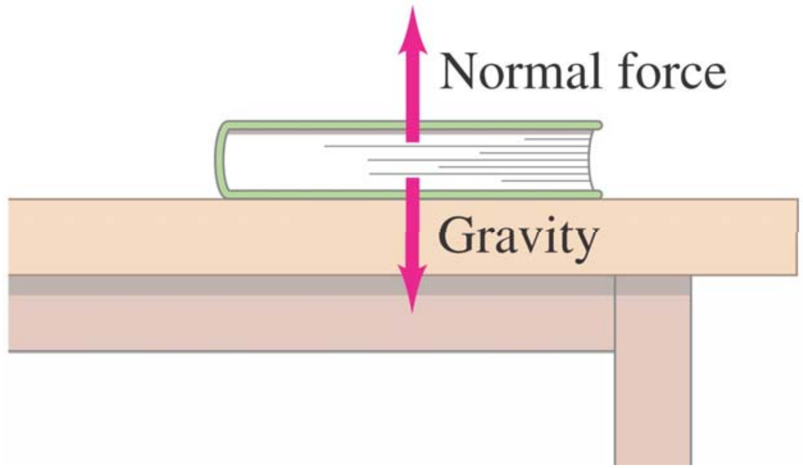
Chapter 12
Static Equilibrium; Elasticity and Fracture

Units of Chapter 12

- The Conditions for Equilibrium
- Solving Statics Problems
- Stability and Balance
- Elasticity; Stress and Strain
- Fracture
- Trusses and Bridges
- Arches and Domes

12-1 The Conditions for Equilibrium

An object with forces acting on it, but with zero net force, is said to be in **equilibrium**.



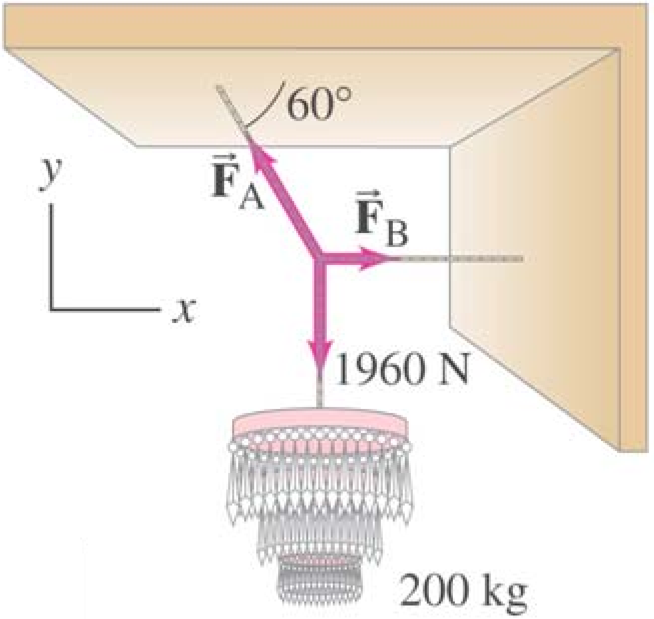
The first condition for equilibrium:

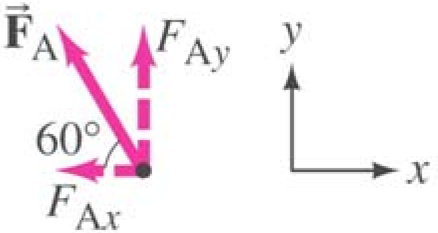
$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \Sigma F_z = 0.$$

12-1 The Conditions for Equilibrium

Example 12-1:
Chandelier cord tension.

Calculate the tensions $\vec{F}_A$ and $\vec{F}_B$ in the two cords that are connected to the vertical cord supporting the 200-kg chandelier shown. Ignore the mass of the cords.

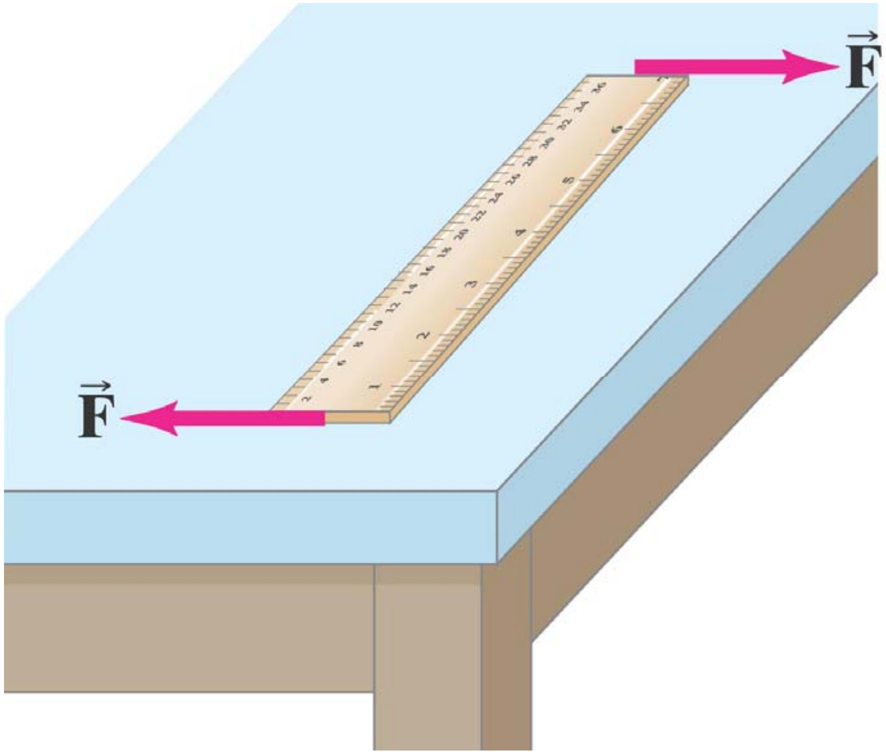




12-1 The Conditions for Equilibrium

The second condition of equilibrium is that there be no torque around any axis; the choice of axis is arbitrary.

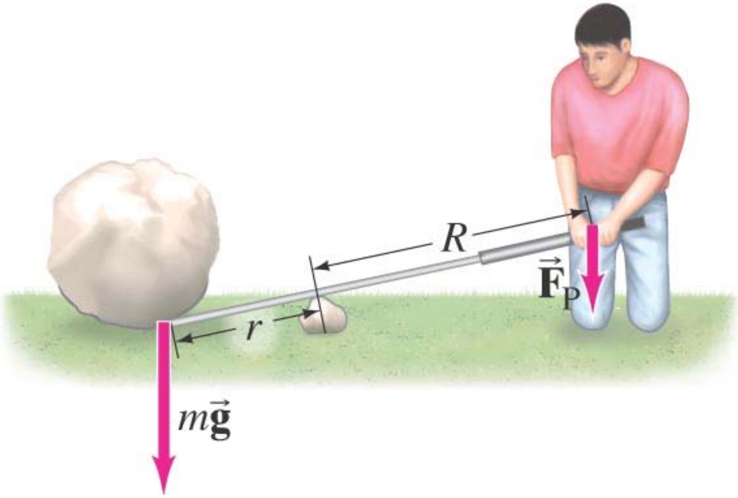
$$\Sigma\tau = 0$$



12-1 The Conditions for Equilibrium

Conceptual Example 12-2: A lever.

This bar is being used as a lever to pry up a large rock. The small rock acts as a fulcrum (pivot point). The force required at the long end of the bar can be quite a bit smaller than the rock's weight *mg*, since it is the torques that balance in the rotation about the fulcrum. If, however, the leverage isn't sufficient, and the large rock isn't budged, what are two ways to increase the leverage?



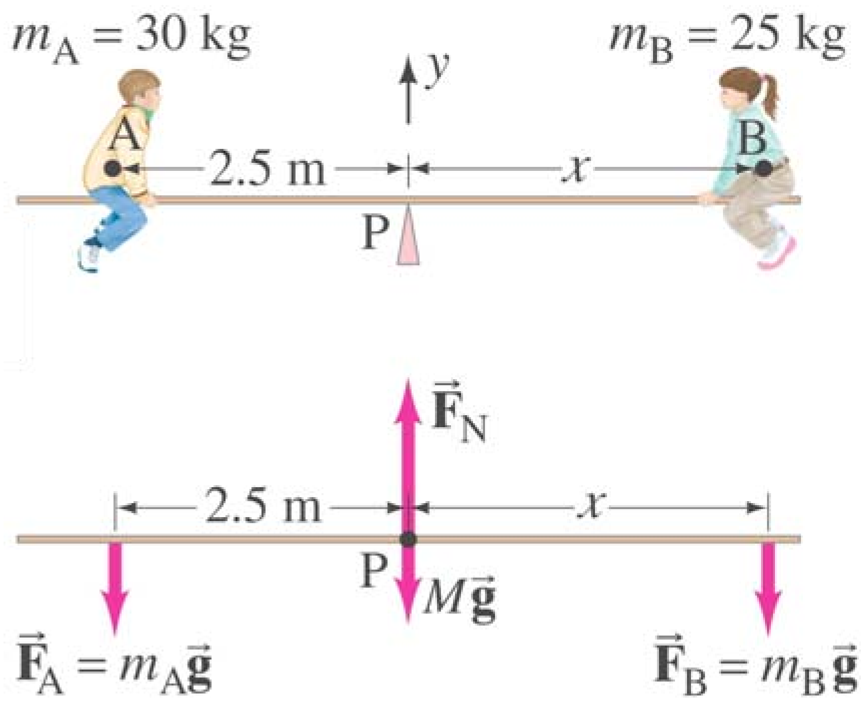
12-2 Solving Statics Problems

1. Choose one **object** at a time, and make a **free-body diagram** by showing all the forces on it and where they act.
2. Choose a **coordinate** system and resolve forces into **components**.
3. Write **equilibrium** equations for the forces.
4. Choose any axis perpendicular to the plane of the forces and write the **torque** equilibrium equation. A clever choice here can simplify the problem enormously.
5. **Solve**.

12-2 Solving Statics Problems

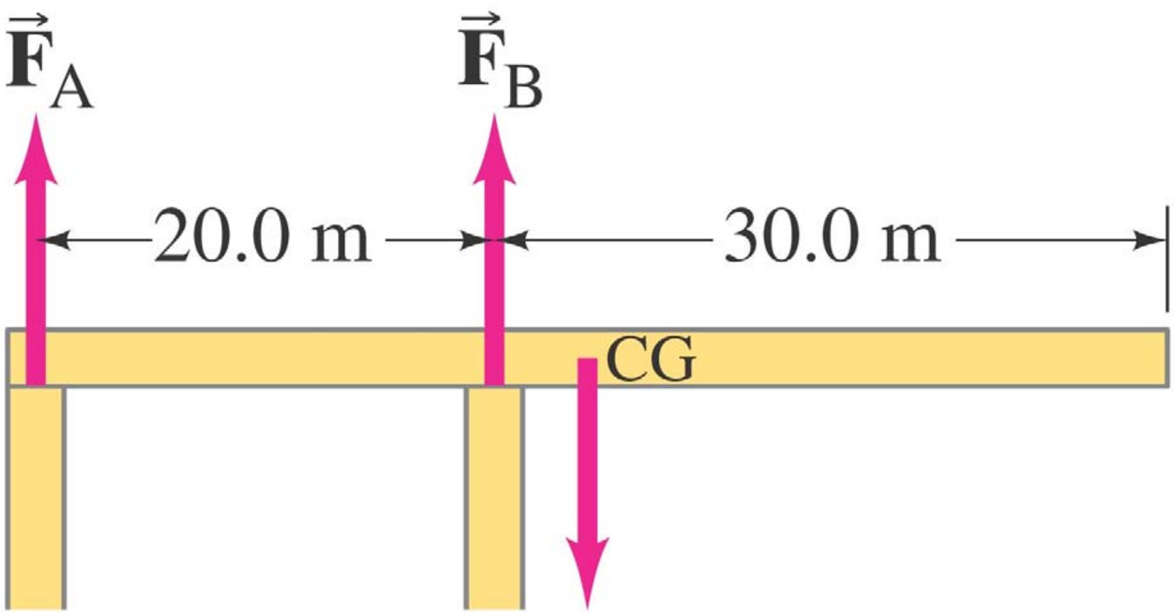
Example 12-3: Balancing a seesaw.

A board of mass M = 2.0 kg serves as a seesaw for two children. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P (his center of gravity is 2.5 m from the pivot). At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? Assume the board is uniform and centered over the pivot.



12-2 Solving Statics Problems

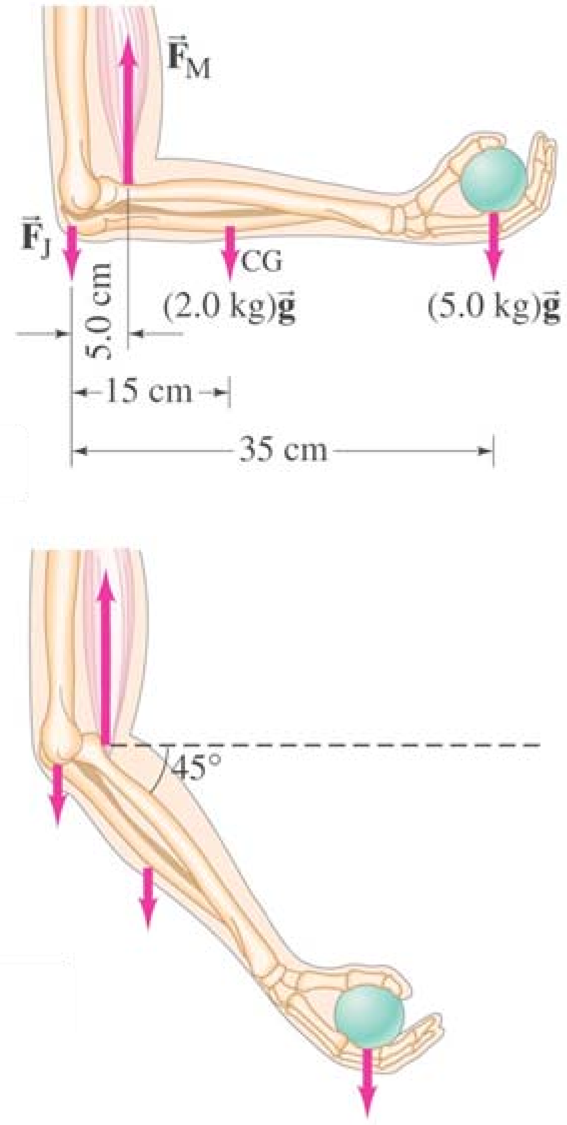
If a force in your solution comes out **negative** (as $\vec{F}_A$ will here), it just means that it's in the **opposite** direction from the one you chose. This is trivial to fix, so don't worry about getting all the signs of the forces right before you start solving.



12-2 Solving Statics Problems

Example 12-4: Force exerted by biceps muscle.

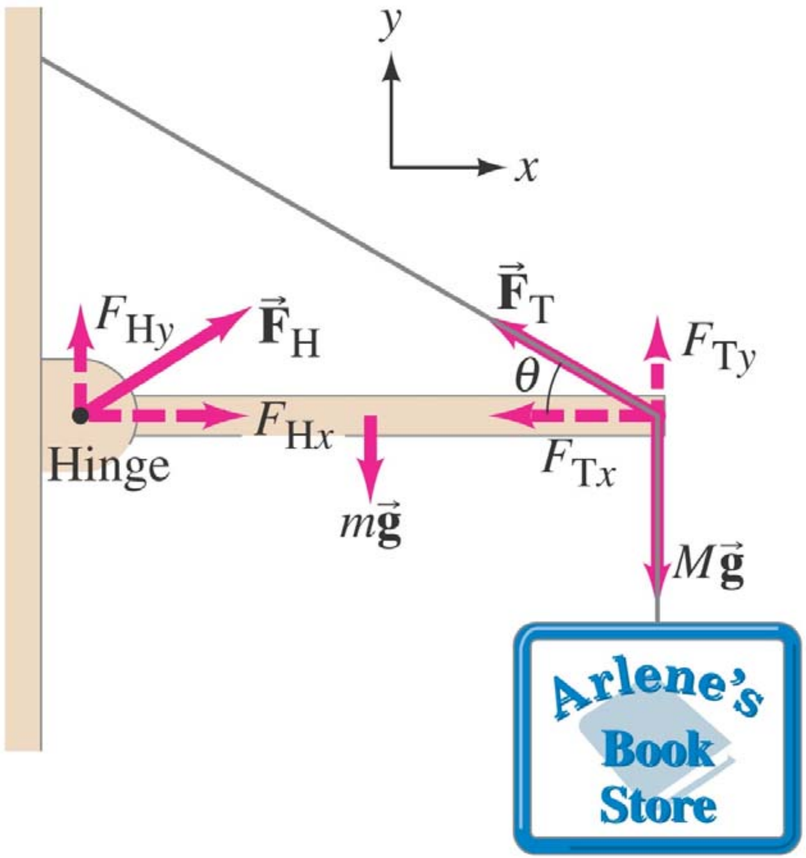
How much force must the biceps muscle exert when a 5.0-kg ball is held in the hand (a) with the arm horizontal, and (b) when the arm is at a 45° angle? The biceps muscle is connected to the forearm by a tendon attached 5.0 cm from the elbow joint. Assume that the mass of forearm and hand together is 2.0 kg and their CG is as shown.



12-2 Solving Statics Problems

Example 12-5: Hinged beam and cable.

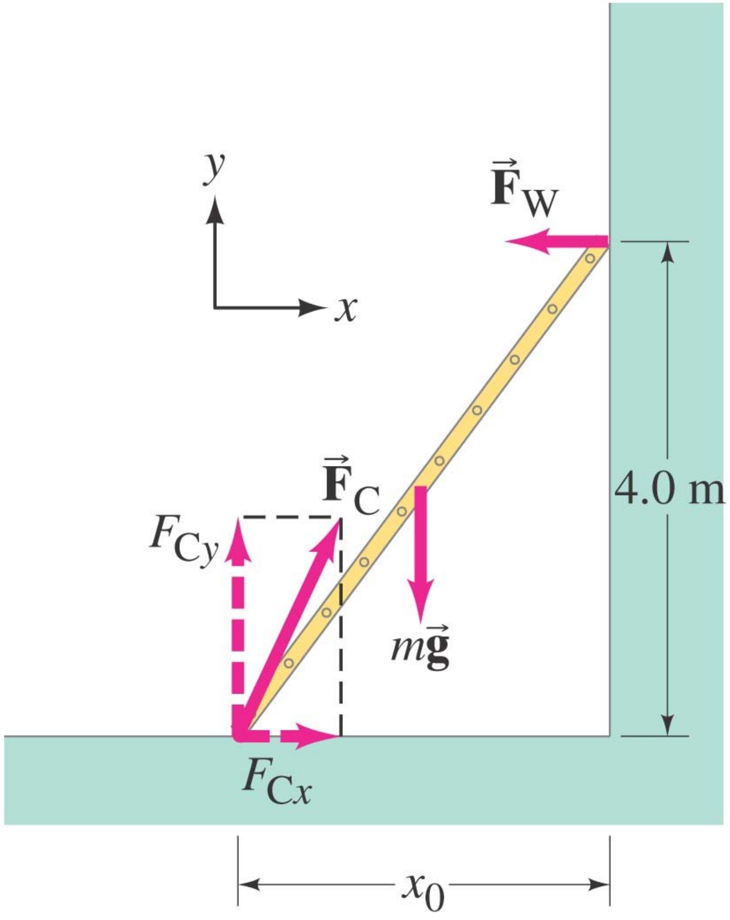
A uniform beam, 2.20 m long with mass m = 25.0 kg, is mounted by a small hinge on a wall. The beam is held in a horizontal position by a cable that makes an angle θ = 30.0°. The beam supports a sign of mass M = 28.0 kg suspended from its end. Determine the components of the force $\vec{F}_H$ that the (smooth) hinge exerts on the beam, and the tension F_T in the supporting cable.



12-2 Solving Statics Problems

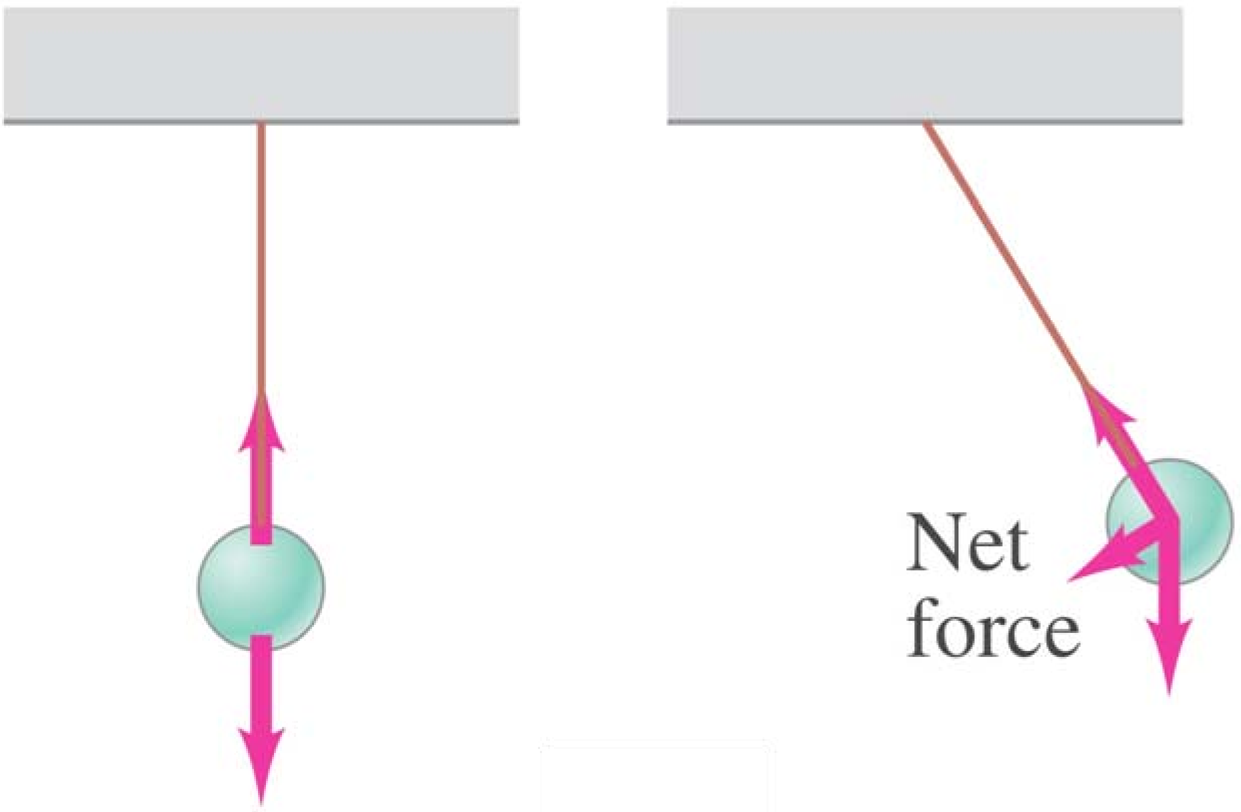
Example 12-6: Ladder.

A 5.0-m-long ladder leans against a smooth wall at a point 4.0 m above a cement floor. The ladder is uniform and has mass m = 12.0 kg. Assuming the wall is frictionless (but the floor is not), determine the forces exerted on the ladder by the floor and by the wall.



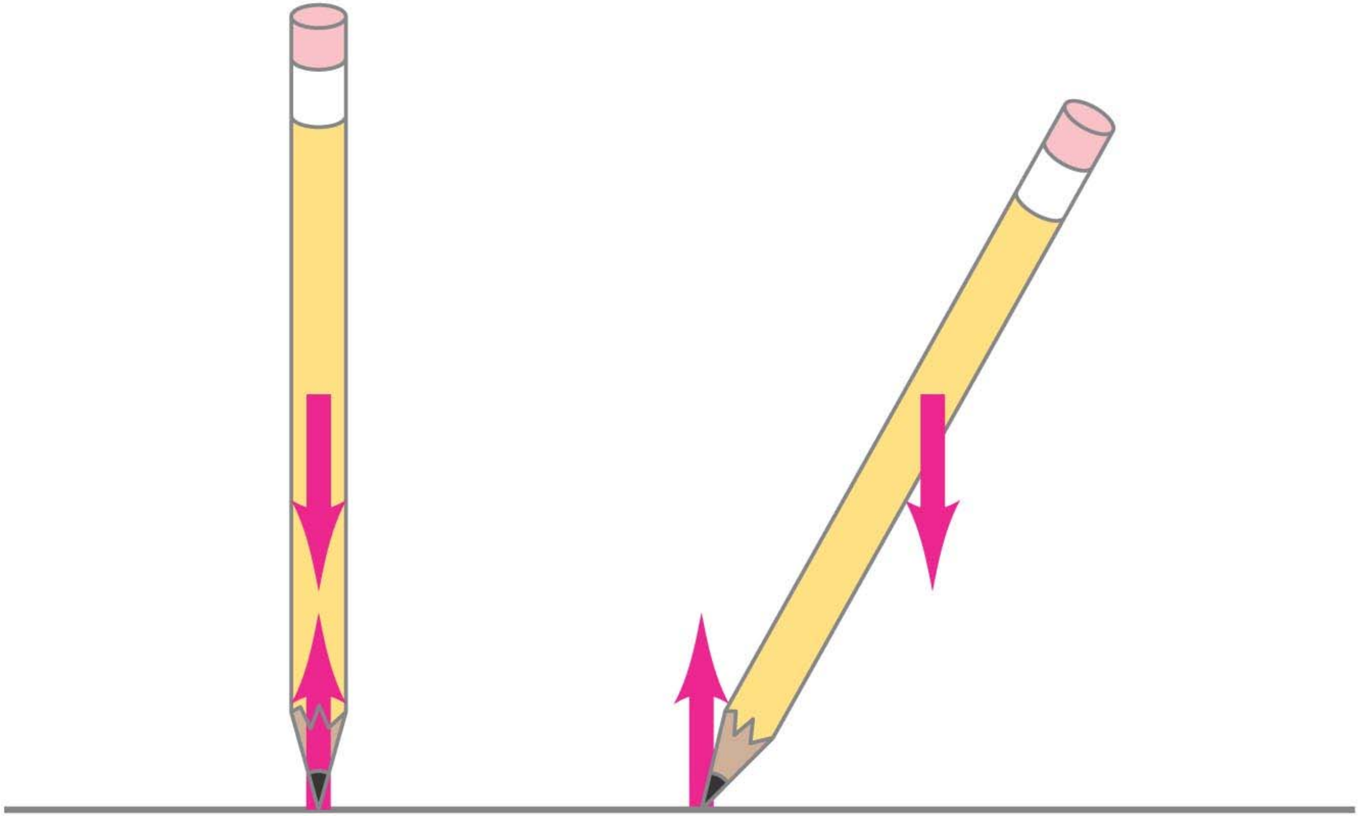
12-3 Stability and Balance

If the forces on an object are such that they tend to **return** it to its equilibrium position, it is said to be in **stable** equilibrium.



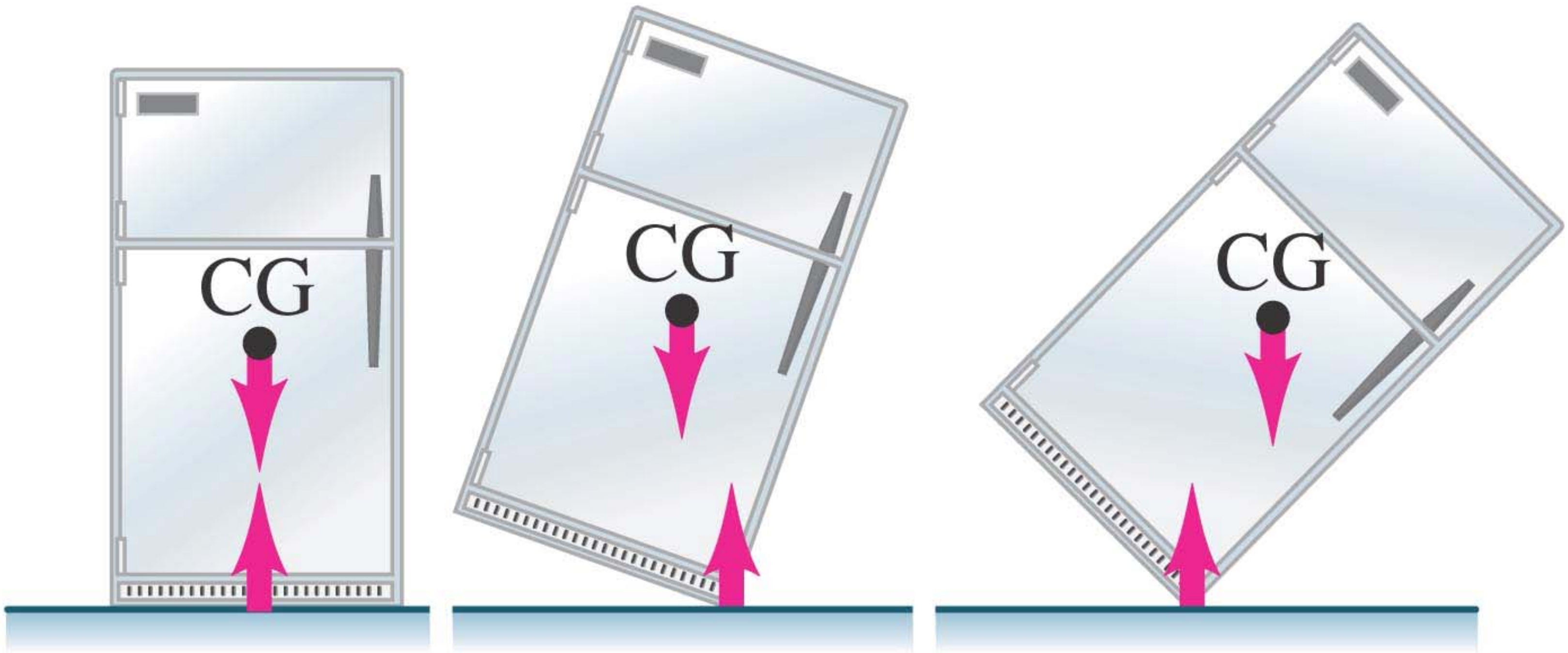
12-3 Stability and Balance

If, however, the forces tend to move it **away** from its equilibrium point, it is said to be in **unstable** equilibrium.



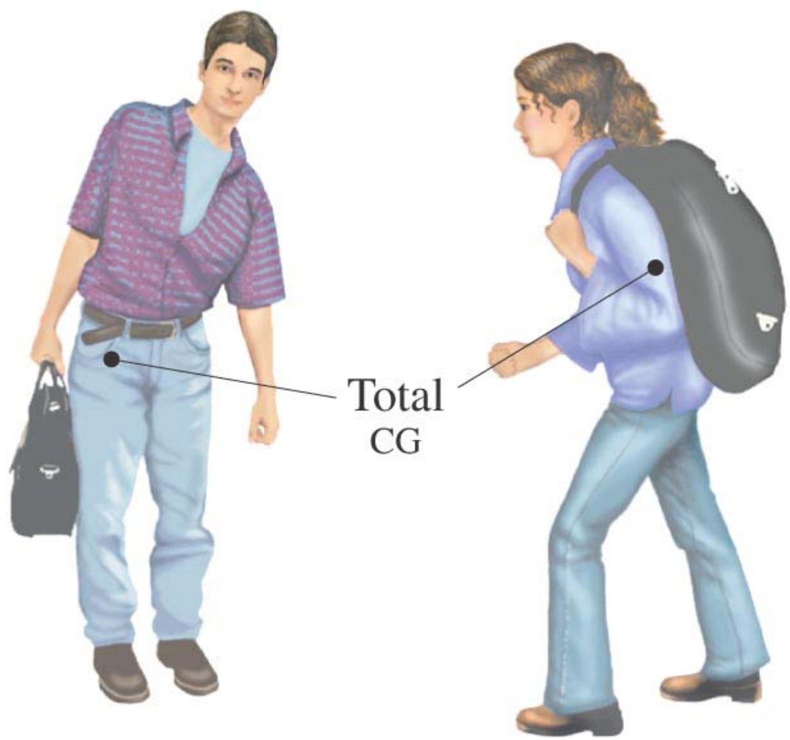
12-3 Stability and Balance

An object in **stable** equilibrium may become **unstable** if it is **tipped** so that its center of gravity is **outside** the pivot point. Of course, it will be stable again once it lands!

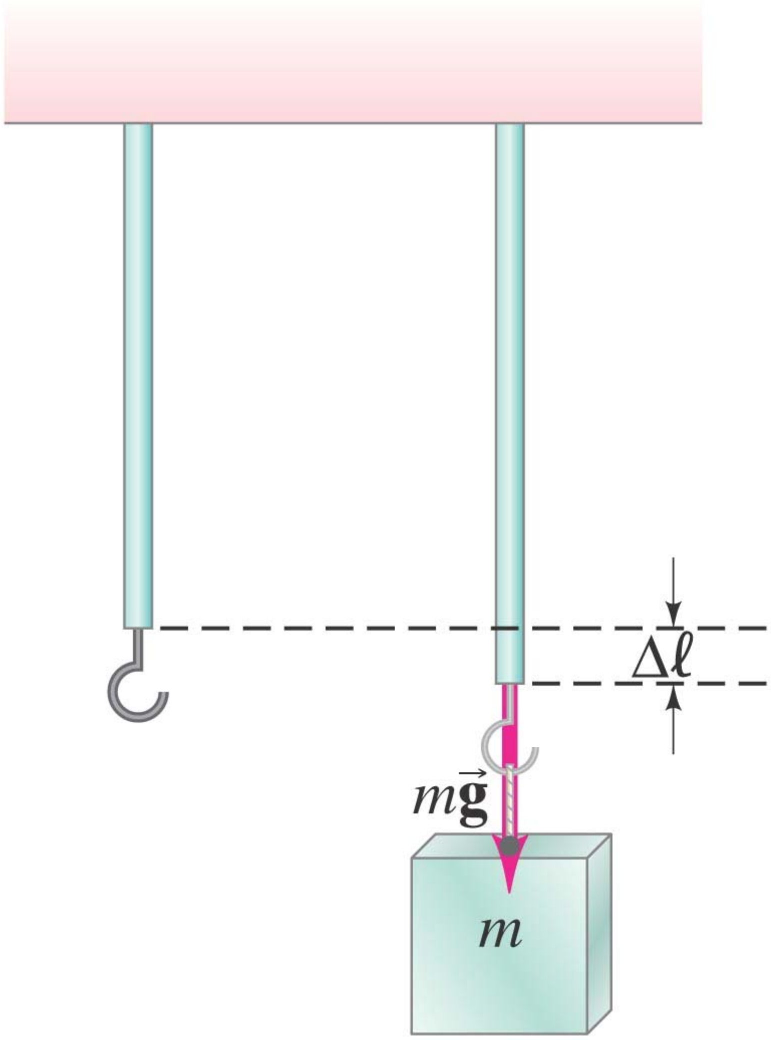


12-3 Stability and Balance

People carrying heavy loads automatically **adjust** their posture so their **center of mass** is over their **feet**. This can lead to injury if the contortion is too great.



12-4 Elasticity; Stress and Strain

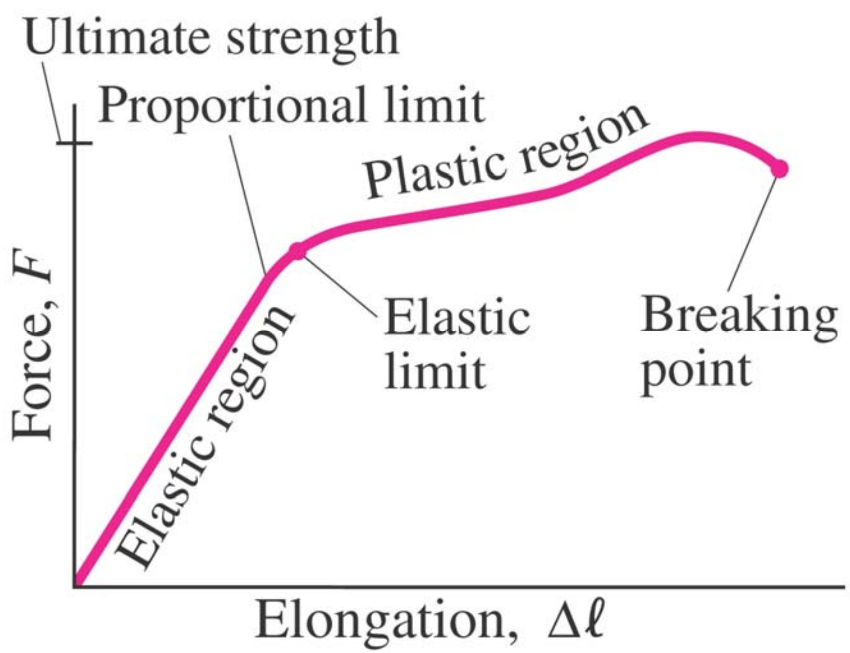


Hooke's law: the change in length is proportional to the applied force.

$$F = k\,\Delta\ell$$

12-4 Elasticity; Stress and Strain

This proportionality holds until the force reaches the **proportional limit**. Beyond that, the object will still return to its original shape up to the **elastic limit**. Beyond the elastic limit, the material is permanently **deformed**, and it **breaks** at the breaking point.



12-4 Elasticity; Stress and Strain

The **change** in length of a stretched object depends not only on the applied **force**, but also on its **length**, cross-sectional **area** and the **material** from which it is made.

The material factor, E, is called the elastic modulus or **Young's modulus**, and it has been measured for many materials.

$$\Delta \ell = \frac{1}{E}\frac{F}{A}\ell_0.$$

12-4 Elasticity; Stress and Strain

TABLE 12–1 Elastic Moduli

Material	Young's Modulus, E (N/m²)	Shear Modulus, G (N/m²)	Bulk Modulus, B (N/m²)
Solids			
Iron, cast	100×10^9	40×10^9	90×10^9
Steel	200×10^9	80×10^9	140×10^9
Brass	100×10^9	35×10^9	80×10^9
Aluminum	70×10^9	25×10^9	70×10^9
Concrete	20×10^9		
Brick	14×10^9		
Marble	50×10^9		70×10^9
Granite	45×10^9		45×10^9
Wood (pine) (parallel to grain)	10×10^9		
(perpendicular to grain)	1×10^9		
Nylon	5×10^9		
Bone (limb)	15×10^9	80×10^9	
Liquids			
Water			2.0×10^9
Alcohol (ethyl)			1.0×10^9
Mercury			2.5×10^9
Gases†			
Air, H_2, He, CO_2			1.01×10^5

†At normal atmospheric pressure; no variation in temperature during process.

12-4 Elasticity; Stress and Strain

Example 12-7: Tension in piano wire.

A 1.60-m-long steel piano wire has a diameter of 0.20 cm. How great is the tension in the wire if it stretches 0.25 cm when tightened?

12-4 Elasticity; Stress and Strain

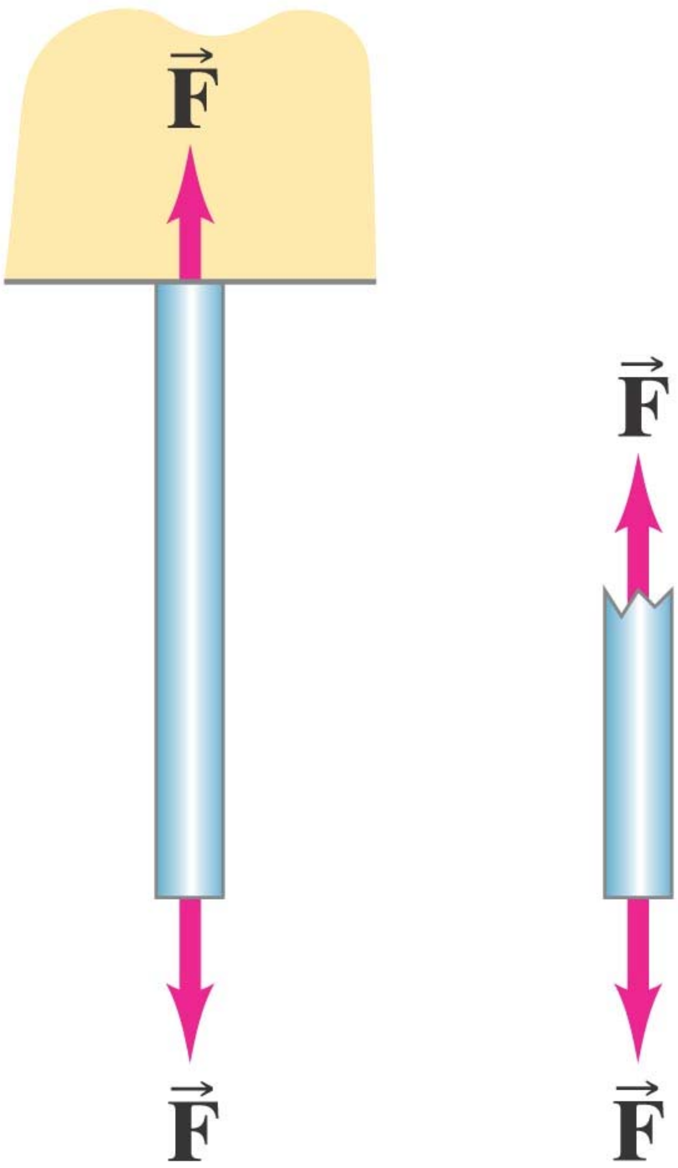
Stress is defined as the force per unit area.

Strain is defined as the ratio of the change in length to the original length.

Therefore, the elastic modulus is equal to the stress divided by the strain:

$$E = \frac{F/A}{\Delta \ell / \ell_0} = \frac{\text{stress}}{\text{strain}}.$$

12-4 Elasticity; Stress and Strain



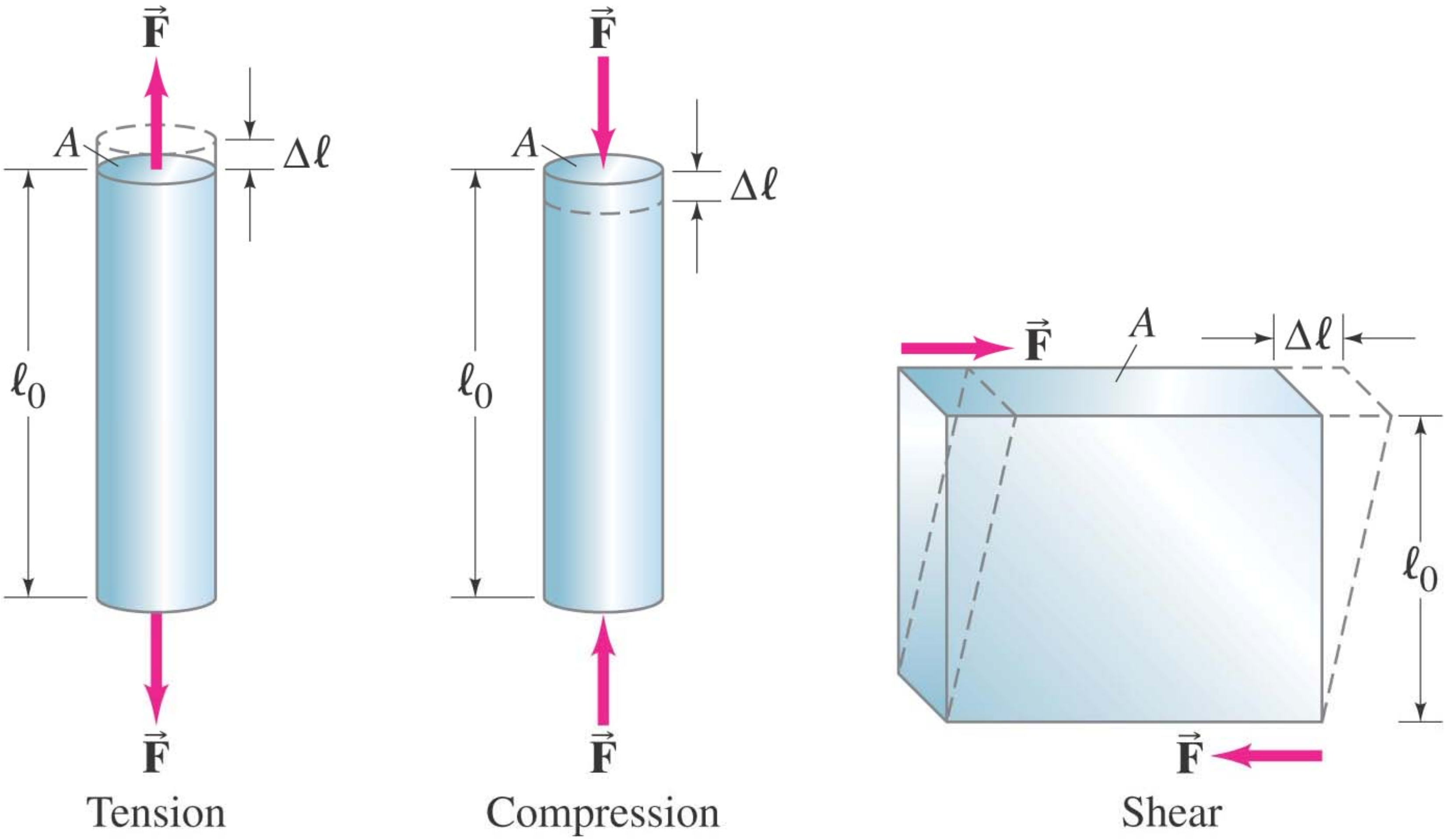
In **tensile** stress, forces tend to **stretch** the object.

12-4 Elasticity; Stress and Strain

Compressional stress is exactly the opposite of tensional stress. These columns are under **compression**.

12-4 Elasticity; Stress and Strain

The three types of stress for rigid objects:



12-4 Elasticity; Stress and Strain

The shear strain, where *G* is the shear modulus:

$$\Delta \ell = \frac{1}{G}\frac{F}{A}\ell_0 .$$

12-4 Elasticity; Stress and Strain

If an object is subjected to inward forces on all sides, its volume changes depending on its bulk modulus. This is the only deformation that applies to fluids.

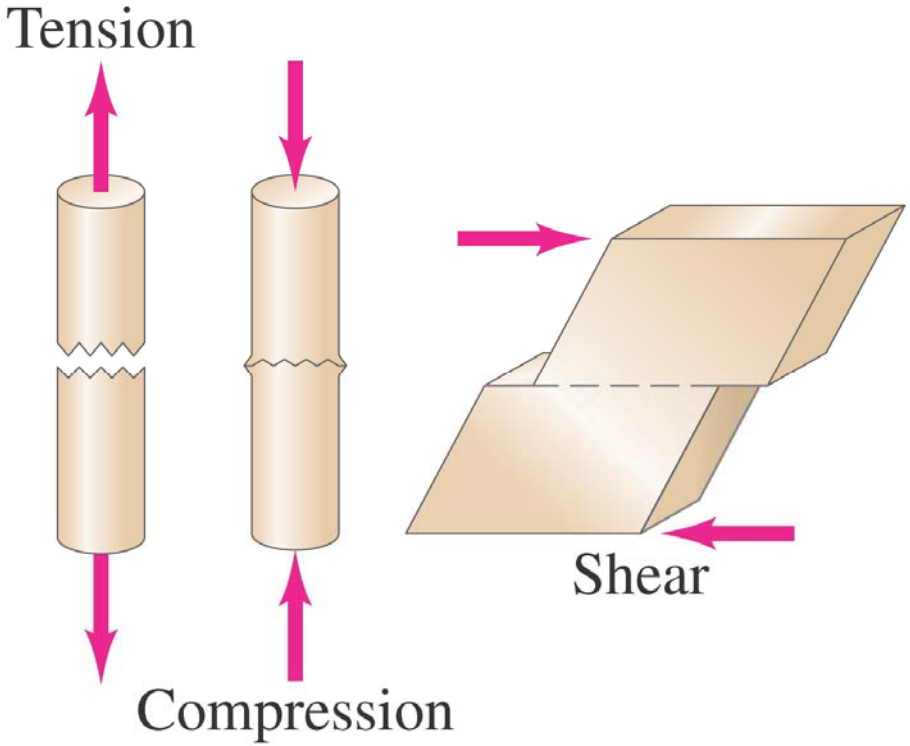
$$\frac{\Delta V}{V_0} = -\frac{1}{B}\Delta P$$

or

$$B = -\frac{\Delta P}{\Delta V/V_0}.$$

12-5 Fracture

If the stress on an object is too great, the object will fracture. The ultimate strengths of materials under tensile stress, compressional stress, and shear stress have been measured.



When designing a structure, it is a good idea to keep anticipated stresses less than 1/3 to 1/10 of the ultimate strength.

12-5 Fracture

TABLE 12–2 Ultimate Strengths of Materials (force/area)

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)	Shear Strength (N/m^2)
Iron, cast	170×10^6	550×10^6	170×10^6
Steel	500×10^6	500×10^6	250×10^6
Brass	250×10^6	250×10^6	200×10^6
Aluminum	200×10^6	200×10^6	200×10^6
Concrete	2×10^6	20×10^6	2×10^6
Brick		35×10^6	
Marble		80×10^6	
Granite		170×10^6	
Wood (pine) (parallel to grain)	40×10^6	35×10^6	5×10^6
(perpendicular to grain)		10×10^6	
Nylon	500×10^6		
Bone (limb)	130×10^6	170×10^6	

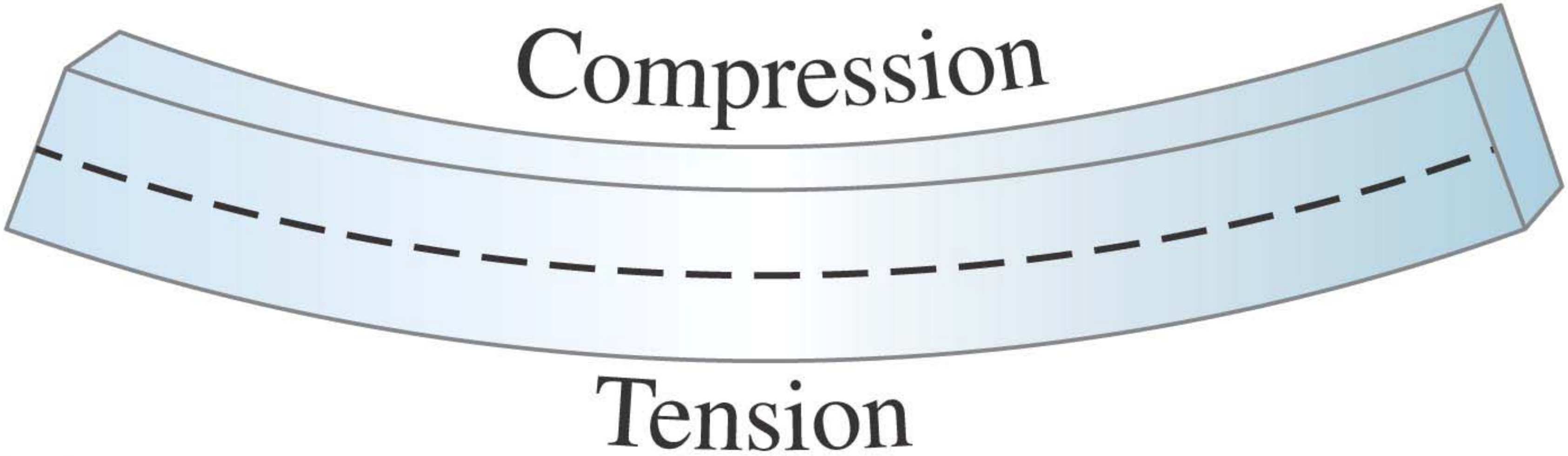
12-5 Fracture

Example 12-8: Breaking the piano wire.

A steel piano wire is 1.60 m long with a diameter of 0.20 cm. Approximately what tension force would break it?

12-5 Fracture

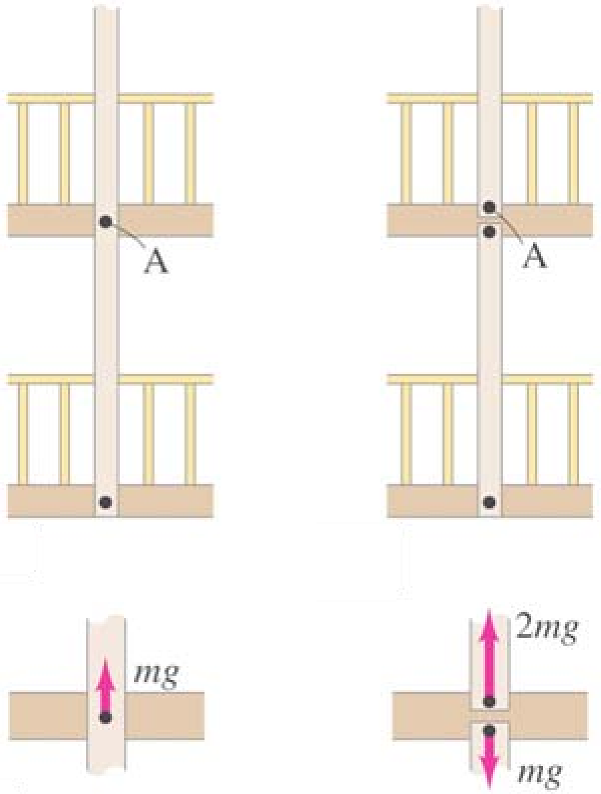
A **horizontal** beam will be under both **tensile** and **compressive** stress due to its own **weight**. Therefore, it must be made of a material that is strong under both compression and tension.



12-5 Fracture

Conceptual Example 12-9: A tragic substitution.

Two walkways, one above the other, are suspended from vertical rods attached to the ceiling of a high hotel lobby. The original design called for single rods 14 m long, but when such long rods proved to be unwieldy to install, it was decided to replace each long rod with two shorter ones as shown. Determine the net force exerted by the rods on the supporting pin A (assumed to be the same size) for each design. Assume each vertical rod supports a mass m of each bridge.



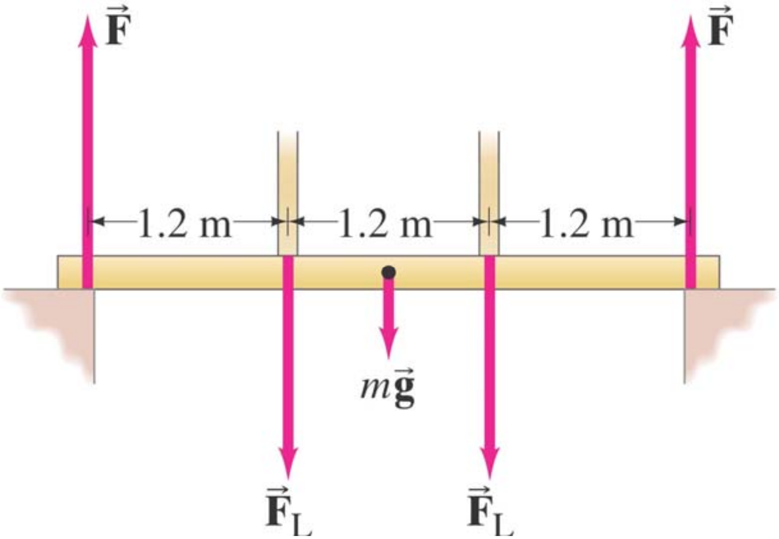
Force on pin A exerted by vertical rod

Forces on pins at A exerted by vertical rods

12-5 Fracture

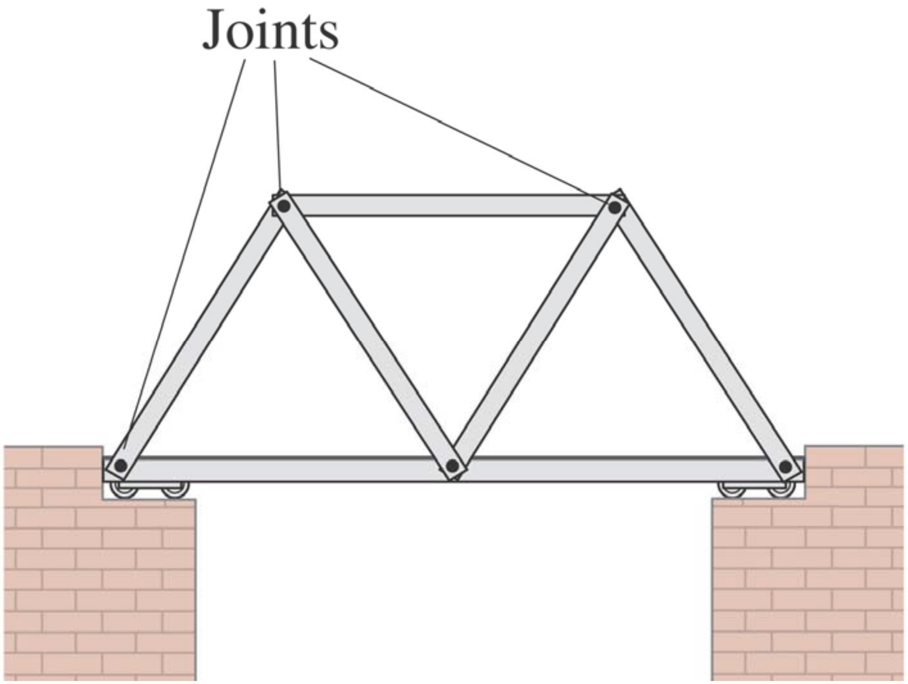
Example 12-10: Shear on a beam.

A uniform pine beam, 3.6 m long and 9.5 cm x 14 cm in cross section, rests on two supports near its ends, as shown. The beam's mass is 25 kg and two vertical roof supports rest on it, each one-third of the way from the ends. What maximum load force F_L can each of the roof supports exert without shearing the pine beam at its supports? Use a safety factor of 5.0.



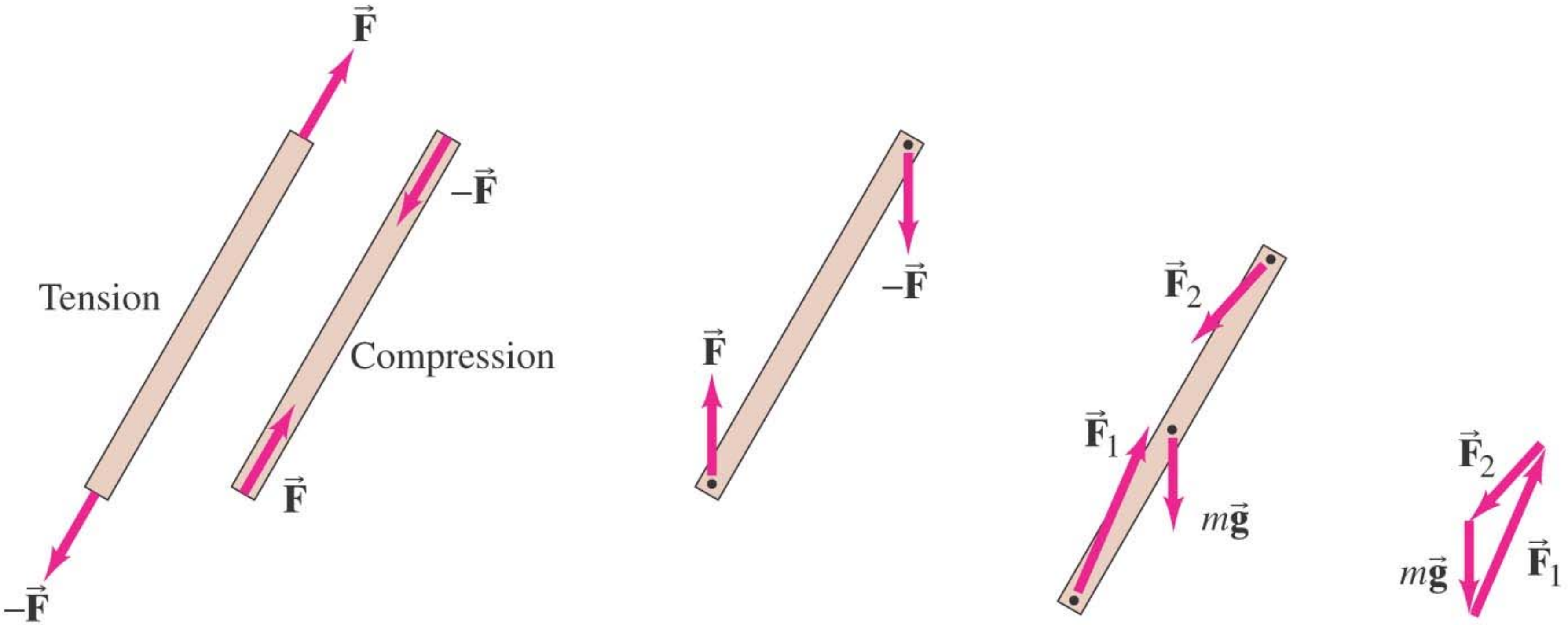
12-6 Trusses and Bridges

One way to span a wide space is to use a truss—a framework of rods or struts joined at their ends into triangles.



12-6 Trusses and Bridges

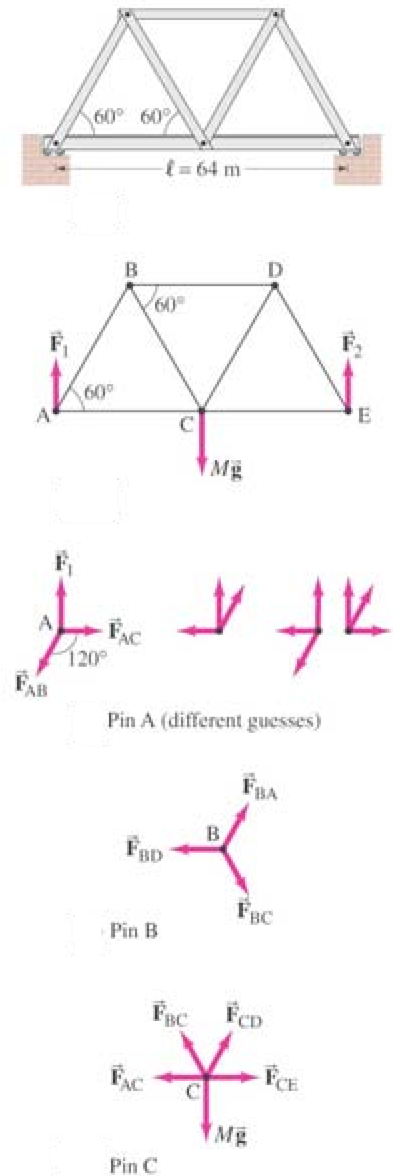
Each truss member is under either tension or compression; if the mass is small, these forces act along the strut.



12-6 Trusses and Bridges

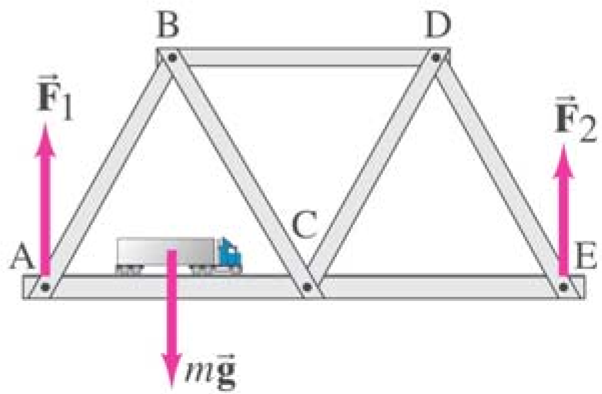
Example 12-11: A truss bridge.

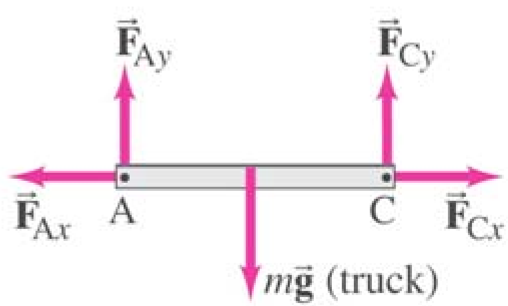
Determine the tension or compression in each of the struts of the truss bridge shown. The bridge is 64 m long and supports a uniform level concrete roadway whose total mass is 1.40 x 10^6 kg. Use the method of joints, which involves (1) drawing a free-body diagram of the truss as a whole, and (2) drawing a free-body diagram for each of the pins (joints), one by one, and setting $\Sigma \vec{F} = 0$ for each pin. Ignore the mass of the struts. Assume all triangles are equilateral.



12-6 Trusses and Bridges

On a real bridge, the load will not, in general, be centered. The maximum load rating for a bridge must take this into account.





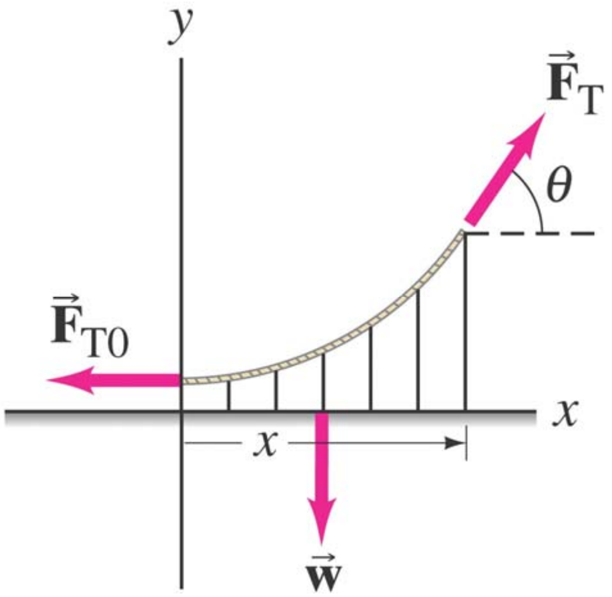
12-6 Trusses and Bridges

For larger bridges, trusses are too heavy. Suspension bridges are one solution; the roadway is suspended from towers by closely spaced vertical wires.

12-6 Trusses and Bridges

Example 12-12: Suspension bridge.

Determine the shape of the cable between the two towers of a suspension bridge, assuming the weight of the roadway is supported uniformly along its length. Ignore the weight of the cable.

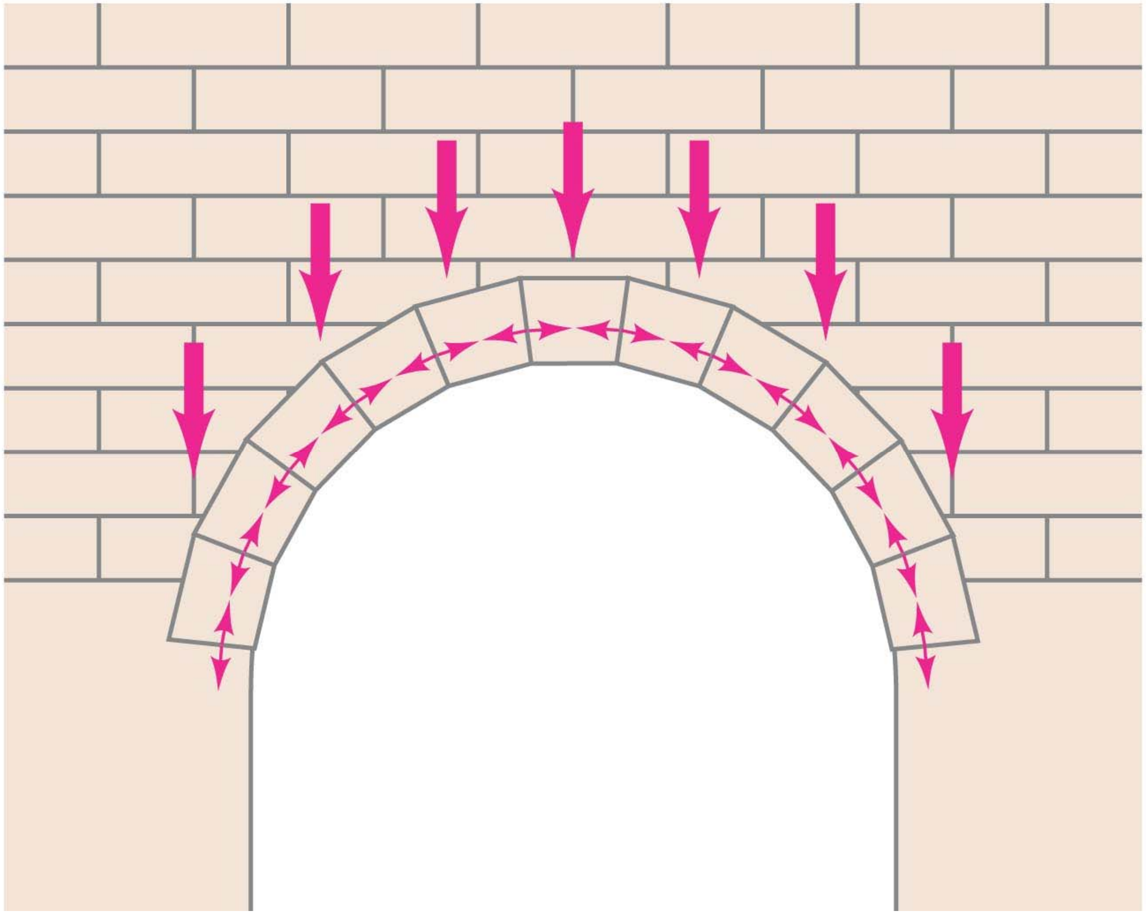


12-7: Arches and Domes

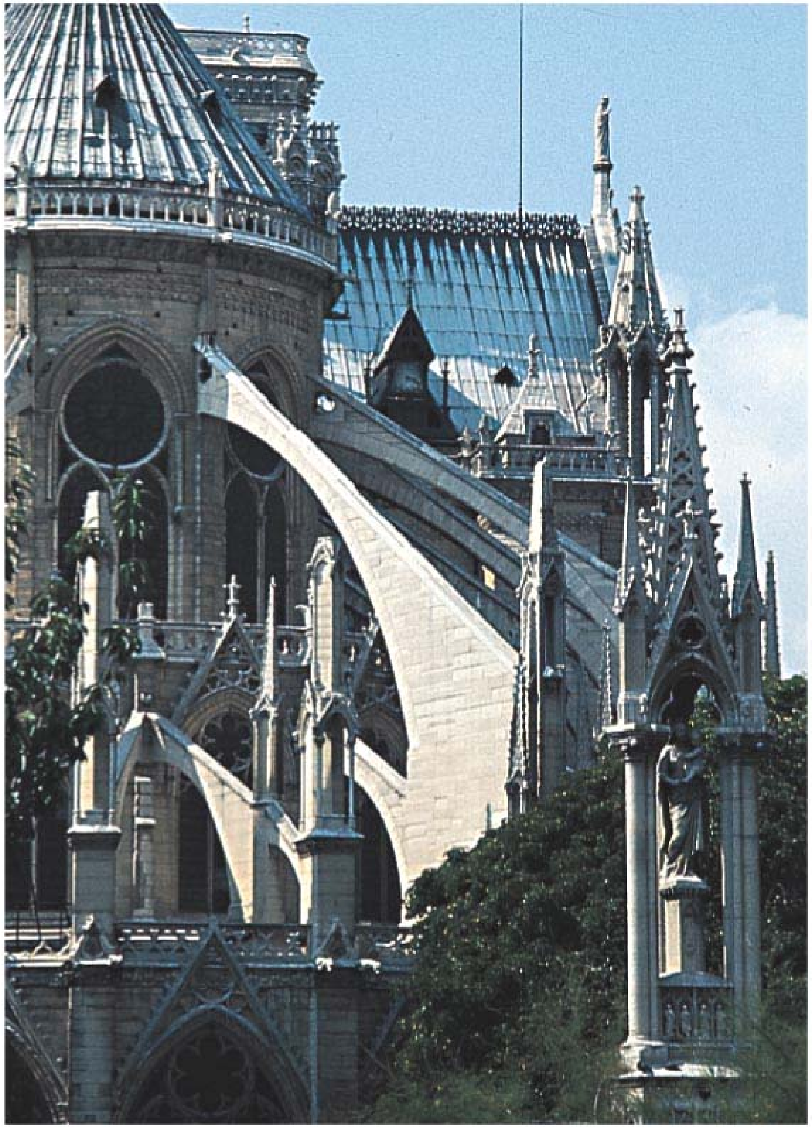
The Romans developed the **semicircular** arch about 2000 years ago. This allowed **wider** spans than could be built with **stone** or **brick** slabs.

12-7: Arches and Domes

The stones or bricks in a round arch are mainly under compression, which tends to strengthen the structure.



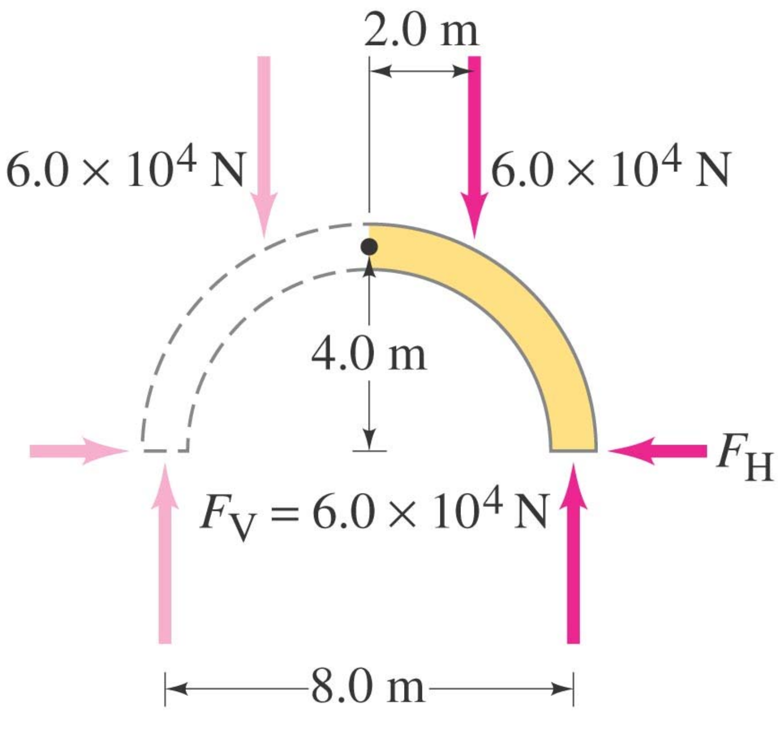
12-7: Arches and Domes

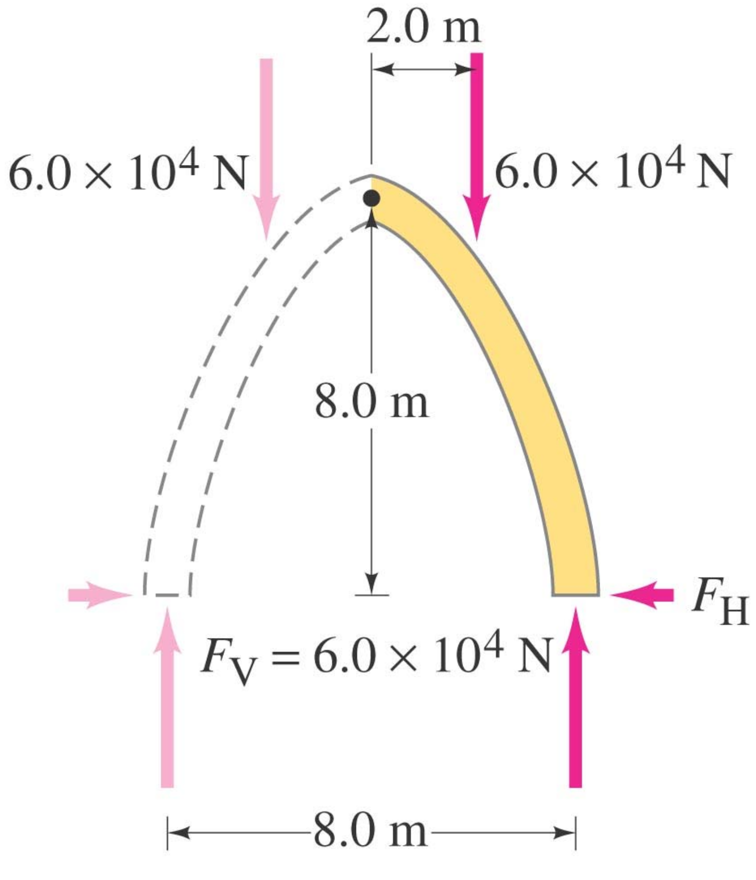


Unfortunately, the **horizontal** forces required for a semicircular arch can become quite large. The pointed arch was an improvement, but still needed external supports, or "**flying buttresses.**"

12-7: Arches and Domes

The **pointed** arches require considerably less horizontal force than a round arch.





12-7: Arches and Domes

A **dome** is similar to an **arch**, but spans a **two-dimensional** space.

Summary of Chapter 12

- An object at rest is in equilibrium; the study of such objects is called statics.

- In order for an object to be in equilibrium, there must be no net force on it along any coordinate, and there must be no net torque around any axis.

- An object in static equilibrium can be in stable, unstable, or neutral equilibrium.

Summary of Chapter 12

- Materials can be under compression, tension, or shear stress.

- If the force is too great, the material will exceed its elastic limit; if the force continues to increase, the material will fracture.