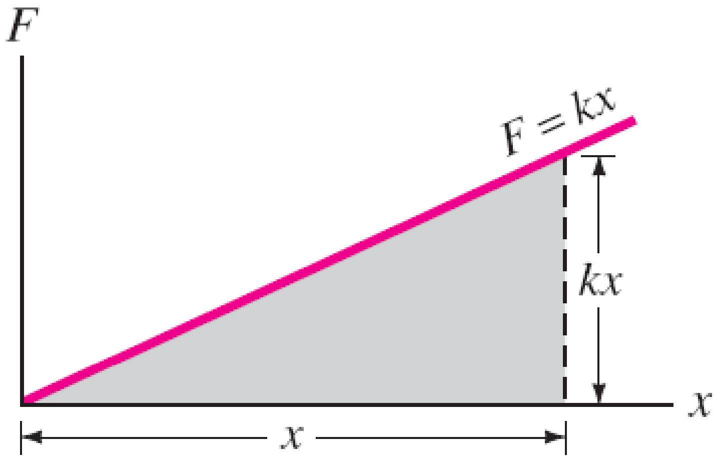
Work done by a spring force:



The force exerted by a spring is given by:

$$F_S = -kx.$$

7-3 Work Done by a Varying Force



Plot of F vs. x. Work done is equal to the shaded area.

$$W_P = \int_{x_a=0}^{x_b=x} [F_P(x)\,\hat{\mathbf{i}}] \cdot [dx\hat{\mathbf{i}}] = \int_0^x F_P(x)\,dx$$

$$= \int_0^x kx\,dx = \tfrac{1}{2}kx^2\Big|_0^x = \tfrac{1}{2}kx^2$$

7-3 Work Done by a Varying Force

Example 7-5: Work done on a spring.

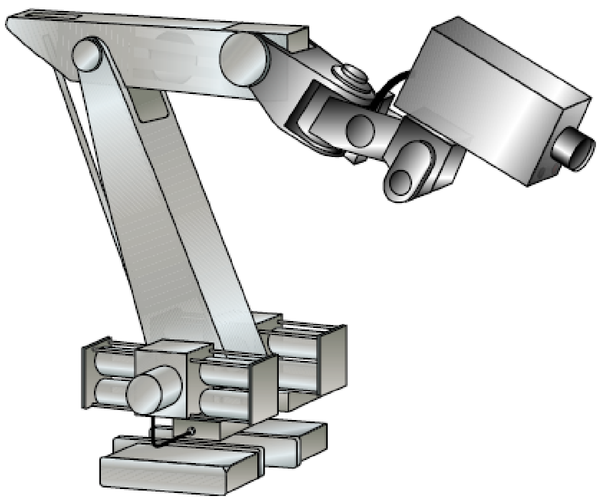
(a) A person pulls on a spring, stretching it 3.0 cm, which requires a maximum force of 75 N. How much work does the person do? (b) If, instead, the person compresses the spring 3.0 cm, how much work does the person do?

7-3 Work Done by a Varying Force

Example 7-6: Force as a function of x.

A robot arm that controls the position of a video camera in an automated surveillance system is manipulated by a motor that exerts a force on the arm. The force is given by

$$F(x) = F_0\left(1 + \frac{1}{6}\frac{x^2}{x_0^2}\right),$$



where F_0 = 2.0 N, x_0 = 0.0070 m, and x is the position of the end of the arm. If the arm moves from x_1 = 0.010 m to x_2 = 0.050 m, how much work did the motor do?

7-4 Kinetic Energy and the Work-Energy Principle

Energy was traditionally defined as the **ability to do work**. We now know that not all forces are able to do work; however, we are dealing in these chapters with **mechanical** energy, which does follow this definition.

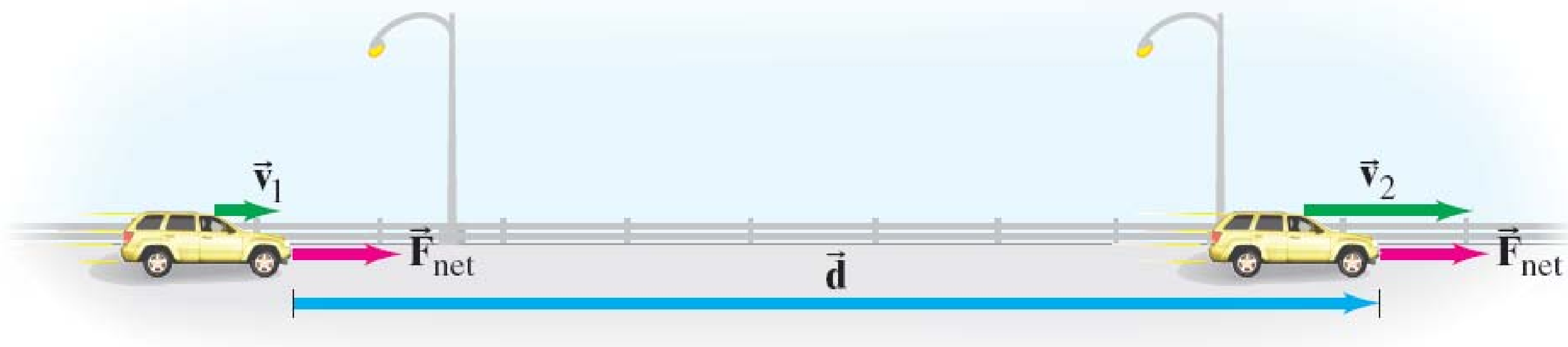
7-4 Kinetic Energy and the Work-Energy Principle

If we write the **acceleration** in terms of the **velocity** and the **distance**, we find that the work done here is

$$W_{net} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2.$$

We define the **kinetic energy** as:

$$K = \tfrac{1}{2}mv^2.$$



7-4 Kinetic Energy and the Work-Energy Principle

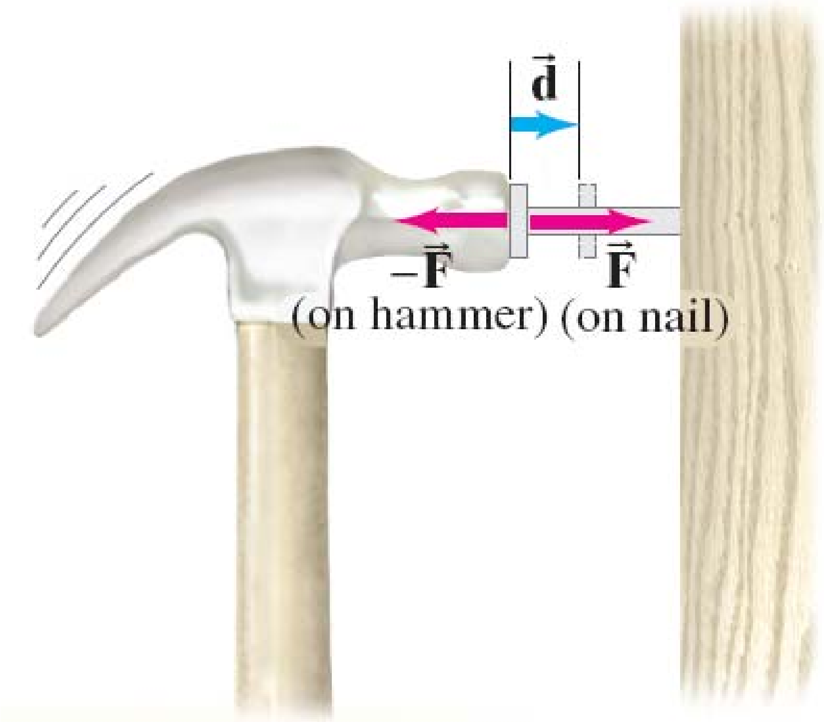
This means that the **work** done is equal to the **change** in the kinetic energy:

$$W_{net} = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2.$$

- If the net work is **positive**, the kinetic energy **increases**.
- If the net work is **negative**, the kinetic energy **decreases**.

7-4 Kinetic Energy and the Work-Energy Principle

Because **work** and **kinetic energy** can be equated, they must have the same **units**: kinetic energy is measured in **joules**. Energy can be considered as the ability to do work:



7-4 Kinetic Energy and the Work-Energy Principle

Example 7-7: Kinetic energy and work done on a baseball.

A 145-g baseball is thrown so that it acquires a speed of 25 m/s. (a) What is its kinetic energy? (b) What was the net work done on the ball to make it reach this speed, if it started from rest?

7-4 Kinetic Energy and the Work-Energy Principle

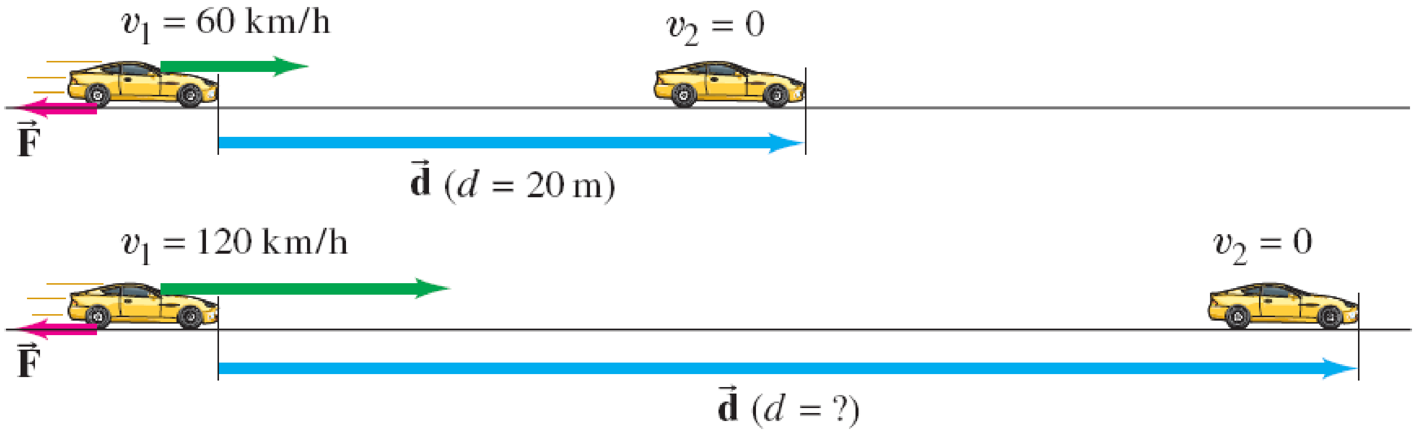
Example 7-8: Work on a car, to increase its kinetic energy.

How much net work is required to accelerate a 1000-kg car from 20 m/s to 30 m/s?

7-4 Kinetic Energy and the Work-Energy Principle

Example 7-9: Work to stop a car.

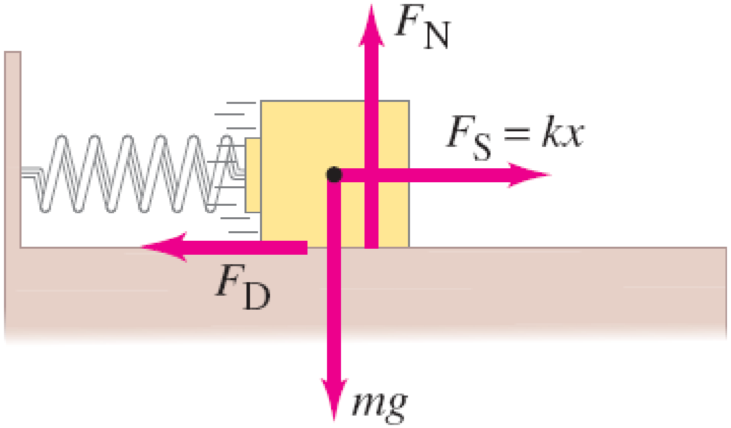
A car traveling 60 km/h can brake to a stop within a distance *d* of 20 m. If the car is going twice as fast, 120 km/h, what is its stopping distance? Assume the maximum braking force is approximately independent of speed.



7-4 Kinetic Energy and the Work-Energy Principle

Example 7-10: A compressed spring.

A horizontal spring has spring constant k = 360 N/m. (a) How much work is required to compress it from its uncompressed length (x = 0) to x = 11.0 cm? (b) If a 1.85-kg block is placed against the spring and the spring is released, what will be the speed of the block when it separates from the spring at x = 0? Ignore friction. (c) Repeat part (b) but assume that the block is moving on a table and that some kind of constant drag force F_D = 7.0 N is acting to slow it down, such as friction (or perhaps your finger).



Summary of Chapter 7

- **Work:** $W = Fd\cos\theta = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}.$
- **Work done by a variable force:**

$$W = \int_a^b \vec{\mathbf{F}} \cdot d\vec{\boldsymbol{\ell}} = \int_a^b F\cos\theta\, d\ell.$$

- **Kinetic energy is energy of motion:**

$$K = \tfrac{1}{2}mv^2.$$

Summary of Chapter 7

• **Work-energy principle: The net work done on an object equals the change in its kinetic energy.**

$$W_{net} = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2.$$