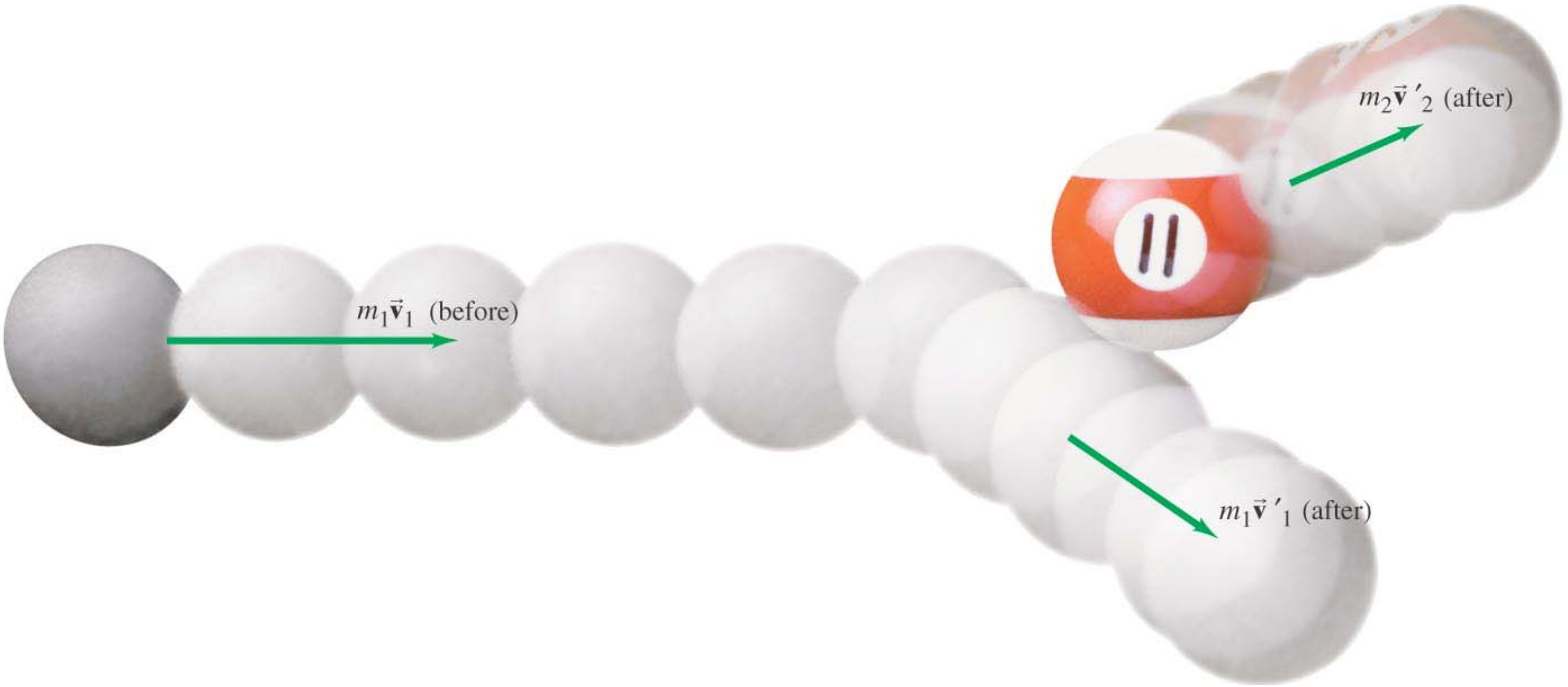
Chapter 9

Linear Momentum



Units of Chapter 9

- Momentum and Its Relation to Force
- Conservation of Momentum
- Collisions and Impulse
- Conservation of Energy and Momentum in Collisions
- Elastic Collisions in One Dimension

Units of Chapter 9

- Inelastic Collisions
- Collisions in Two or Three Dimensions
- Center of Mass (CM)
- Center of Mass and Translational Motion
- Systems of Variable Mass; Rocket Propulsion

9-1 Momentum and Its Relation to Force

Momentum is a vector symbolized by the symbol $\vec{\mathbf{p}}$, and is defined as

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}.$$

The **rate** of change of momentum is equal to the net **force**:

$$\Sigma\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}.$$

This can be shown using Newton's **second** law.

9-1 Momentum and Its Relation to Force

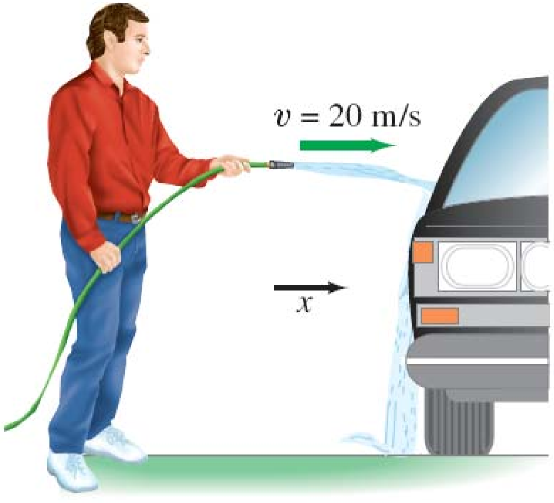
Example 9-1: Force of a tennis serve.

For a top player, a tennis ball may leave the racket on the serve with a speed of 55 m/s (about 120 mi/h). If the ball has a mass of 0.060 kg and is in contact with the racket for about 4 ms (4 x 10^{-3} s), estimate the average force on the ball. Would this force be large enough to lift a 60-kg person?

9-1 Momentum and Its Relation to Force

Example 9-2: Washing a car: momentum change and force.

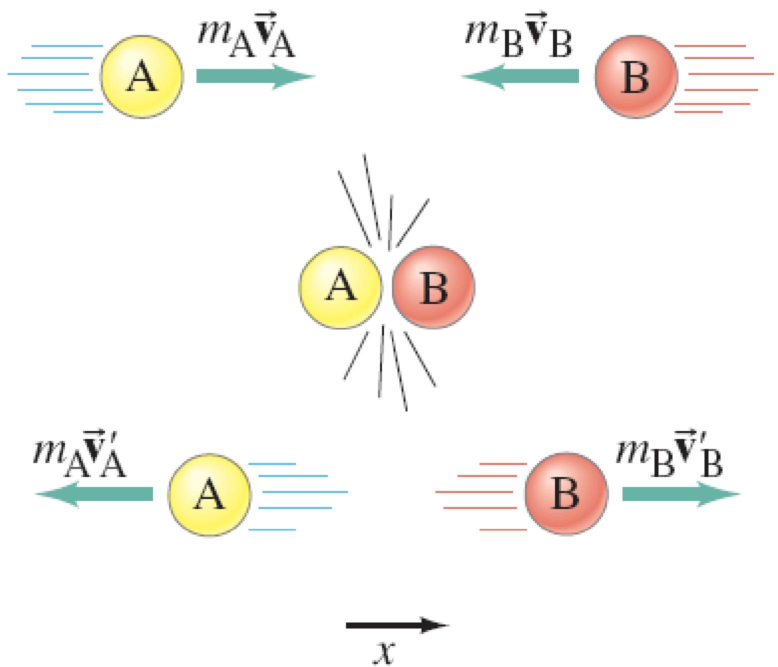
Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it. (That is, we ignore any splashing back.) What is the force exerted by the water on the car?



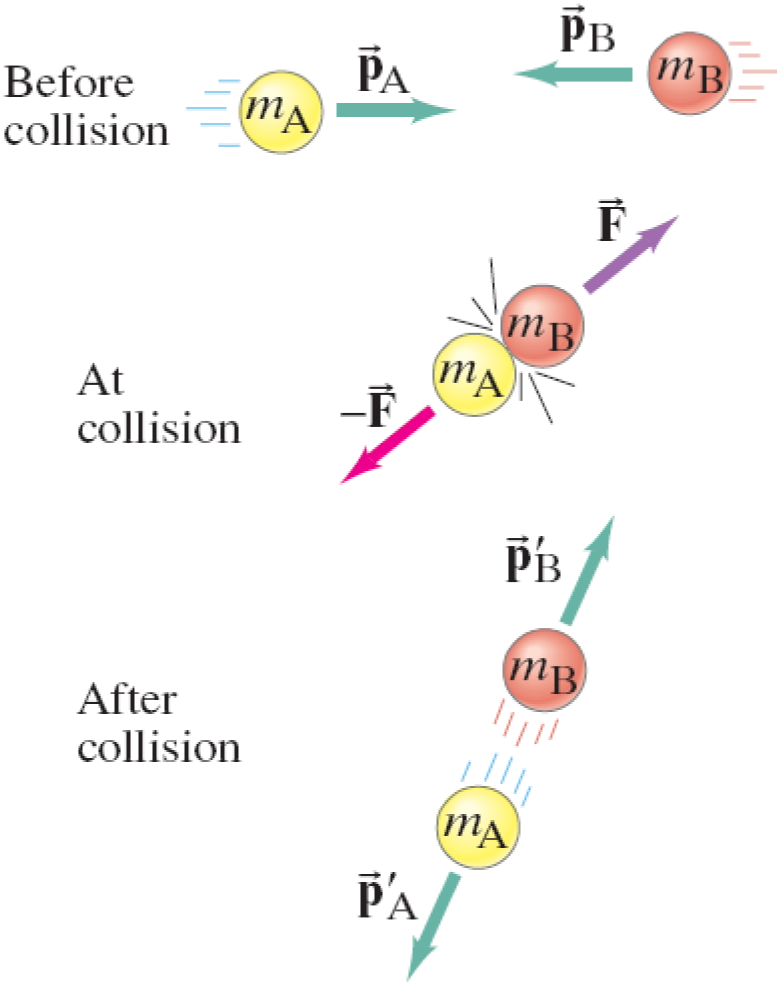
9-2 Conservation of Momentum

During a **collision**, measurements show that the total **momentum** does not change:

$$m_A \vec{\mathbf{v}}_A + m_B \vec{\mathbf{v}}_B = m_A \vec{\mathbf{v}}'_A + m_B \vec{\mathbf{v}}'_B .$$



9-2 Conservation of Momentum



Conservation of momentum can also be derived from Newton's laws. A collision takes a short enough time that we can ignore external forces. Since the internal forces are equal and opposite, the total momentum is constant.

9-2 Conservation of Momentum

For more than two objects,

$$\frac{d\vec{\mathbf{P}}}{dt} = \sum \frac{d\vec{\mathbf{p}}_i}{dt} = \sum \vec{\mathbf{F}}_i .$$

Or, since the internal forces cancel,

$$\frac{d\vec{\mathbf{P}}}{dt} = \sum \vec{\mathbf{F}}_{\text{ext}} .$$

9-2 Conservation of Momentum

This is the law of conservation of linear momentum:

when the net external force on a system of objects is zero, the total momentum of the system remains constant.

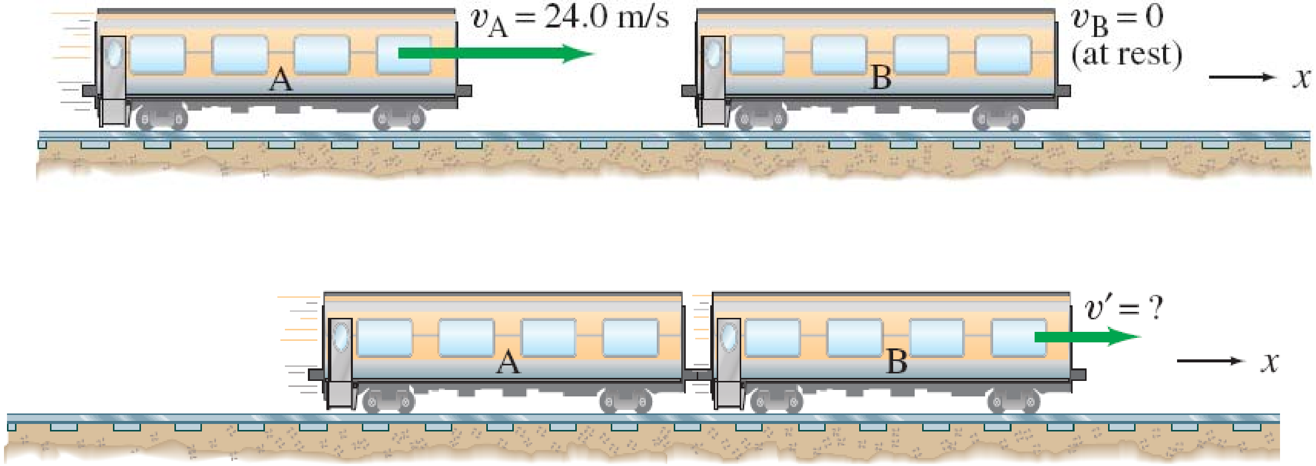
Equivalently,

the total momentum of an isolated system remains constant.

9-2 Conservation of Momentum

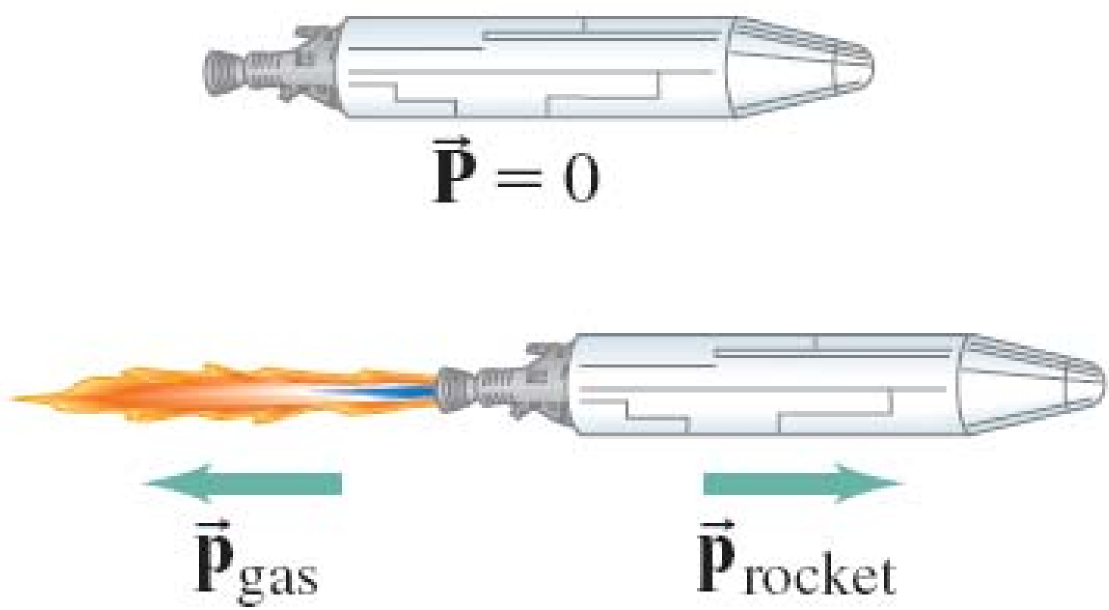
Example 9-3: Railroad cars collide: momentum conserved.

A 10,000-kg railroad car, A, traveling at a speed of 24.0 m/s strikes an identical car, B, at rest. If the cars lock together as a result of the collision, what is their common speed immediately after the collision?



9-2 Conservation of Momentum

Momentum conservation works for a rocket as long as we consider the rocket and its fuel to be one system, and account for the mass loss of the rocket.



9-2 Conservation of Momentum

Example 9-4: Rifle recoil.

Calculate the recoil velocity of a 5.0-kg rifle that shoots a 0.020-kg bullet at a speed of 620 m/s.

9-2 Conservation of Momentum

Conceptual Example 9-5: Falling on or off a sled.

(a) An empty sled is sliding on frictionless ice when Susan drops vertically from a tree above onto the sled. When she lands, does the sled speed up, slow down, or keep the same speed? (b) Later: Susan falls sideways off the sled. When she drops off, does the sled speed up, slow down, or keep the same speed?

9-3 Collisions and Impulse

During a collision, objects are **deformed** due to the large forces involved.

Since $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$, we can write $d\vec{\mathbf{p}} = \vec{\mathbf{F}}\,dt.$

Integrating,

$$\int_{i}^{f} d\vec{\mathbf{p}} = \vec{\mathbf{p}}_{f} - \vec{\mathbf{p}}_{i} = \int_{t_i}^{t_f} \vec{\mathbf{F}}\,dt.$$

9-3 Collisions and Impulse

This quantity is defined as the impulse, J:

$$\vec{\mathbf{J}} = \int_{t_i}^{t_f} \vec{\mathbf{F}}\, dt.$$

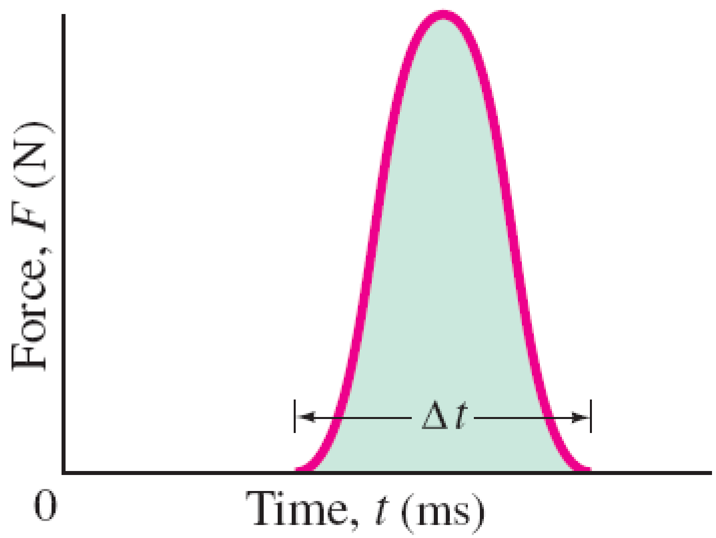
The impulse is equal to the change in momentum:

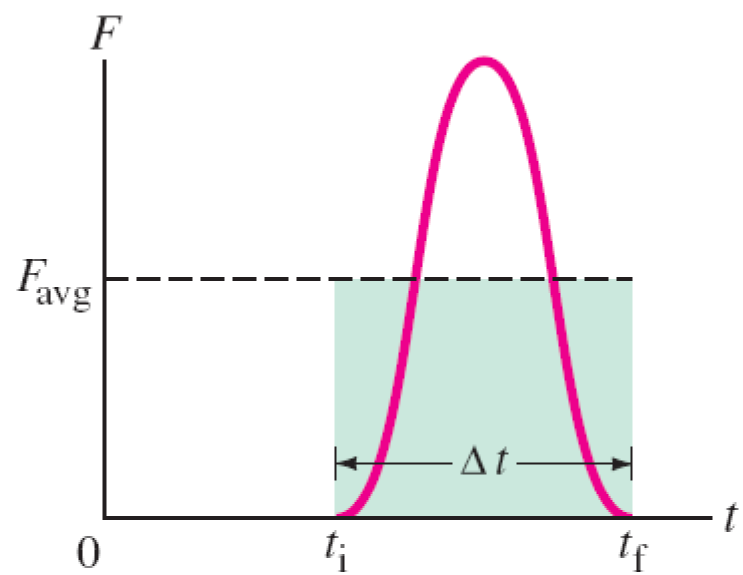
$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \int_{t_i}^{t_f} \vec{\mathbf{F}}\, dt = \vec{\mathbf{J}}.$$

9-3 Collisions and Impulse

Since the time of the collision is often very short, we may be able to use the average force, which would produce the same impulse over the same time interval.

$$\vec{\mathbf{F}}_{avg}\,\Delta t = \int_{t_i}^{t_f} \vec{\mathbf{F}}\,dt.$$



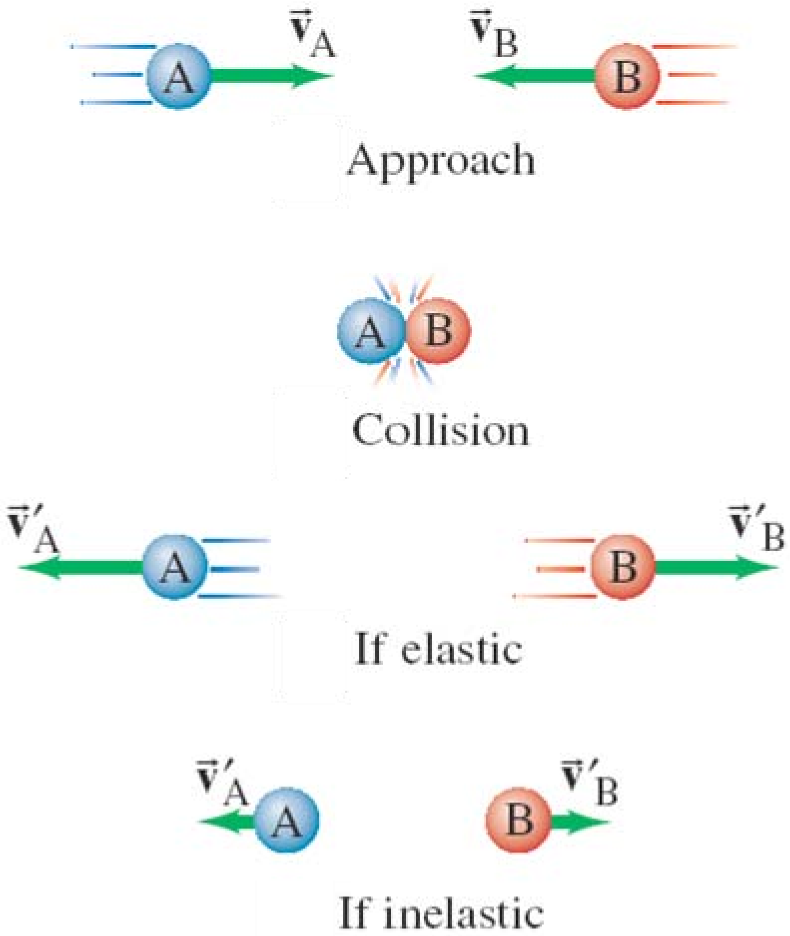


9-3 Collisions and Impulse

Example 9-6: Karate blow.

Estimate the impulse and the average force delivered by a karate blow that breaks a board a few cm thick. Assume the hand moves at roughly 10 m/s when it hits the board.

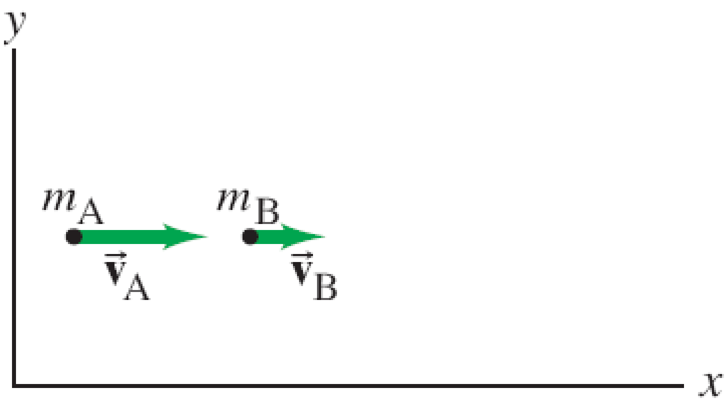
9-4 Conservation of Energy and Momentum in Collisions

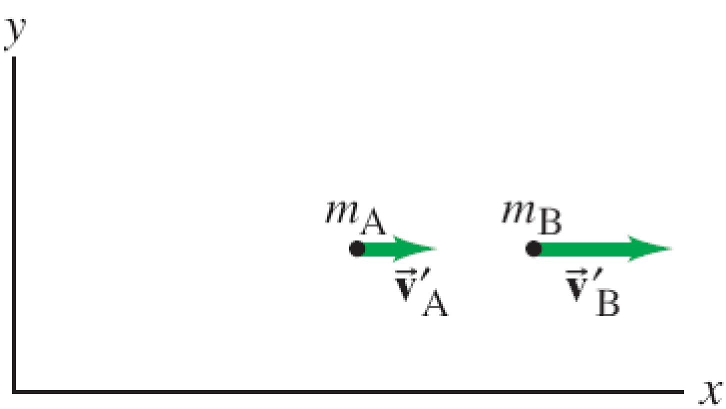


Momentum is **conserved** in all **collisions**.

Collisions in which **kinetic energy** is conserved as well are called **elastic** collisions, and those in which it is not are called **inelastic**.

9-5 Elastic Collisions in One Dimension





Here we have two objects colliding **elastically**. We know the masses and the initial speeds.

Since both **momentum** and **kinetic energy** are conserved, we can write **two** equations. This allows us to solve for the **two** unknown final speeds.

9-5 Elastic Collisions in One Dimension

Example 9-7: Equal masses.

Billiard ball A of mass m moving with speed v_A collides head-on with ball B of equal mass. What are the speeds of the two balls after the collision, assuming it is elastic? Assume (a) both balls are moving initially (v_A and v_B), (b) ball B is initially at rest ($v_B = 0$).

9-5 Elastic Collisions in One Dimension

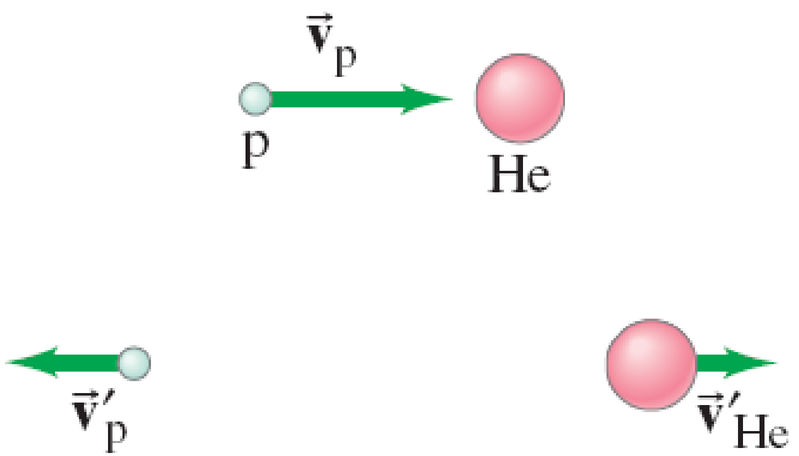
Example 9-8: Unequal masses, target at rest.

A very common practical situation is for a moving object (m_A) to strike a second object (m_B, the "target") at rest ($v_B = 0$). Assume the objects have unequal masses, and that the collision is elastic and occurs along a line (head-on). (a) Derive equations for v_B' and v_A' in terms of the initial velocity v_A of mass m_A and the masses m_A and m_B. (b) Determine the final velocities if the moving object is much more massive than the target ($m_A >> m_B$). (c) Determine the final velocities if the moving object is much less massive than the target ($m_A << m_B$).

9-5 Elastic Collisions in One Dimension

Example 9-9: A nuclear collision.

A proton (p) of mass 1.01 u (unified atomic mass units) traveling with a speed of 3.60 x 10^4 m/s has an elastic head-on collision with a helium (He) nucleus (m_{He} = 4.00 u) initially at rest. What are the velocities of the proton and helium nucleus after the collision? Assume the collision takes place in nearly empty space.



9-6 Inelastic Collisions

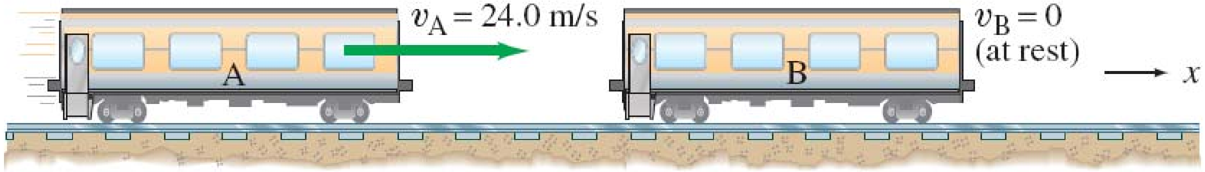
With **inelastic** collisions, some of the initial kinetic energy is lost to **thermal** or **potential** energy. Kinetic energy may also be gained during **explosions**, as there is the addition of **chemical** or **nuclear** energy.

A **completely inelastic** collision is one in which the objects **stick together** afterward, so there is only one final velocity.

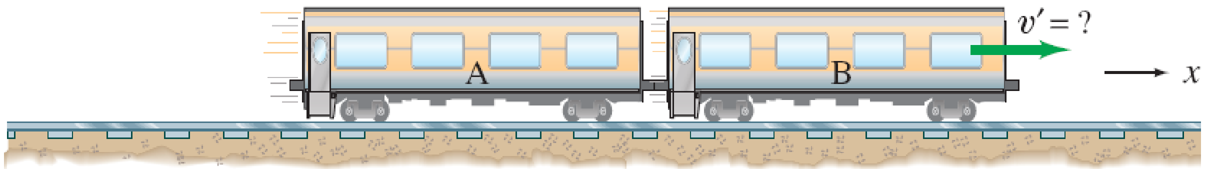
9-6 Inelastic Collisions

Example 9-10: Railroad cars again.

A 10,000-kg railroad car, A, traveling at a speed of 24.0 m/s strikes an identical car, B, at rest. If the cars lock together as a result of the collision, how much of the initial kinetic energy is transformed to thermal or other forms of energy?

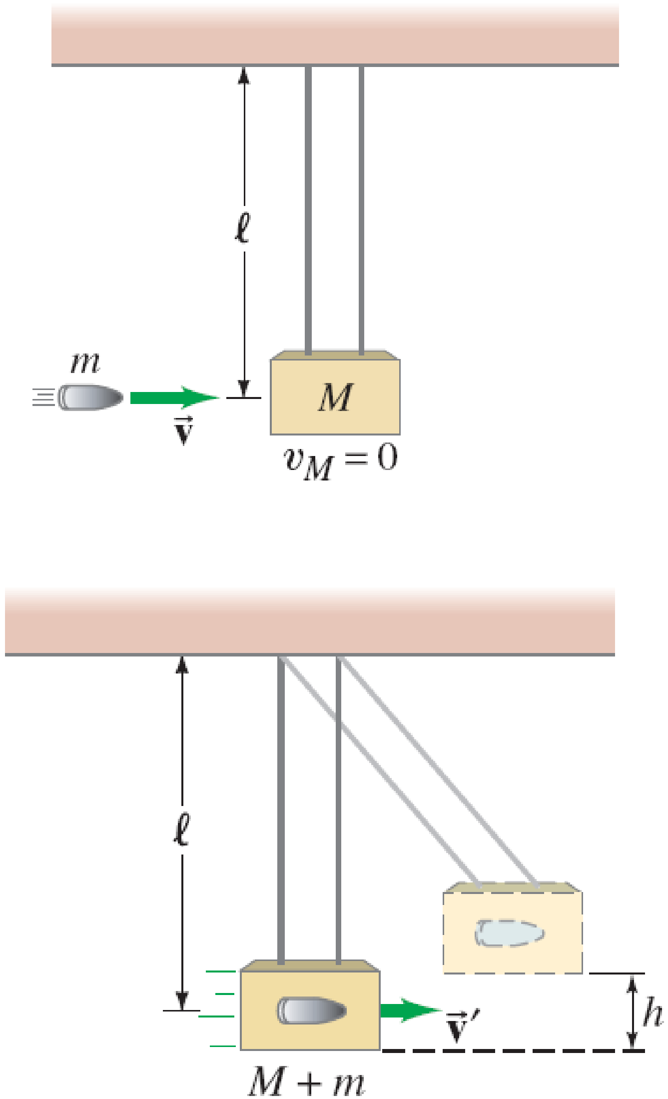


Before collision



After collision

9-6 Inelastic Collisions



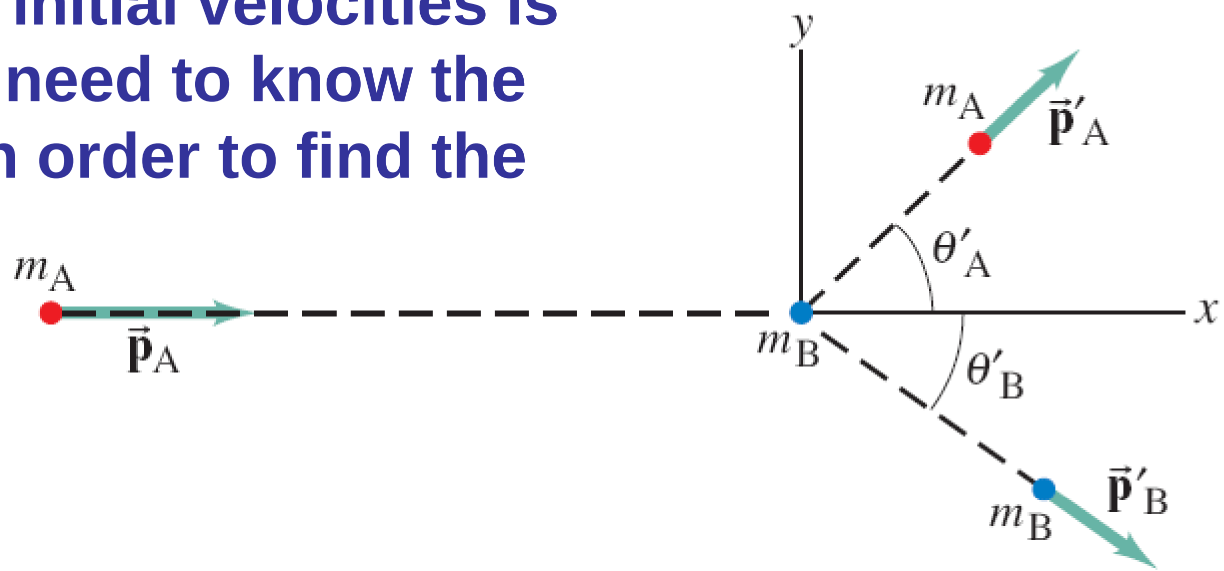
Example 9-11: Ballistic pendulum.

The ballistic pendulum is a device used to measure the speed of a projectile, such as a bullet. The projectile, of mass m, is fired into a large block of mass M, which is suspended like a pendulum. As a result of the collision, the pendulum and projectile together swing up to a maximum height h. Determine the relationship between the initial horizontal speed of the projectile, v, and the maximum height h.

9-7 Collisions in Two or Three Dimensions

Conservation of energy and momentum can also be used to analyze collisions in **two** or **three** dimensions, but unless the situation is very simple, the math quickly becomes unwieldy.

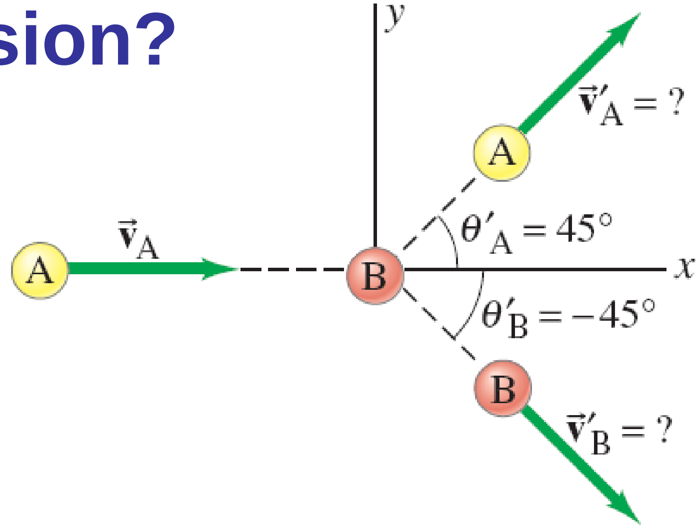
Here, a **moving** object collides with an object initially at **rest**. Knowing the masses and initial velocities is not enough; we need to know the **angles** as well in order to find the final velocities.



9-7 Collisions in Two or Three Dimensions

Example 9-12: Billiard ball collision in 2-D.

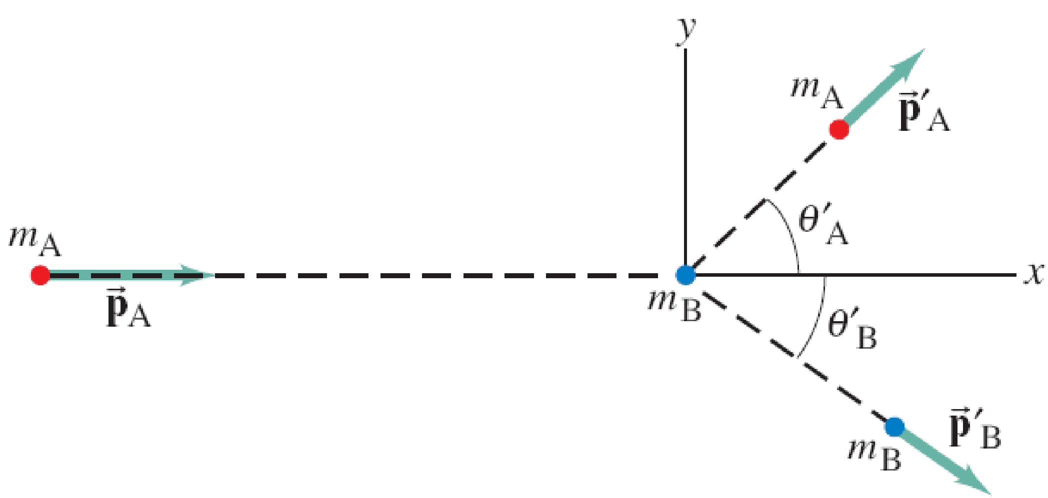
Billiard ball A moving with speed v_A = 3.0 m/s in the +x direction strikes an equal-mass ball B initially at rest. The two balls are observed to move off at 45° to the x axis, ball A above the x axis and ball B below. That is, θ_A' = 45° and θ_B' = -45 °. What are the speeds of the two balls after the collision?



9-7 Collisions in Two or Three Dimensions

Example 9-13: Proton-proton collision.

A proton traveling with speed 8.2×10^5 m/s collides elastically with a stationary proton in a hydrogen target. One of the protons is observed to be scattered at a 60° angle. At what angle will the second proton be observed, and what will be the velocities of the two protons after the collision?



9-7 Collisions in Two or Three Dimensions

Problem solving:

1. Choose the **system**. If it is complex, **subsystems** may be chosen where one or more conservation laws apply.

2. Is there an **external** force? If so, is the collision time short enough that you can **ignore** it?

3. **Draw** diagrams of the initial and final situations, with **momentum** vectors labeled.

4. Choose a **coordinate** system.

9-7 Collisions in Two or Three Dimensions

5. Apply **momentum conservation**; there will be one equation for each dimension.

6. If the collision is **elastic**, apply conservation of **kinetic energy** as well.

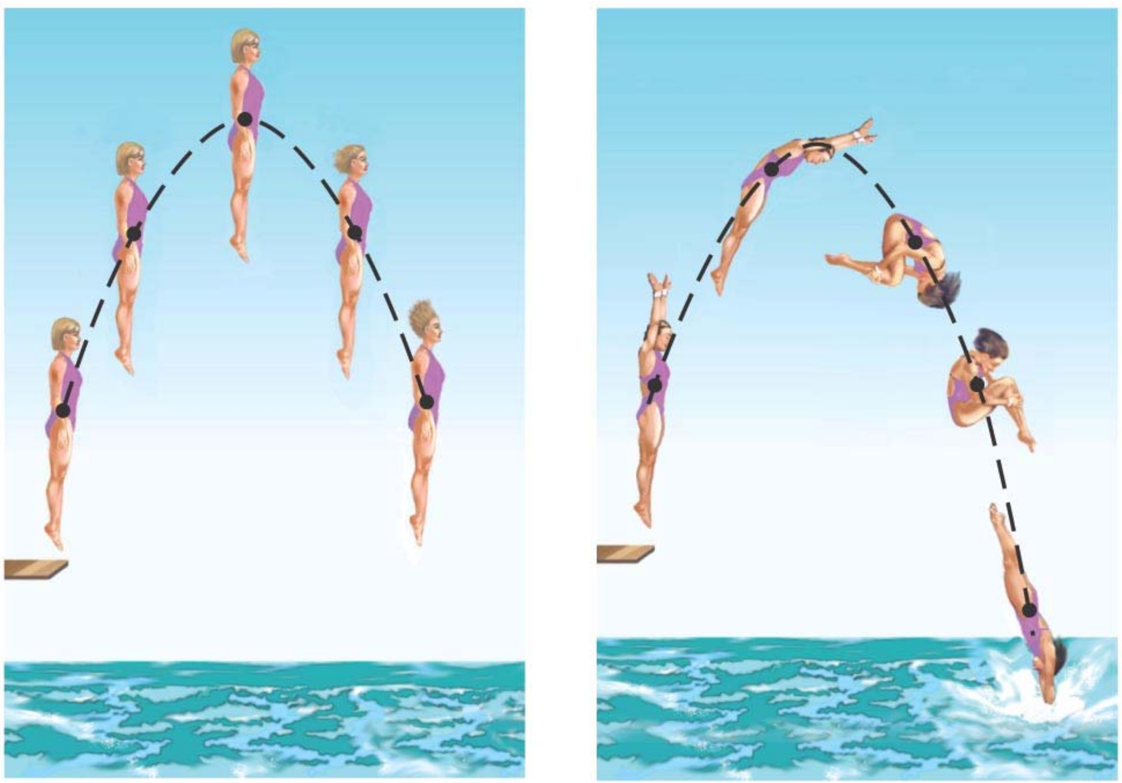
7. **Solve**.

8. **Check** units and magnitudes of result.

9-8 Center of Mass (CM)

In (a), the diver's motion is pure **translation**; in (b) it is **translation** plus **rotation**.

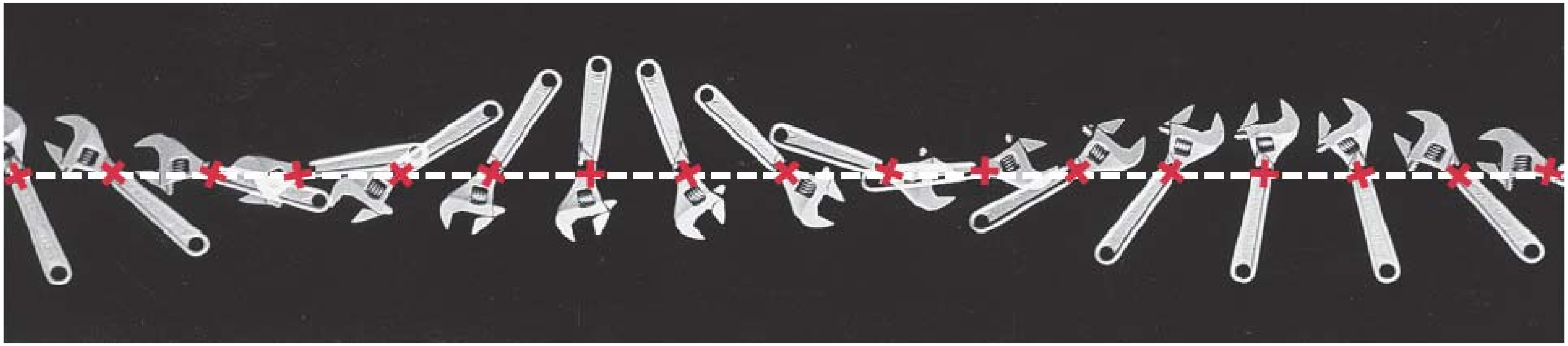
There is one point that moves in the **same** path a particle would take if subjected to the same force as the diver. This point is called the **center of mass** (CM).



(a) (b)

9-8 Center of Mass (CM)

The **general** motion of an object can be considered as the **sum** of the **translational** motion of the **CM**, plus **rotational**, **vibrational**, or other forms of motion about the CM.

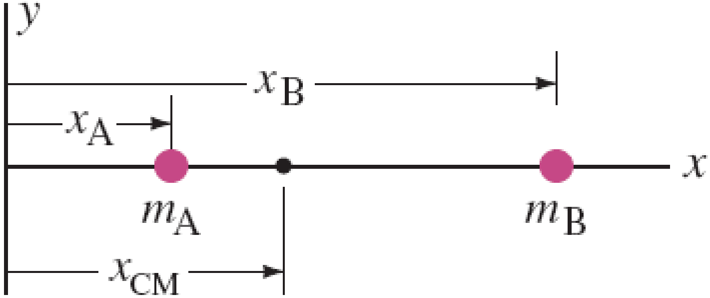


9-8 Center of Mass (CM)

For two particles, the **center of mass** lies closer to the one with the most mass:

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M},$$

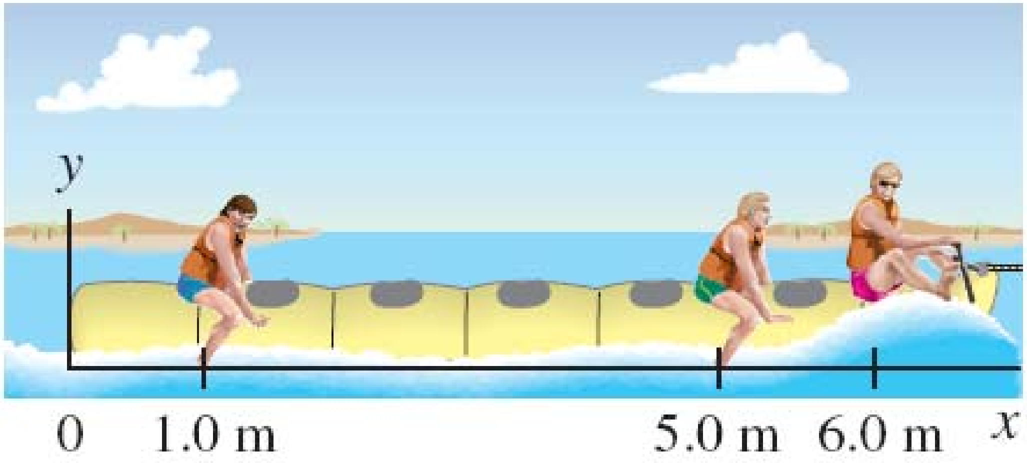
where *M* is the **total** mass.



9-8 Center of Mass (CM)

Example 9-14: CM of three guys on a raft.

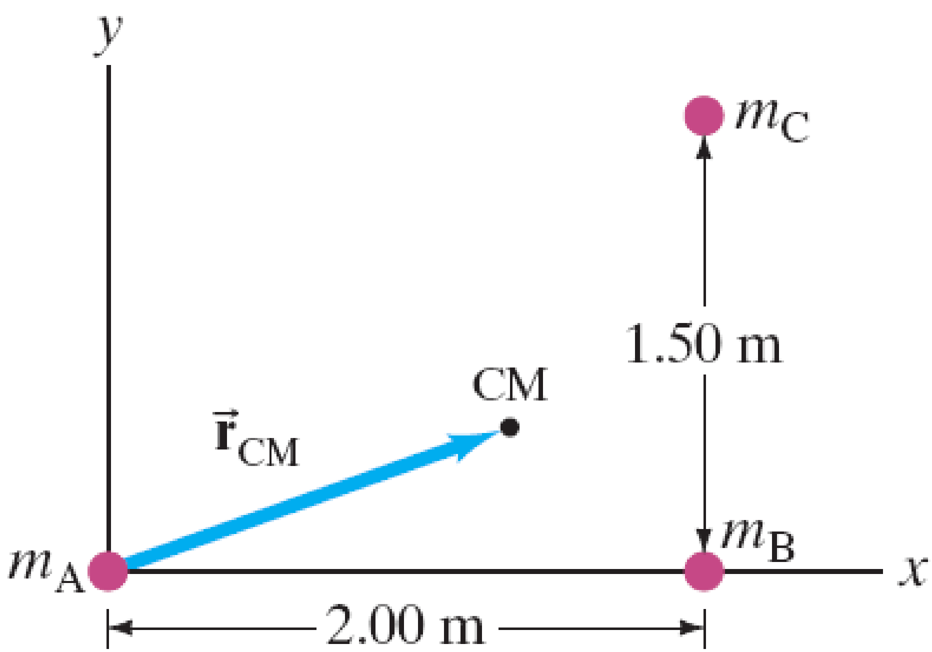
Three people of roughly equal masses m on a lightweight (air-filled) banana boat sit along the x axis at positions x_A = 1.0 m, x_B = 5.0 m, and x_C = 6.0 m, measured from the left-hand end. Find the position of the CM. Ignore the boat's mass.



9-8 Center of Mass (CM)

Exercise 9-15: Three particles in 2-D.

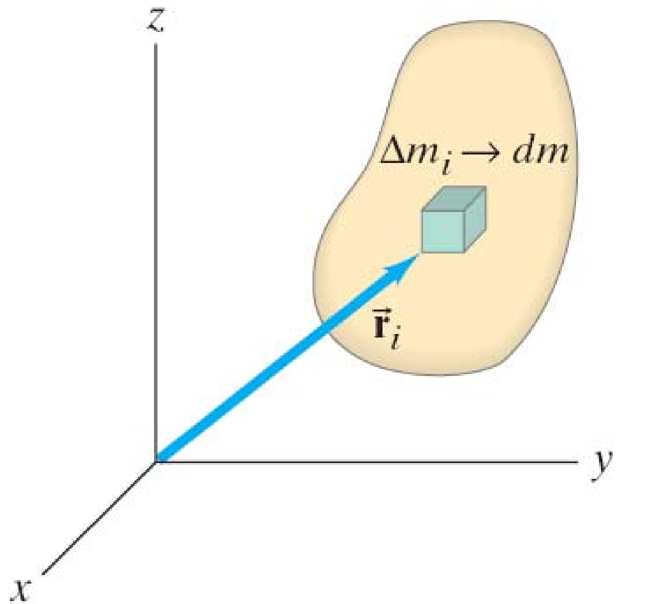
Three particles, each of mass 2.50 kg, are located at the corners of a right triangle whose sides are 2.00 m and 1.50 m long, as shown. Locate the center of mass.



9-8 Center of Mass (CM)

For an extended object, we imagine making it up of tiny particles, each of tiny mass, and adding up the product of each particle's mass with its position and dividing by the total mass. In the limit that the particles become infinitely small, this gives:

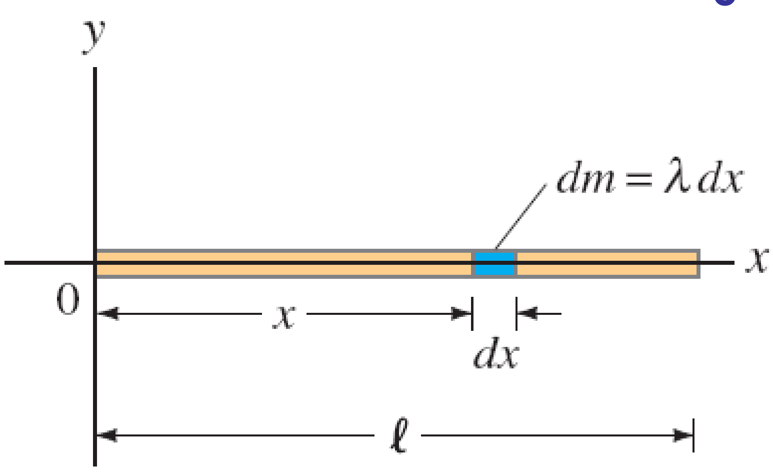
$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\int \vec{\mathbf{r}}\, dm.$$



9-8 Center of Mass (CM)

Example 9-16: CM of a thin rod.

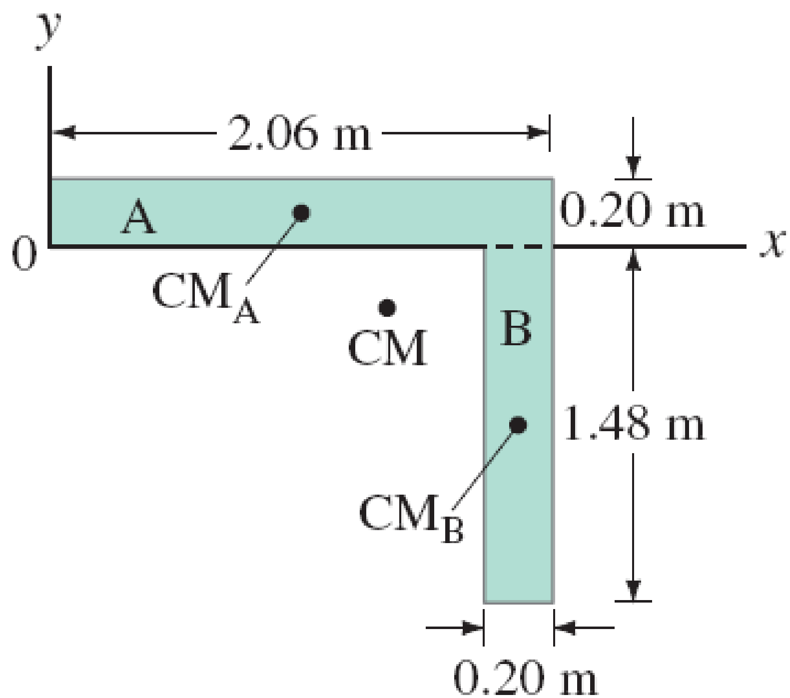
(a) Show that the CM of a uniform thin rod of length *l* and mass *M* is at its center. (b) Determine the CM of the rod assuming its linear mass density λ (its mass per unit length) varies linearly from $\lambda = \lambda_0$ at the left end to double that value, $\lambda = 2\lambda_0$, at the right end.



9-8 Center of Mass (CM)

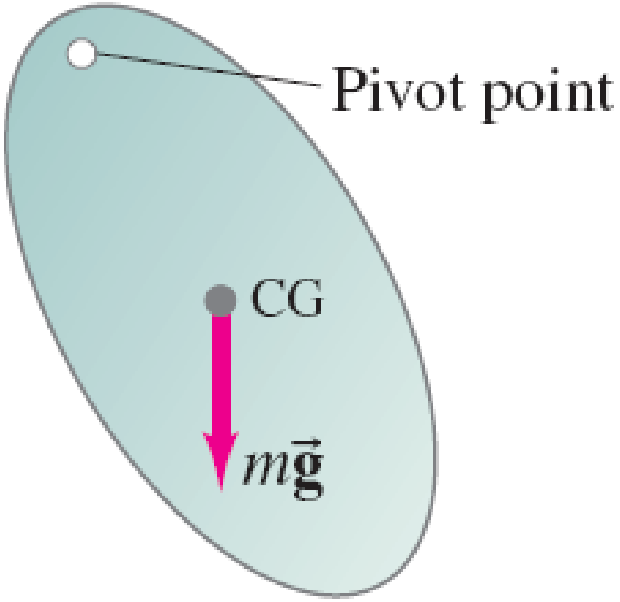
Example 9-17: CM of L-shaped flat object.

Determine the CM of the uniform thin L-shaped construction brace shown.



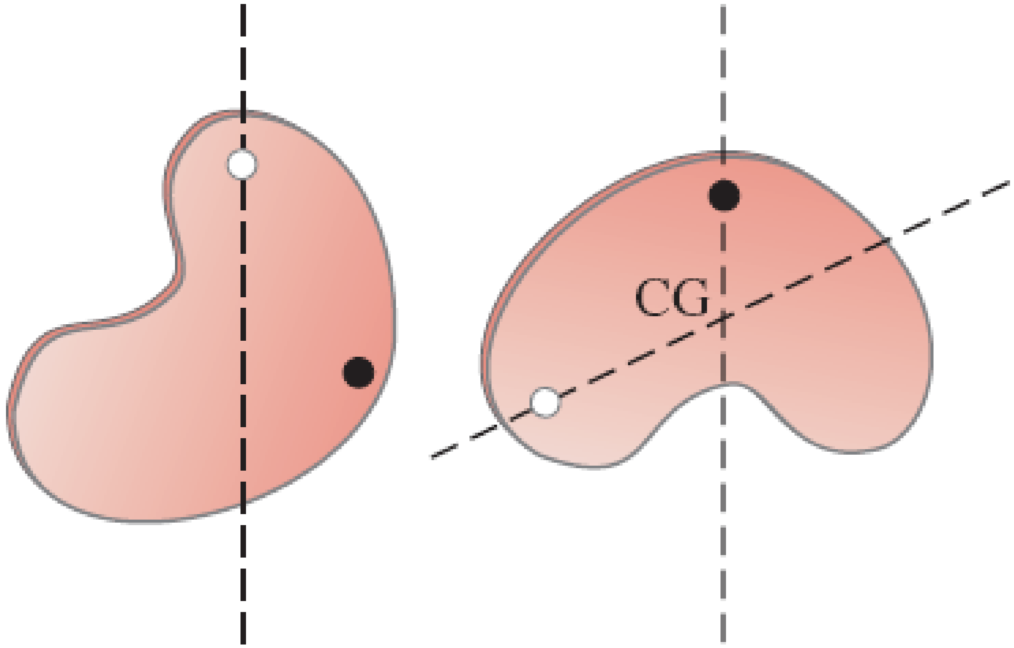
9-8 Center of Mass (CM)

The **center of gravity** is the point at which the gravitational force can be considered to act. It is the same as the **center of mass** as long as the gravitational force does not **vary** among different parts of the object.



9-8 Center of Mass (CM)

The center of gravity can be found experimentally by suspending an object from different points. The CM need not be within the actual object—a doughnut's CM is in the center of the hole.



9-9 Center of Mass and Translational Motion

The **total momentum** of a system of particles is equal to the product of the **total mass** and the **velocity** of the **center of mass**.

The **sum** of all the **forces** acting on a system is equal to the **total mass** of the system multiplied by the **acceleration** of the **center of mass**:

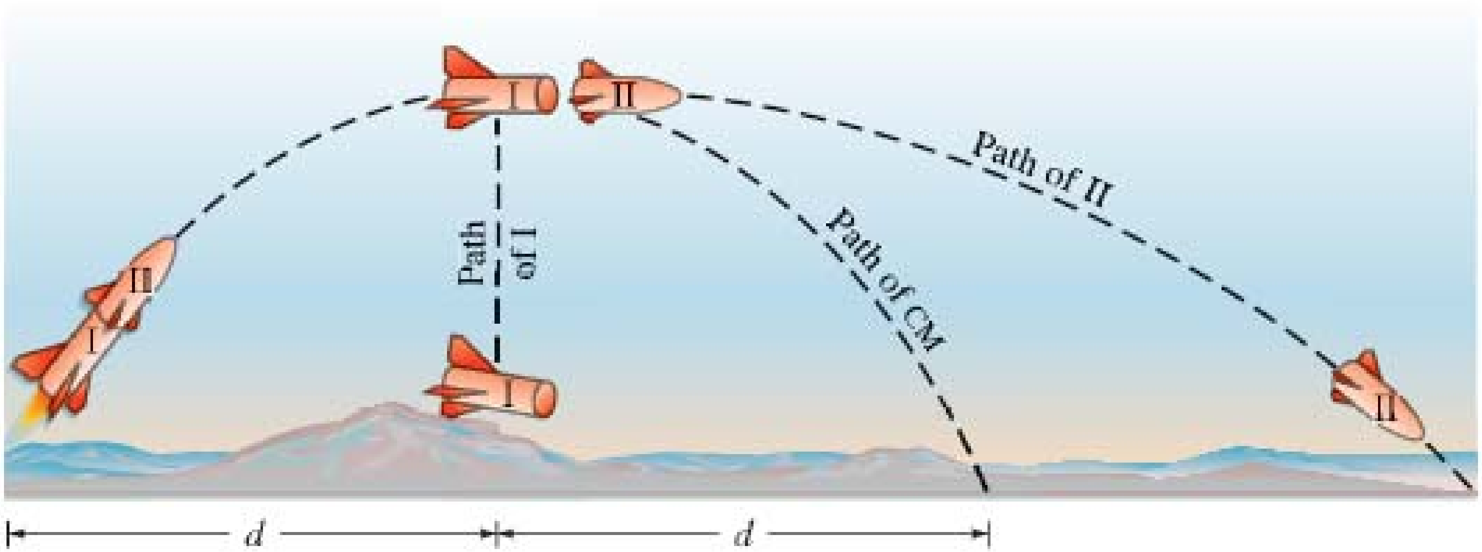
$$M\vec{\mathbf{a}}_{\text{CM}} = \Sigma\vec{\mathbf{F}}_{\text{ext}}.$$

Therefore, the center of mass of a system of particles (or objects) with total mass M moves like a single particle of mass M acted upon by the same net external force.

9-9 Center of Mass and Translational Motion

Conceptual Example 9-18: A two-stage rocket.

A rocket is shot into the air as shown. At the moment it reaches its highest point, a horizontal distance *d* from its starting point, a prearranged explosion separates it into two parts of equal mass. Part I is stopped in midair by the explosion and falls vertically to Earth. Where does part II land? Assume g = constant.

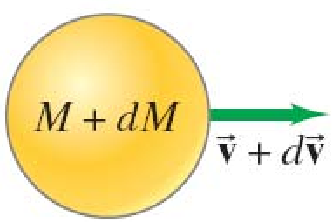


9-10 Systems of Variable Mass; Rocket Propulsion

Applying Newton's second law to the system shown gives:

$$d\vec{\mathbf{P}}/dt = \Sigma \vec{\mathbf{F}}_{\text{ext}}.$$

Therefore,



$$\Sigma \vec{\mathbf{F}}_{\text{ext}} = M\frac{d\vec{\mathbf{v}}}{dt} - (\vec{\mathbf{u}} - \vec{\mathbf{v}})\frac{dM}{dt},$$

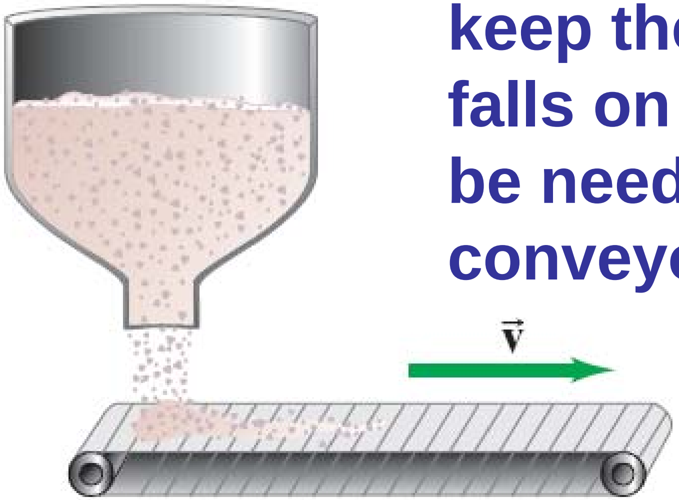
or

$$M\frac{d\vec{\mathbf{v}}}{dt} = \Sigma \vec{\mathbf{F}}_{\text{ext}} + \vec{\mathbf{v}}_{\text{rel}}\frac{dM}{dt}.$$

9-10 Systems of Variable Mass; Rocket Propulsion

Example 9-19: Conveyor belt.

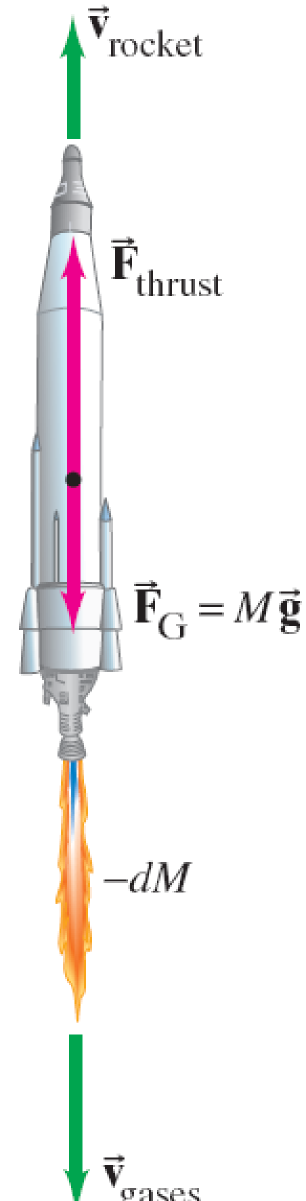
You are designing a conveyor system for a gravel yard. A hopper drops gravel at a rate of 75.0 kg/s onto a conveyor belt that moves at a constant speed v = 2.20 m/s. (a) Determine the additional force (over and above internal friction) needed to keep the conveyor belt moving as gravel falls on it. (b) What power output would be needed from the motor that drives the conveyor belt?



9-10 Systems of Variable Mass; Rocket Propulsion

Example 9-20: Rocket propulsion.

A fully fueled rocket has a mass of 21,000 kg, of which 15,000 kg is fuel. The burned fuel is spewed out the rear at a rate of 190 kg/s with a speed of 2800 m/s relative to the rocket. If the rocket is fired vertically upward calculate: (a) the thrust of the rocket; (b) the net force on the rocket at blastoff, and just before burnout (when all the fuel has been used up); (c) the rocket's velocity as a function of time, and (d) its final velocity at burnout. Ignore air resistance and assume the acceleration due to gravity is constant at g = 9.80 m/s^2.



Summary of Chapter 9

- **Momentum of an object:** $\vec{\mathbf{p}} = m\vec{\mathbf{v}}.$
- **Newton's second law:** $\Sigma\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}.$

- **Total momentum of an isolated system of objects is conserved.**

- **During a collision, the colliding objects can be considered to be an isolated system even if external forces exist, as long as they are not too large.**

- **Momentum will therefore be conserved during collisions.**

Summary of Chapter 9, cont.

- Impulse: $\vec{\mathbf{J}} = \int \vec{\mathbf{F}}\, dt.$
- In an elastic collision, total kinetic energy is also conserved.
- In an inelastic collision, some kinetic energy is lost.
- In a completely inelastic collision, the two objects stick together after the collision.
- The center of mass of a system is the point at which external forces can be considered to act.