



Chapter 8

Conservation of Energy

Units of Chapter 8

- **Conservative and Nonconservative Forces**
- **Potential Energy**
- **Mechanical Energy and Its Conservation**
- **Problem Solving Using Conservation of Mechanical Energy**
- **The Law of Conservation of Energy**
- **Energy Conservation with Dissipative Forces: Solving Problems**

Units of Chapter 8

- **Gravitational Potential Energy and Escape Velocity**
- **Power**
- **Potential Energy Diagrams; Stable and Unstable Equilibrium**

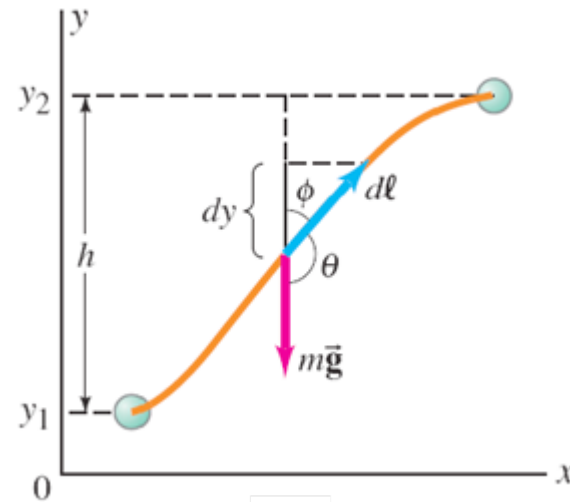
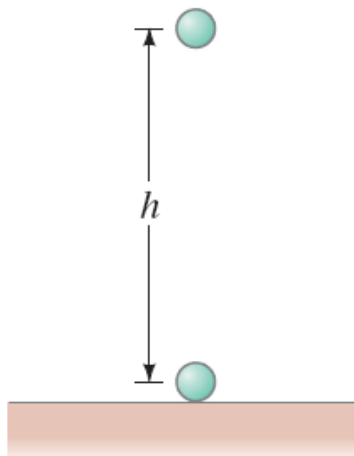


8-1 Conservative and Nonconservative Forces

A force is conservative if:

the work done by the force on an object moving from one point to another depends only on the initial and final positions of the object, and is independent of the particular path taken.

Example: gravity.

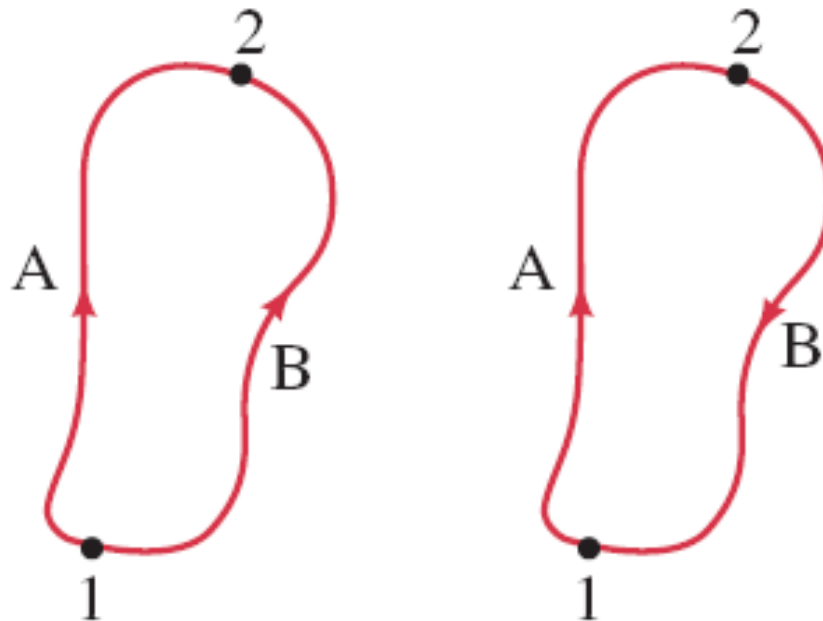




8-1 Conservative and Nonconservative Forces

Another definition of a conservative force:

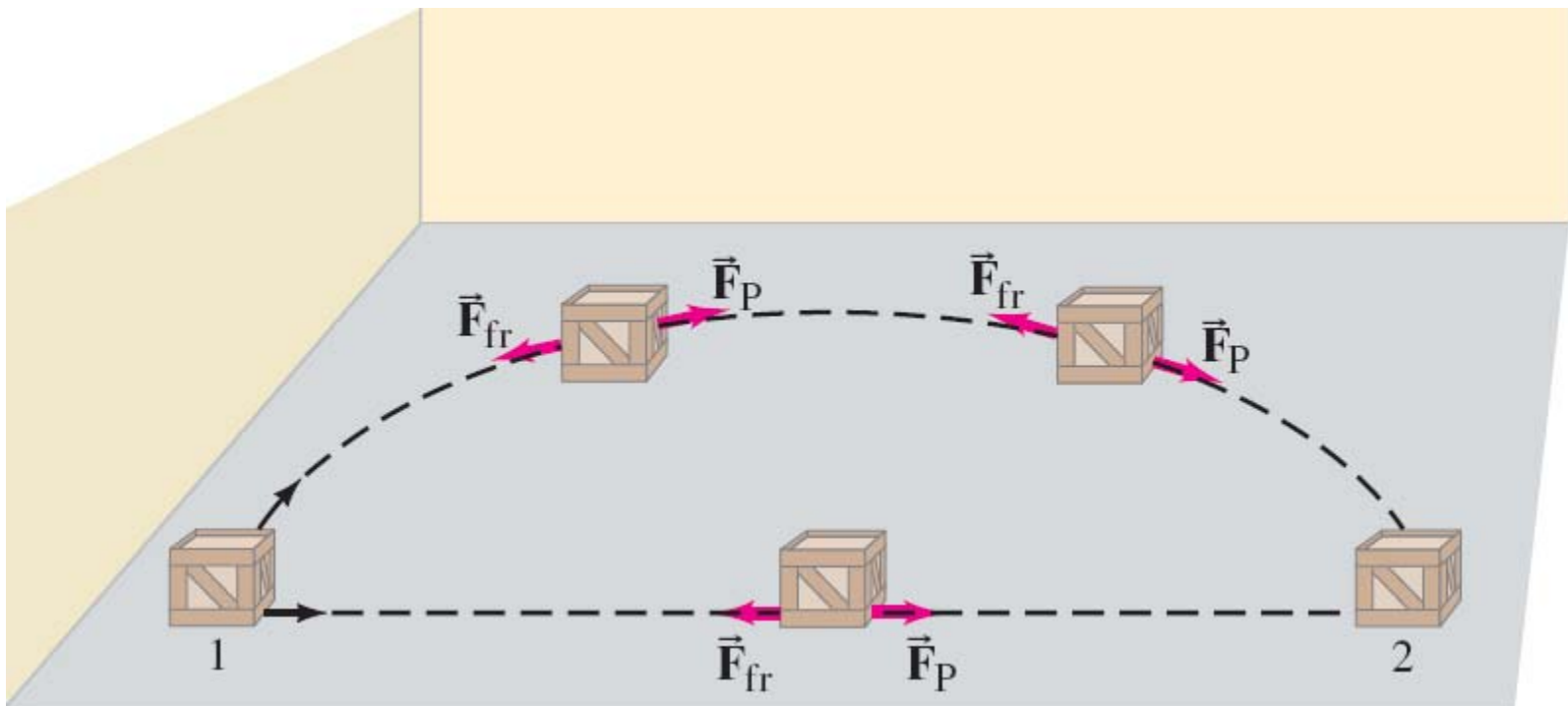
a force is conservative if the net work done by the force on an object moving around any closed path is zero.





8-1 Conservative and Nonconservative Forces

If **friction** is present, the work done depends not only on the starting and ending points, but also on the **path** taken. Friction is called a **nonconservative force**.



8-1 Conservative and Nonconservative Forces

TABLE 8–1 Conservative and Nonconservative Forces

| Conservative Forces | Nonconservative Forces |
|----------------------------|-------------------------------|
| Gravitational | Friction |
| Elastic | Air resistance |
| Electric | Tension in cord |
| | Motor or rocket propulsion |
| | Push or pull by a person |

Potential energy can only be defined for conservative forces.

8-2 Potential Energy

An object can have potential energy by virtue of its surroundings.

Familiar examples of potential energy:

- **A wound-up spring**
- **A stretched elastic band**
- **An object at some height above the ground**



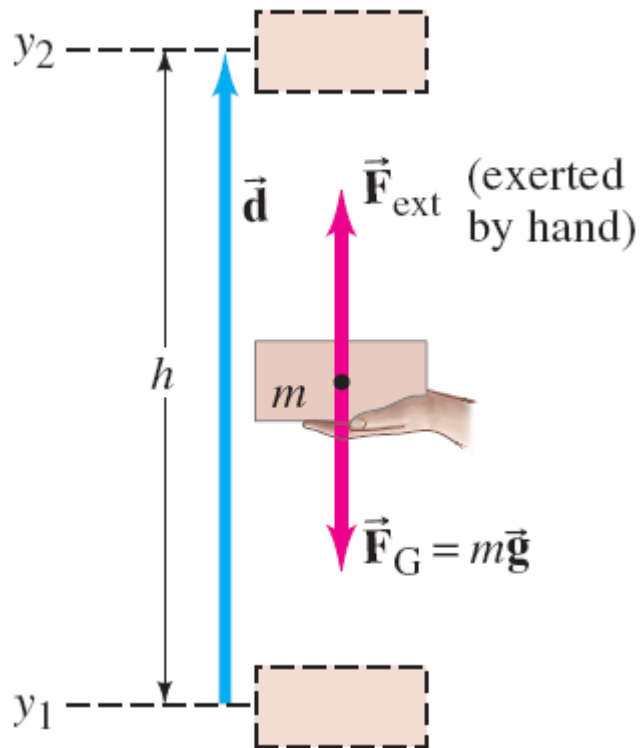
8-2 Potential Energy

In raising a mass m to a height h , the work done by the external force is

$$\begin{aligned} W_{\text{ext}} &= \vec{\mathbf{F}}_{\text{ext}} \cdot \vec{\mathbf{d}} = mgh \cos 0^\circ \\ &= mgh = mg(y_2 - y_1). \end{aligned}$$

We therefore define the gravitational potential energy at a height y above some reference point:

$$U_{\text{grav}} = mgy.$$



8-2 Potential Energy

This potential energy can become kinetic energy if the object is dropped.

Potential energy is a property of a system as a whole, not just of the object (because it depends on external forces).

If $U_{\text{grav}} = mgy$, where do we measure y from?

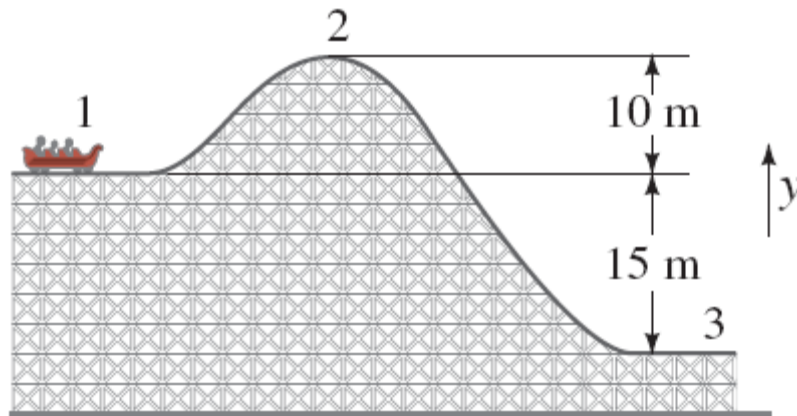
It turns out not to matter, as long as we are consistent about where we choose $y = 0$. Only changes in potential energy can be measured.



8-2 Potential Energy

Example 8-1: Potential energy changes for a roller coaster.

A 1000-kg roller-coaster car moves from point 1 to point 2 and then to point 3. (a) What is the gravitational potential energy at points 2 and 3 relative to point 1? That is, take $y = 0$ at point 1. (b) What is the change in potential energy when the car goes from point 2 to point 3? (c) Repeat parts (a) and (b), but take the reference point ($y = 0$) to be at point 3.



8-2 Potential Energy

General definition of gravitational potential energy:

$$\Delta U = -W_G = -\int_1^2 \vec{\mathbf{F}}_G \cdot d\vec{\ell}.$$

For any conservative force:

$$\Delta U = U_2 - U_1 = -\int_1^2 \vec{\mathbf{F}} \cdot d\vec{\ell} = -W.$$

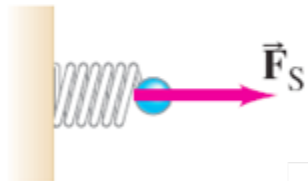
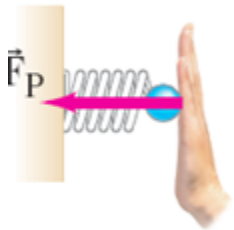
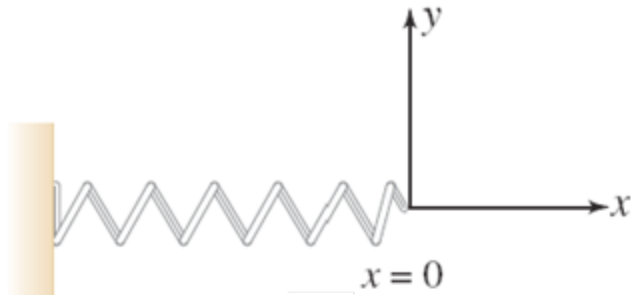


8-2 Potential Energy

A spring has potential energy, called elastic potential energy, when it is compressed. The force required to compress or stretch a spring is:

$$F_S = -kx,$$

where k is called the spring constant, and needs to be measured for each spring.



8-2 Potential Energy

Then the potential energy is:

$$\begin{aligned}\Delta U &= U(x) - U(0) \\ &= -\int_1^2 \vec{\mathbf{F}}_S \cdot d\vec{\ell} = -\int_0^x (-kx) dx = \frac{1}{2} kx^2\end{aligned}$$

$$U_{\text{el}}(x) = \frac{1}{2} kx^2.$$

8-2 Potential Energy

In one dimension,

$$U(x) = - \int F(x) dx + C.$$

We can invert this equation to find $U(x)$ if we know $F(x)$:

$$F(x) = - \frac{dU(x)}{dx}.$$

In three dimensions:

$$\vec{\mathbf{F}}(x, y, z) = -\hat{\mathbf{i}} \frac{\partial U}{\partial x} - \hat{\mathbf{j}} \frac{\partial U}{\partial y} - \hat{\mathbf{k}} \frac{\partial U}{\partial z}.$$

8-3 Mechanical Energy and Its Conservation

If there are no nonconservative forces, the sum of the changes in the kinetic energy and in the potential energy is zero—the kinetic and potential energy changes are equal but opposite in sign.

This allows us to define the **total mechanical energy**:

$$E = K + U.$$

And its conservation:

$$K_2 + U_2 = K_1 + U_1.$$

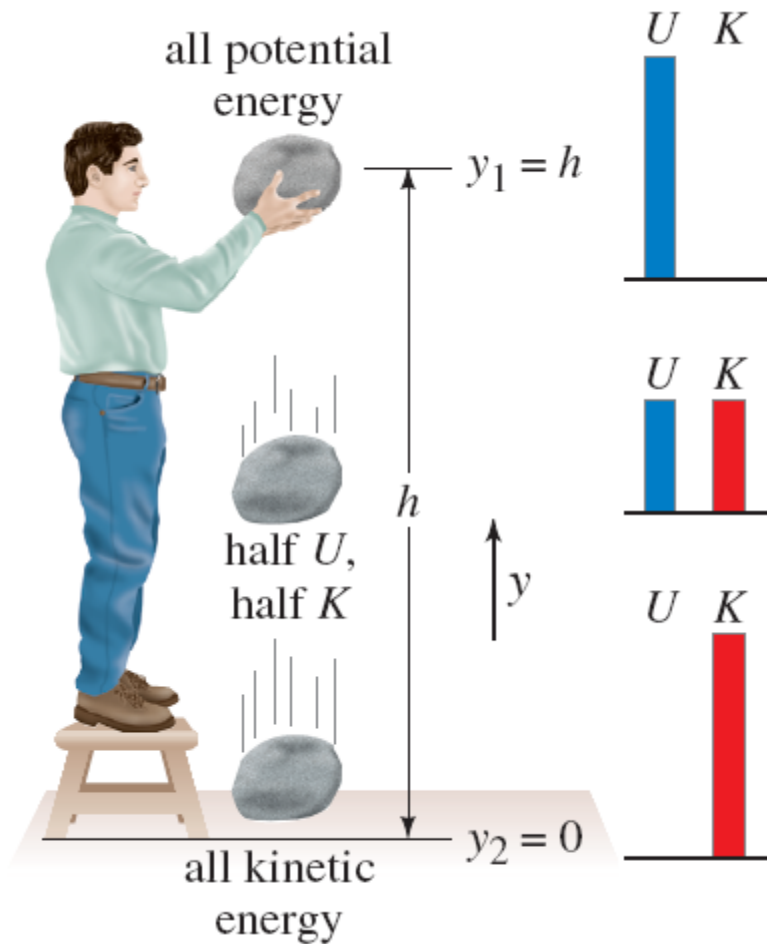
8-3 Mechanical Energy and Its Conservation

The principle of conservation of mechanical energy:

If only conservative forces are doing work, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant—it is conserved.



8-4 Problem Solving Using Conservation of Mechanical Energy



In the image on the left, the total mechanical energy at any point is:

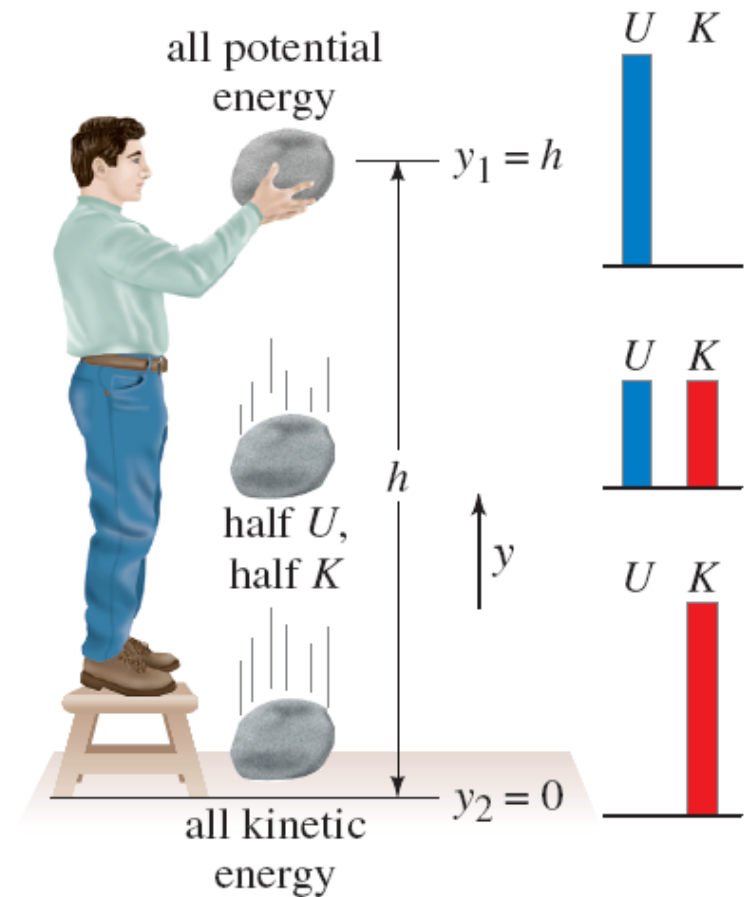
$$\begin{aligned} E &= K + U \\ &= \frac{1}{2}mv^2 + mgy. \end{aligned}$$



8-4 Problem Solving Using Conservation of Mechanical Energy

Example 8-3: Falling rock.

If the original height of the rock is $y_1 = h = 3.0$ m, calculate the rock's speed when it has fallen to 1.0 m above the ground.

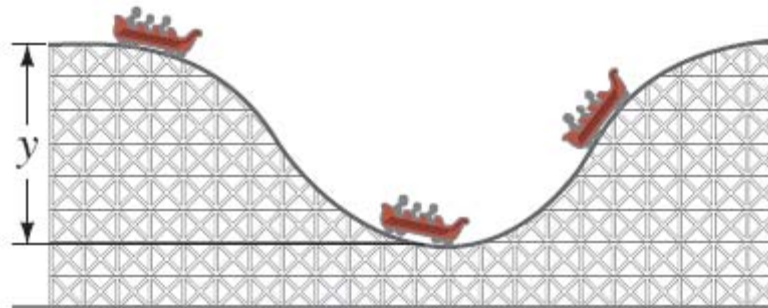




8-4 Problem Solving Using Conservation of Mechanical Energy

Example 8-4: Roller-coaster car speed using energy conservation.

Assuming the height of the hill is 40 m, and the roller-coaster car starts from rest at the top, calculate (a) the speed of the roller-coaster car at the bottom of the hill, and (b) at what height it will have half this speed. Take $y = 0$ at the bottom of the hill.

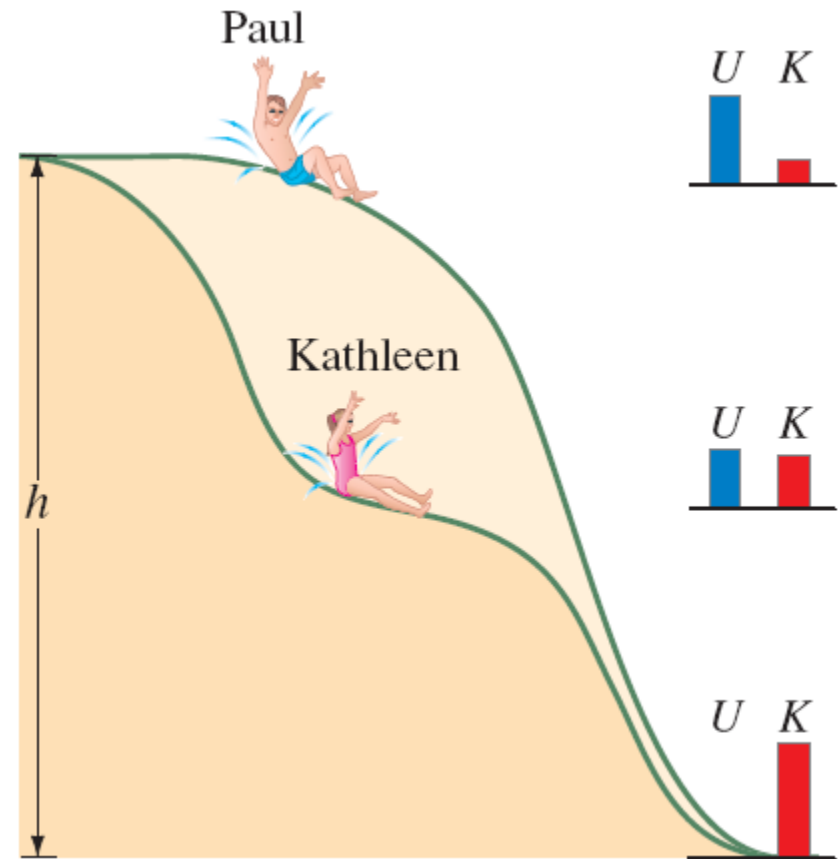




8-4 Problem Solving Using Conservation of Mechanical Energy

Conceptual Example 8-5: Speeds on two water slides.

Two water slides at a pool are shaped differently, but start at the same height h . Two riders, Paul and Kathleen, start from rest at the same time on different slides. (a) Which rider, Paul or Kathleen, is traveling faster at the bottom? (b) Which rider makes it to the bottom first? Ignore friction and assume both slides have the same path length.



8-4 Problem Solving Using Conservation of Mechanical Energy

Which to use for solving problems?

Newton's laws: best when forces are constant

Work and energy: good when forces are constant; also may succeed when forces are not constant



8-4 Problem Solving Using Conservation of Mechanical Energy

Example 8-6: Pole vault.



Estimate the kinetic energy and the speed required for a 70-kg pole vaulter to just pass over a bar 5.0 m high. Assume the vaulter's center of mass is initially 0.90 m off the ground and reaches its maximum height at the level of the bar itself.



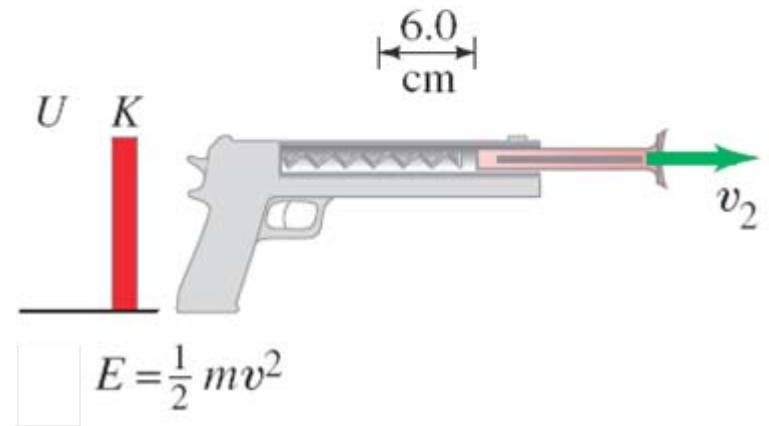
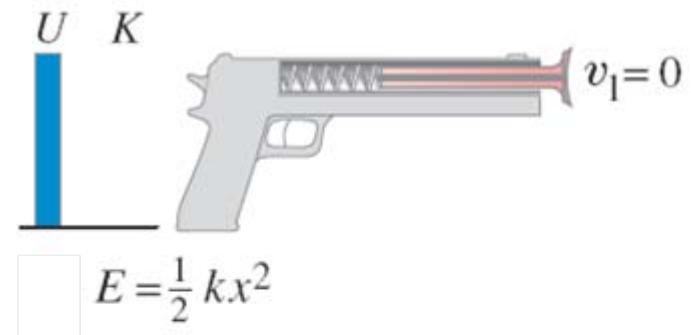
8-4 Problem Solving Using Conservation of Mechanical Energy

For an **elastic force**, conservation of energy tells us:

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2.$$

Example 8-7: Toy dart gun.

A dart of mass 0.100 kg is pressed against the spring of a toy dart gun. The spring (with spring stiffness constant $k = 250$ N/m and ignorable mass) is compressed 6.0 cm and released. If the dart detaches from the spring when the spring reaches its natural length ($x = 0$), what speed does the dart acquire?

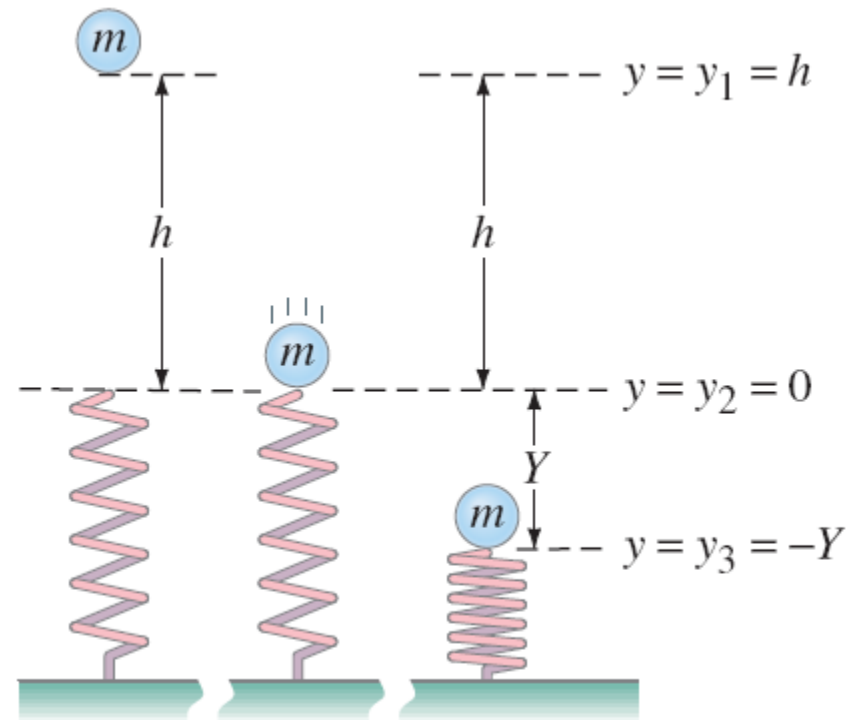




8-4 Problem Solving Using Conservation of Mechanical Energy

Example 8-8: Two kinds of potential energy.

A ball of mass $m = 2.60$ kg, starting from rest, falls a vertical distance $h = 55.0$ cm before striking a vertical coiled spring, which it compresses an amount $Y = 15.0$ cm. Determine the spring stiffness constant of the spring. Assume the spring has negligible mass, and ignore air resistance. Measure all distances from the point where the ball first touches the uncompressed spring ($y = 0$ at this point).

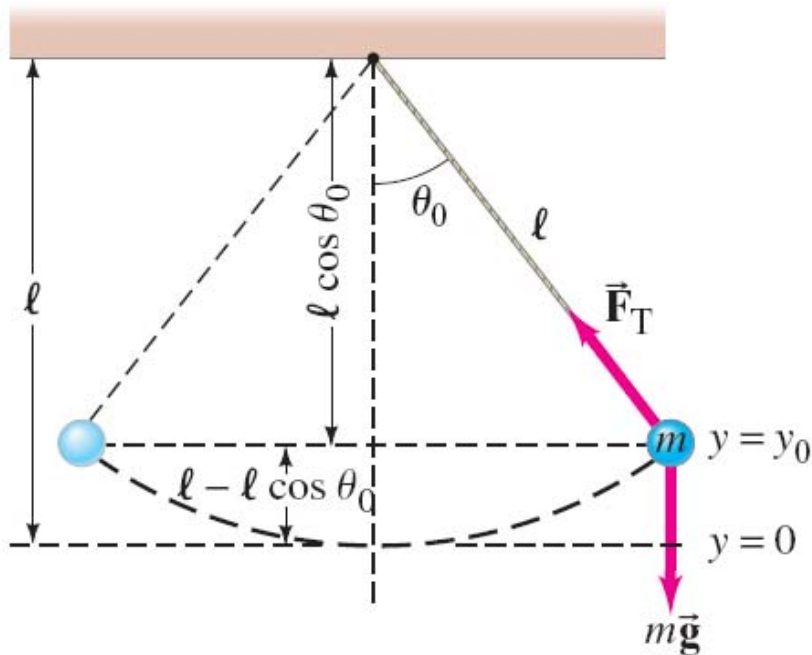




8-4 Problem Solving Using Conservation of Mechanical Energy

Example 8-9: A swinging pendulum.

This simple pendulum consists of a small bob of mass m suspended by a massless cord of length l . The bob is released (without a push) at $t = 0$, where the cord makes an angle $\theta = \theta_0$ to the vertical.



- (a) Describe the motion of the bob in terms of kinetic energy and potential energy. Then
- (b) as a function of position θ as it swings back and forth, and (c) at the lowest point of the swing.
- (d) Find the tension in the cord, \vec{F}_T . Ignore friction and air resistance.

8-5 The Law of Conservation of Energy

Nonconservative, or dissipative, forces:

Friction

Heat

Electrical energy

Chemical energy

and more

do not conserve mechanical energy. However, when these forces are taken into account, the total energy is still conserved:

$$\Delta K + \Delta U + [\text{change in all other forms of energy}] = 0.$$

8-5 The Law of Conservation of Energy

The law of conservation of energy is one of the most important principles in physics.

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one object to another, but the total amount remains constant.

8-6 Energy Conservation with Dissipative Forces: Solving Problems

Problem Solving:

1. **Draw a picture.**
2. **Determine the system for which energy will be conserved.**
3. **Figure out what you are looking for, and decide on the initial and final positions.**
4. **Choose a logical reference frame.**
5. **Apply conservation of energy.**
6. **Solve.**

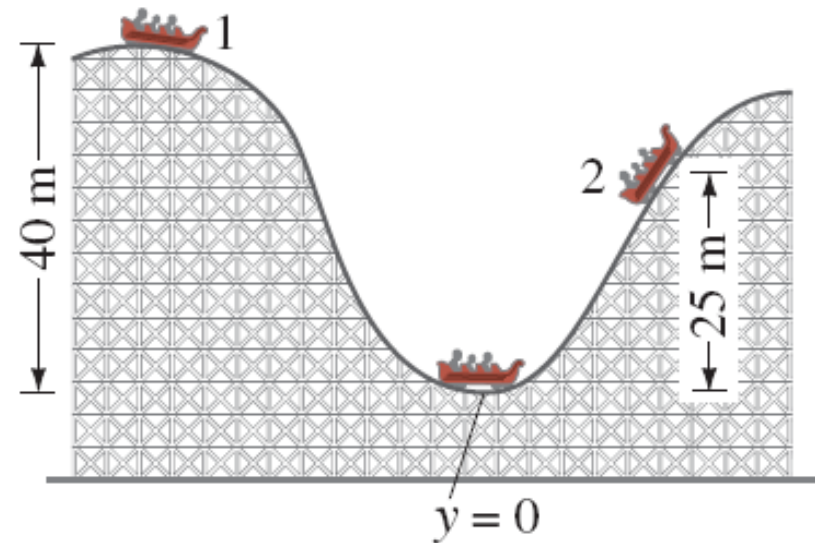


8-6 Energy Conservation with Dissipative Forces: Solving Problems

Example 8-10: Friction on the roller-coaster car.

The roller-coaster car shown reaches a vertical height of only 25 m on the second hill before coming to a momentary stop. It traveled a total distance of 400 m.

Determine the thermal energy produced and estimate the average friction force (assume it is roughly constant) on the car, whose mass is 1000 kg.

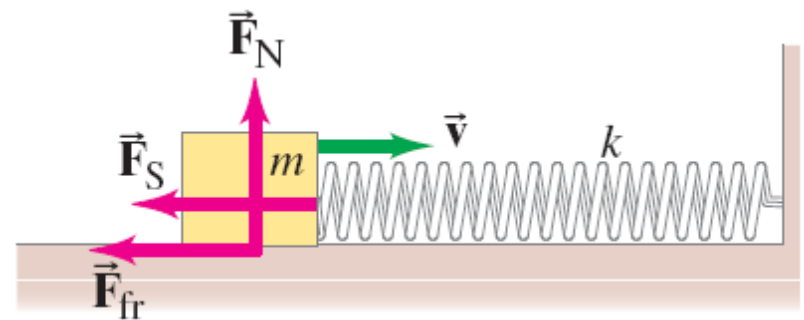
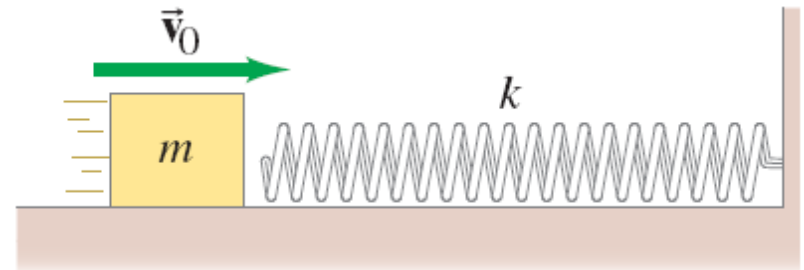




8-6 Energy Conservation with Dissipative Forces: Solving Problems

Example 8-11: Friction with a spring.

A block of mass m sliding along a rough horizontal surface is traveling at a speed v_0 when it strikes a massless spring head-on and compresses the spring a maximum distance X . If the spring has stiffness constant k , determine the coefficient of kinetic friction between block and surface.





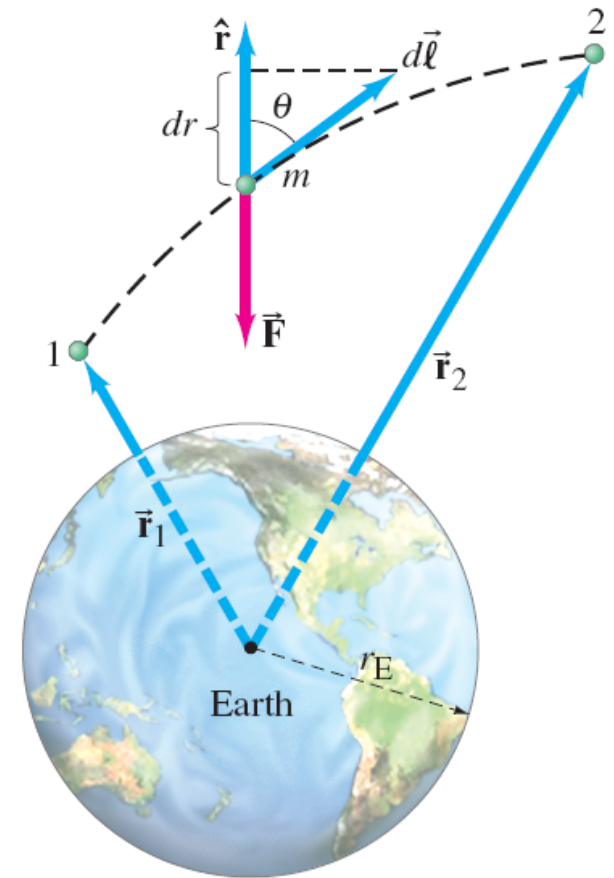
8-7 Gravitational Potential Energy and Escape Velocity

Far from the surface of the Earth, the force of gravity is not constant:

$$\vec{\mathbf{F}} = -G \frac{mM_E}{r^2} \hat{\mathbf{r}}.$$

The work done on an object moving in the Earth's gravitational field is given by:

$$W = \int_1^2 \vec{\mathbf{F}} \cdot d\vec{\ell} = -GmM_E \int_1^2 \frac{\hat{\mathbf{r}} \cdot d\vec{\ell}}{r^2}.$$





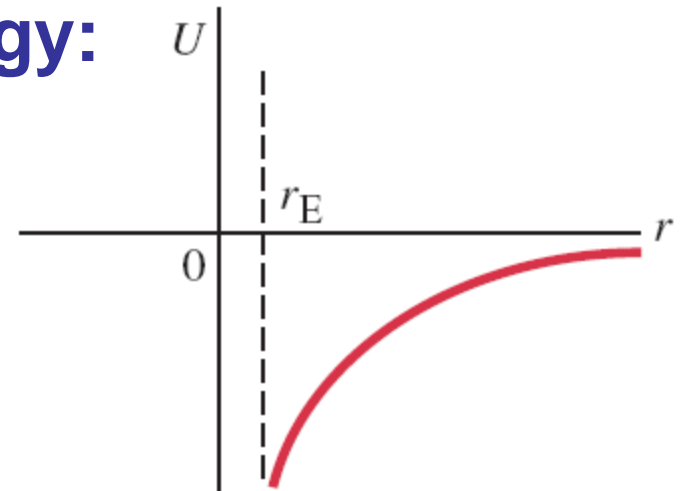
8-7 Gravitational Potential Energy and Escape Velocity

Solving the integral gives:

$$W = \frac{GmM_E}{r_2} - \frac{GmM_E}{r_1}.$$

Because the value of the integral depends only on the end points, the gravitational force is conservative and we can define gravitational potential energy:

$$U(r) = -\frac{GmM_E}{r}.$$





8-7 Gravitational Potential Energy and Escape Velocity

Example 8-12: Package dropped from high-speed rocket.

A box of empty film canisters is allowed to fall from a rocket traveling outward from Earth at a speed of 1800 m/s when 1600 km above the Earth's surface. The package eventually falls to the Earth. Estimate its speed just before impact. Ignore air resistance.

8-7 Gravitational Potential Energy and Escape Velocity

If an object's initial kinetic energy is equal to the potential energy at the Earth's surface, its total energy will be zero. The velocity at which this is true is called the escape velocity; for Earth:

$$v_{\text{esc}} = \sqrt{2GM_{\text{E}}/r_{\text{E}}} = 1.12 \times 10^4 \text{ m/s.}$$



8-7 Gravitational Potential Energy and Escape Velocity

Example 8-13: Escaping the Earth or the Moon.

- (a) Compare the escape velocities of a rocket from the Earth and from the Moon.
- (b) Compare the energies required to launch the rockets. For the Moon, $M_M = 7.35 \times 10^{22}$ kg and $r_M = 1.74 \times 10^6$ m, and for Earth, $M_E = 5.98 \times 10^{24}$ kg and $r_E = 6.38 \times 10^6$ m.

8-8 Power

Power is the rate at which work is done.

Average power:

$$\overline{P} = \frac{W}{t}.$$

Instantaneous power:

$$P = \frac{dW}{dt}.$$

In the SI system, the units of power are watts:

$$1 \text{ W} = 1 \text{ J/s}.$$

8-8 Power

Power can also be described as the rate at which energy is transformed:

$$P = \frac{dE}{dt}.$$

In the British system, the basic unit for power is the foot-pound per second, but more often horsepower is used:

$$1 \text{ hp} = 550 \text{ ft}\cdot\text{lb/s} = 746 \text{ W}.$$



8-8 Power

Example 8-14: Stair-climbing power.

A 60-kg jogger runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. (a) Estimate the jogger's power output in watts and horsepower. (b) How much energy did this require?



8-8 Power

Power is also needed for acceleration and for moving against the force of friction.

The power can be written in terms of the net force and the velocity:

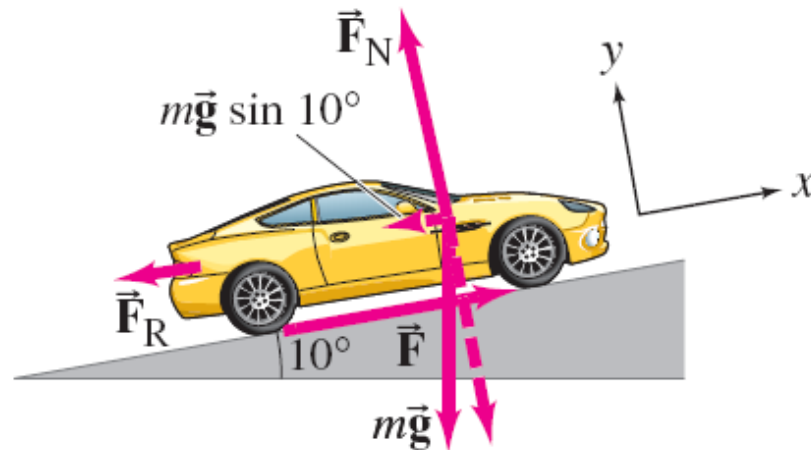
$$P = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \frac{d\vec{\ell}}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}.$$



8-8 Power

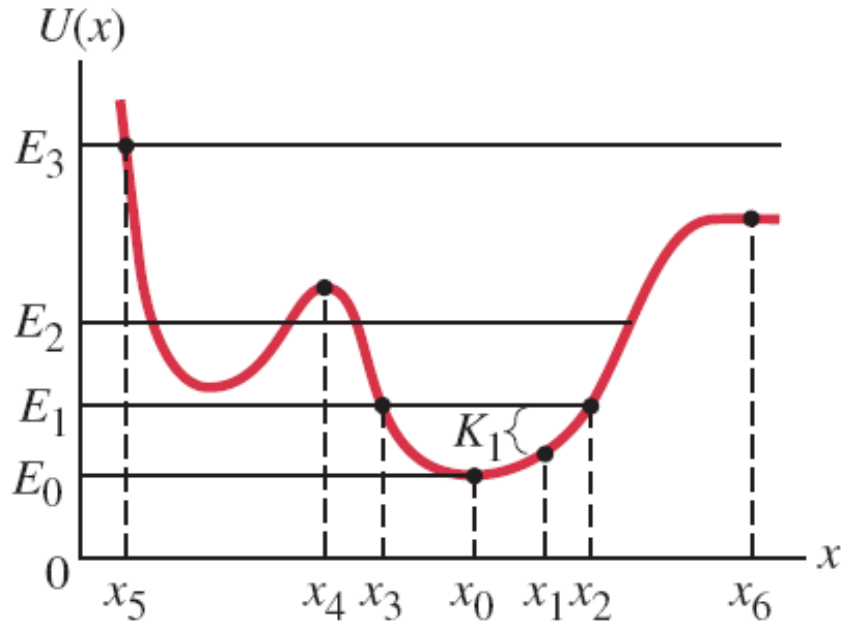
Example 8-15: Power needs of a car.

Calculate the power required of a 1400-kg car under the following circumstances: (a) the car climbs a 10° hill (a fairly steep hill) at a steady 80 km/h; and (b) the car accelerates along a level road from 90 to 110 km/h in 6.0 s to pass another car. Assume that the average retarding force on the car is $F_R = 700$ N throughout.





8-9 Potential Energy Diagrams; Stable and Unstable Equilibrium



This is a potential energy diagram for a particle moving under the influence of a conservative force. Its behavior will be determined by its total energy.

With energy E_1 , the object oscillates between x_3 and x_2 , called turning points. An object with energy E_2 has four turning points; an object with energy E_0 is in stable equilibrium. An object at x_4 is in unstable equilibrium.

Summary of Chapter 8

- **Conservative force:** work depends only on end points
- **Gravitational potential energy:** $U_{\text{grav}} = mgy$.
- **Elastic potential energy:** $U_{\text{el}} = \frac{1}{2} kx^2$.
- **For any conservative force:**

$$\Delta U = U_2 - U_1 = - \int_1^2 \vec{\mathbf{F}} \cdot d\vec{\ell}.$$

- **Inverting,**

$$F = - \frac{dU(x)}{dx}.$$

Summary of Chapter 8

- Total mechanical energy is the sum of kinetic and potential energies.
- Additional types of energy are involved when nonconservative forces act.
- Total energy (including all forms) is conserved.
- Gravitational potential energy:

$$U(r) = -\frac{GmM_E}{r}.$$

Summary of Chapter 8

- **Power:** rate at which work is done, or energy is transformed:

$$P = \frac{dW}{dt} = \frac{dE}{dt},$$

or

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}.$$