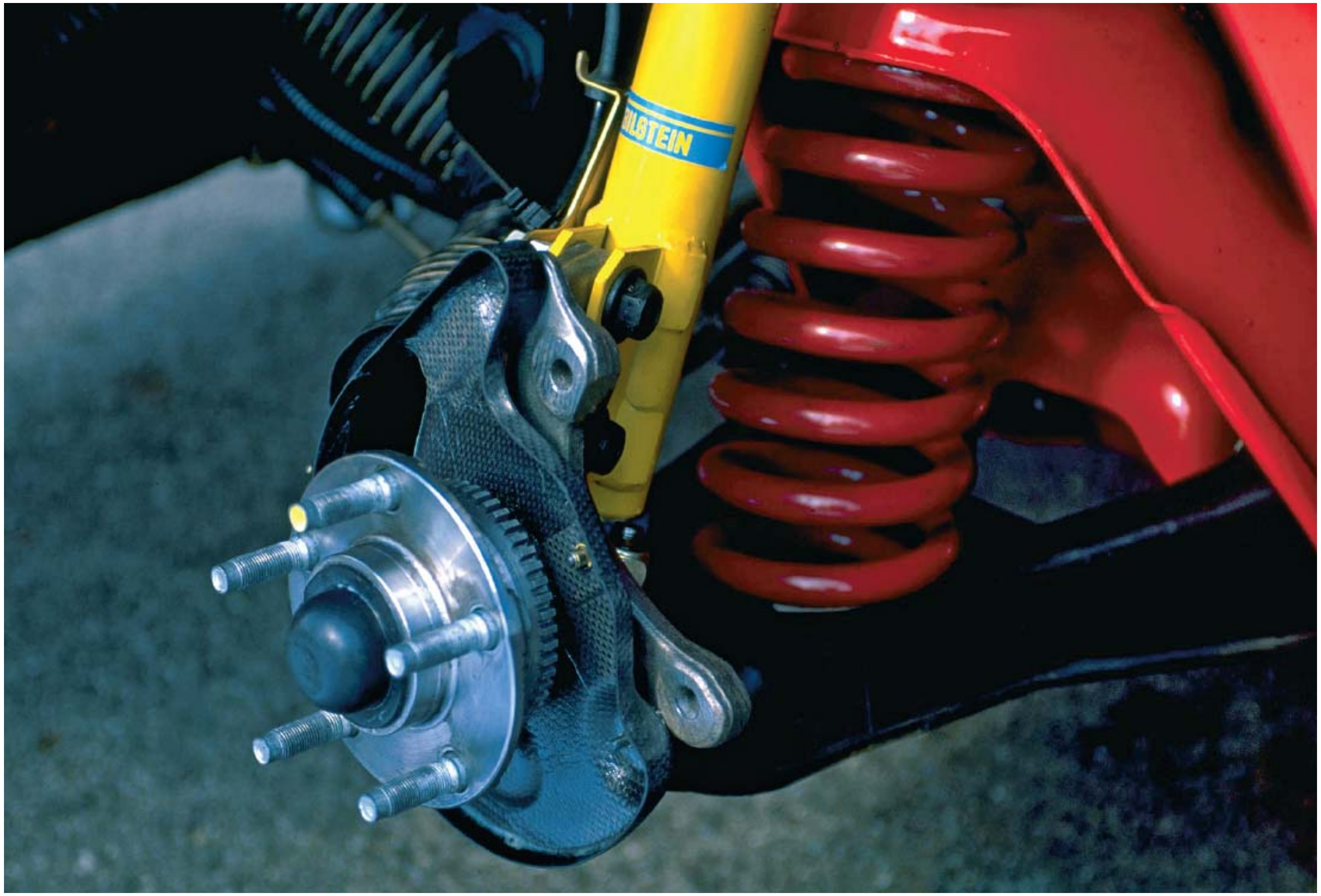
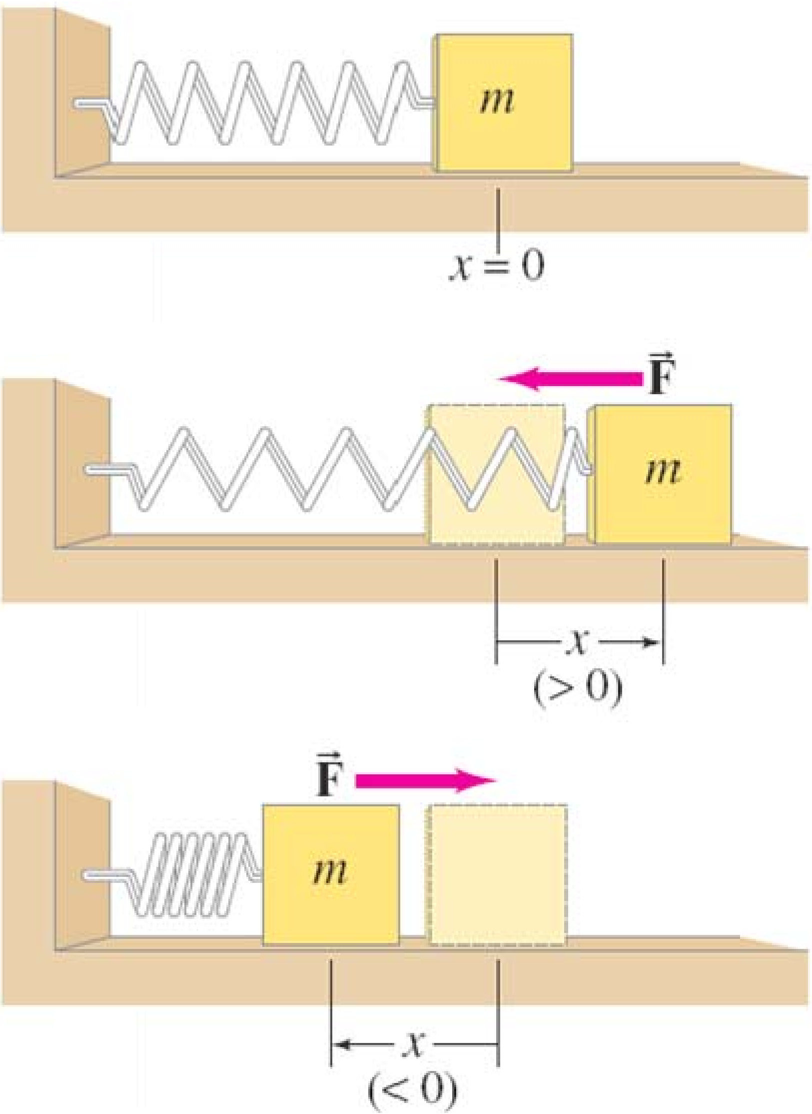
Chapter 14
Oscillations



Units of Chapter 14

- Oscillations of a Spring
- Simple Harmonic Motion
- Energy in the Simple Harmonic Oscillator
- Simple Harmonic Motion Related to Uniform Circular Motion
- The Simple Pendulum
- The Physical Pendulum and the Torsion Pendulum
- Damped Harmonic Motion
- Forced Oscillations; Resonance

14-1 Oscillations of a Spring



If an object **vibrates** or **oscillates** back and forth over the same path, each cycle taking the same amount of time, the motion is called **periodic**. The **mass** and **spring** system is a useful model for a periodic system.

14-1 Oscillations of a Spring

We assume that the surface is **frictionless**. There is a point where the spring is neither stretched nor compressed; this is the **equilibrium** position. We measure **displacement** from that point (x = 0 on the previous figure).

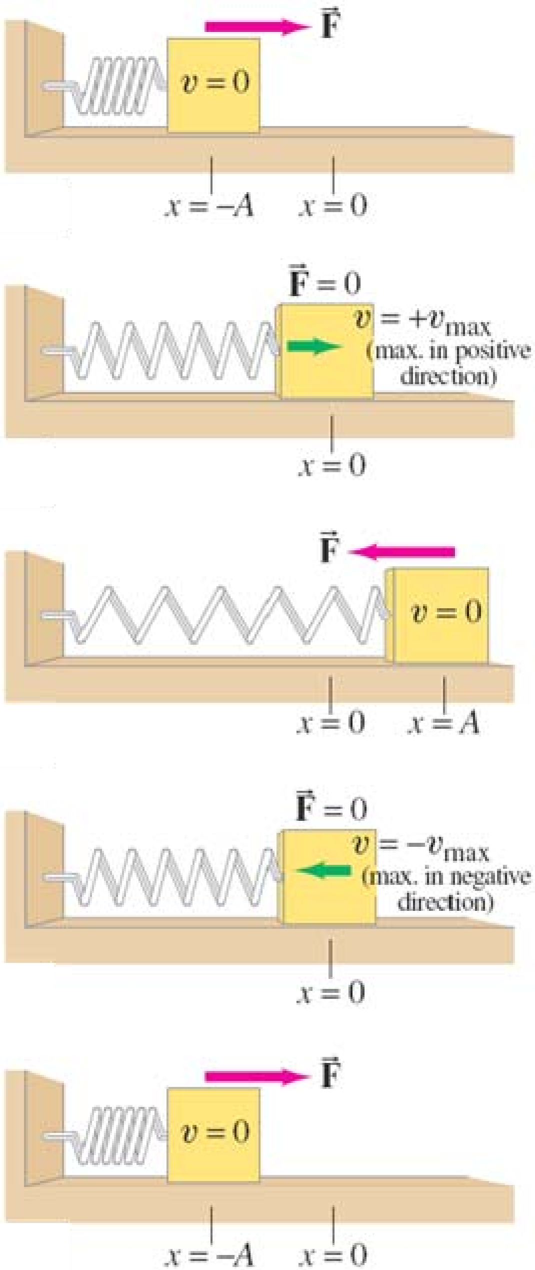
The **force** exerted by the spring depends on the **displacement**:

$$F = -kx.$$

14-1 Oscillations of a Spring

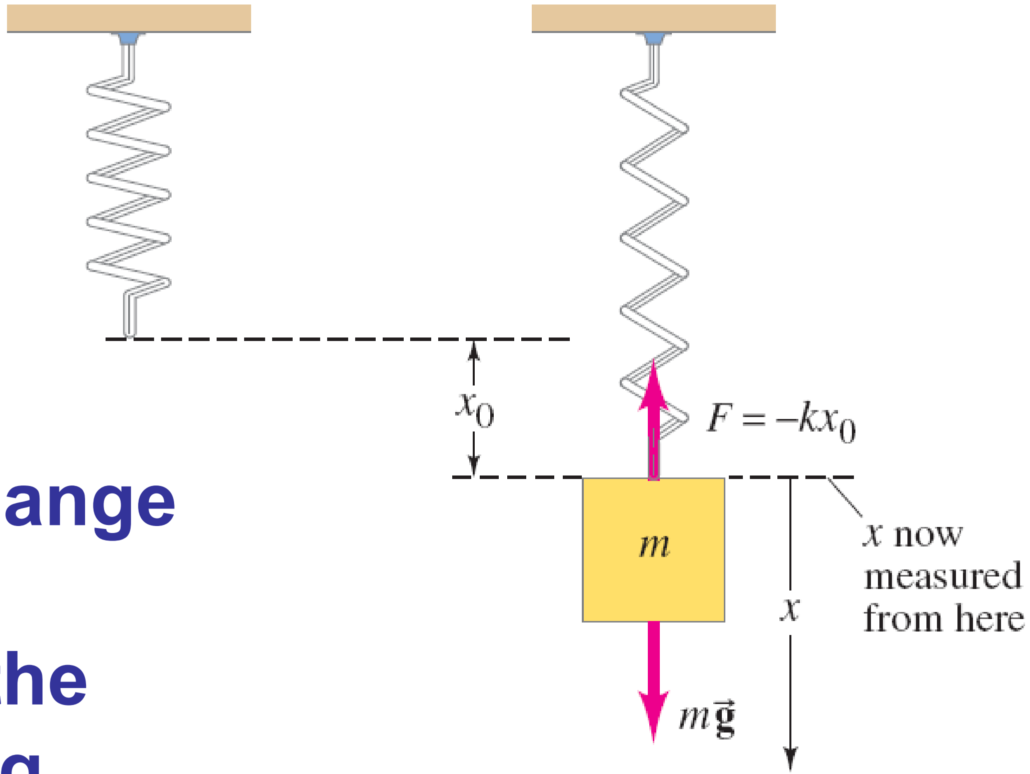
- The **minus** sign on the force indicates that it is a **restoring** force—it is directed to restore the mass to its **equilibrium** position.
- k is the **spring constant.**
- The force is **not constant**, so the acceleration is not constant either.

14-1 Oscillations of a Spring



- **Displacement** is measured from the **equilibrium** point.
- **Amplitude** is the **maximum** displacement.
- A **cycle** is a full to-and-fro motion.
- **Period** is the **time** required to complete one cycle.
- **Frequency** is the **number** of cycles completed per second.

14-1 Oscillations of a Spring



If the spring is hung **vertically**, the only change is in the **equilibrium position**, which is at the point where the spring force equals the gravitational force.

14-1 Oscillations of a Spring

Example 14-1: Car springs.

When a family of four with a total mass of 200 kg step into their 1200-kg car, the car's springs compress 3.0 cm. (a) What is the spring constant of the car's springs, assuming they act as a single spring? (b) How far will the car lower if loaded with 300 kg rather than 200 kg?

14-2 Simple Harmonic Motion

Any vibrating system where the restoring force is proportional to the negative of the displacement is in **simple harmonic motion** (**SHM**), and is often called a **simple harmonic oscillator** (**SHO**).

Substituting $F = kx$ into Newton's second law gives the equation of motion:

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0,$$

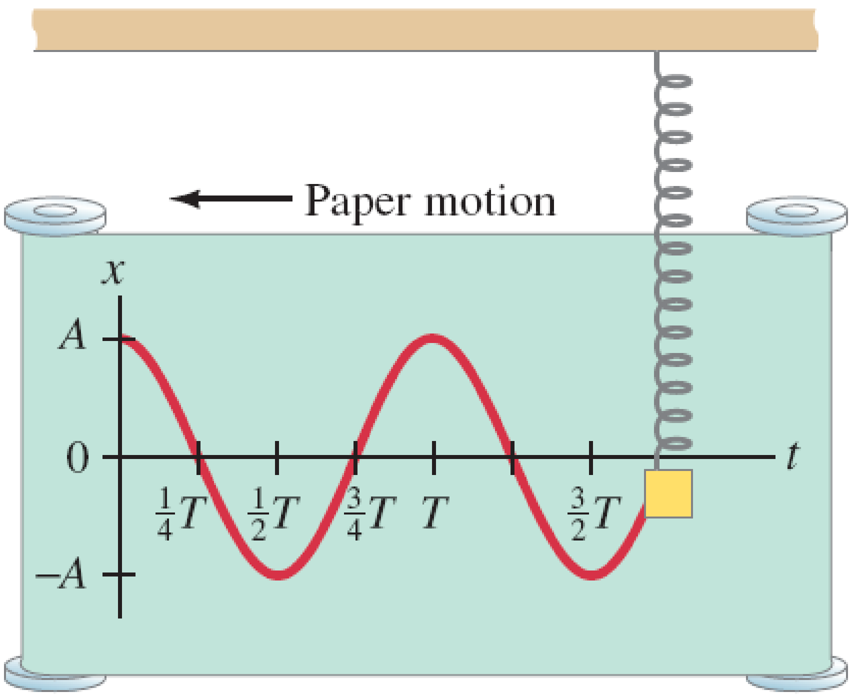
with solutions of the form:

$$x = A\cos(\omega t + \phi).$$

14-2 Simple Harmonic Motion

Substituting, we verify that this solution does indeed satisfy the equation of motion, with:

$$\omega^2 = \frac{k}{m}.$$



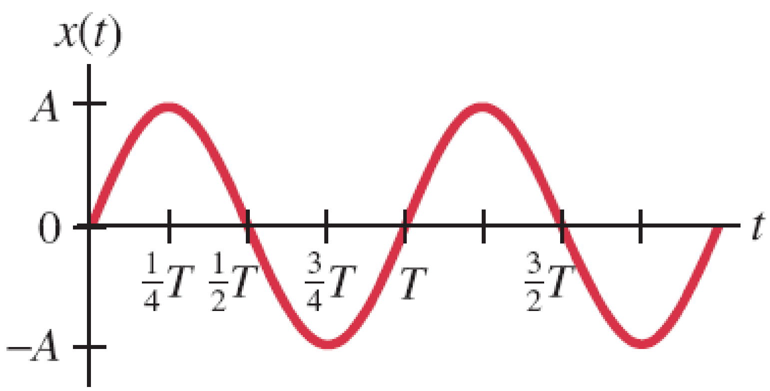
The constants A and φ will be determined by initial conditions; A is the amplitude, and φ gives the phase of the motion at $t = 0$.

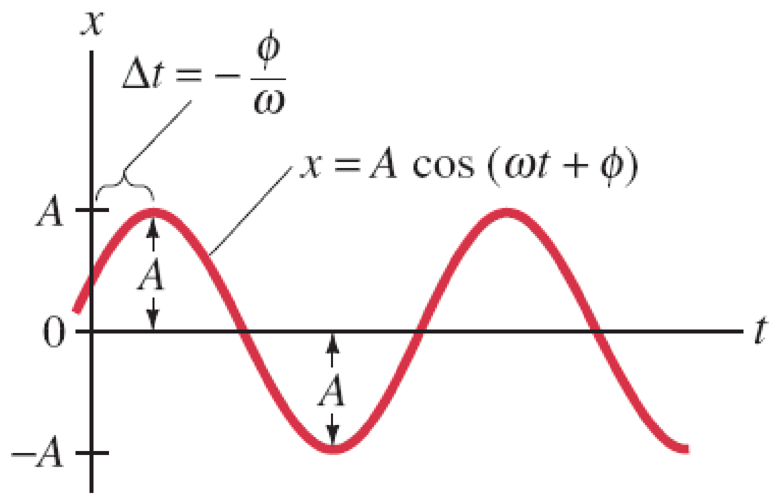
14-2 Simple Harmonic Motion

The velocity can be found by differentiating the displacement:

$$v = \frac{dx}{dt} = \frac{d}{dt}[A\cos(\omega t + \phi)] = -\omega A \sin(\omega t + \phi).$$

These figures illustrate the effect of φ:





14-2 Simple Harmonic Motion

Because $\omega = 2\pi f = \sqrt{k/m}$, **then**

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}},$$

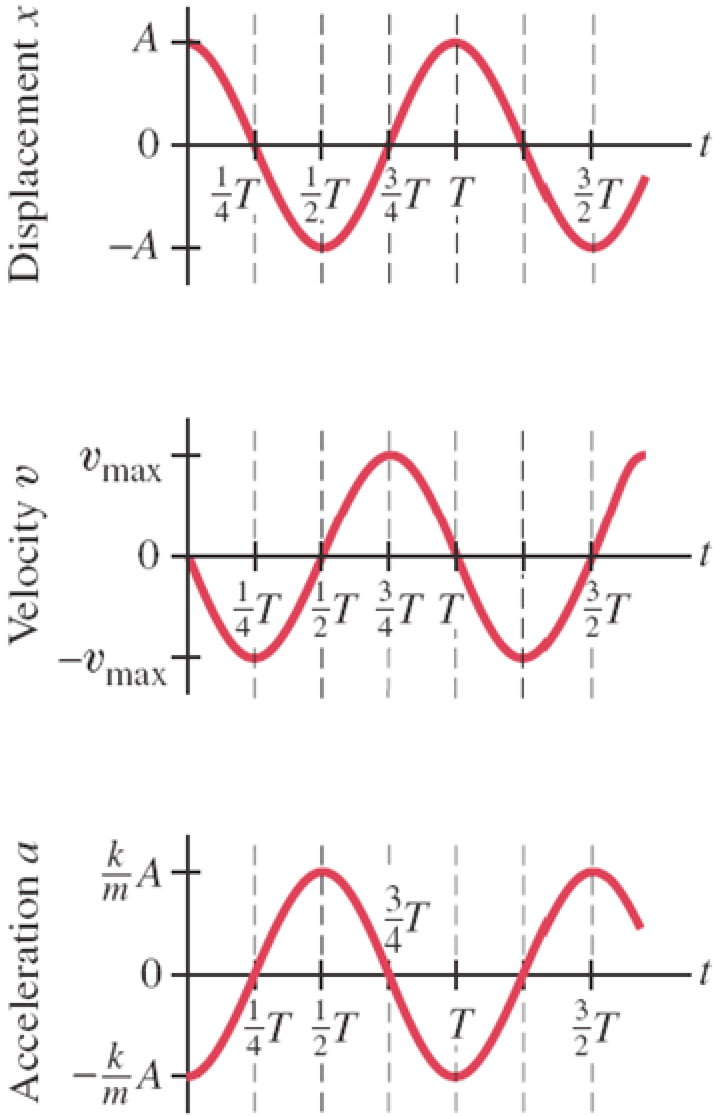
$$T = 2\pi\sqrt{\frac{m}{k}}.$$

14-2 Simple Harmonic Motion

Example 14-2: Car springs again.

Determine the period and frequency of a car whose mass is 1400 kg and whose shock absorbers have a spring constant of 6.5 x 10^4 N/m after hitting a bump. Assume the shock absorbers are poor, so the car really oscillates up and down.

14-2 Simple Harmonic Motion



The velocity and acceleration for simple harmonic motion can be found by differentiating the displacement:

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi).$$

14-2 Simple Harmonic Motion

Example 14-3: A vibrating floor.

A large motor in a factory causes the floor to vibrate at a frequency of 10 Hz. The amplitude of the floor's motion near the motor is about 3.0 mm. Estimate the maximum acceleration of the floor near the motor.

14-2 Simple Harmonic Motion

Example 14-4: Loudspeaker.

The cone of a loudspeaker oscillates in SHM at a frequency of 262 Hz ("middle C"). The amplitude at the center of the cone is $A = 1.5 \times 10^{-4}$ m, and at $t = 0$, $x = A$. (a) What equation describes the motion of the center of the cone? (b) What are the velocity and acceleration as a function of time? (c) What is the position of the cone at $t = 1.00$ ms ($= 1.00 \times 10^{-3}$ s)?

14-2 Simple Harmonic Motion

Example 14-5: Spring calculations.

A spring stretches 0.150 m when a 0.300-kg mass is gently attached to it. The spring is then set up horizontally with the 0.300-kg mass resting on a frictionless table. The mass is pushed so that the spring is compressed 0.100 m from the equilibrium point, and released from rest. Determine: (a) the spring stiffness constant k and angular frequency ω; (b) the amplitude of the horizontal oscillation A; (c) the magnitude of the maximum velocity v_{max}; (d) the magnitude of the maximum acceleration a_{max} of the mass; (e) the period T and frequency f; (f) the displacement x as a function of time; and (g) the velocity at $t = 0.150$ s.

14-2 Simple Harmonic Motion

Example 14-6: Spring is started with a push.

Suppose the spring of Example 14–5 (where ω = 8.08 s^{-1}) is compressed 0.100 m from equilibrium (x_0 = -0.100 m) but is given a shove to create a velocity in the +x direction of v_0 = 0.400 m/s. Determine (a) the phase angle φ, (b) the amplitude A, and (c) the displacement x as a function of time, $x(t)$.

14-3 Energy in the Simple Harmonic Oscillator

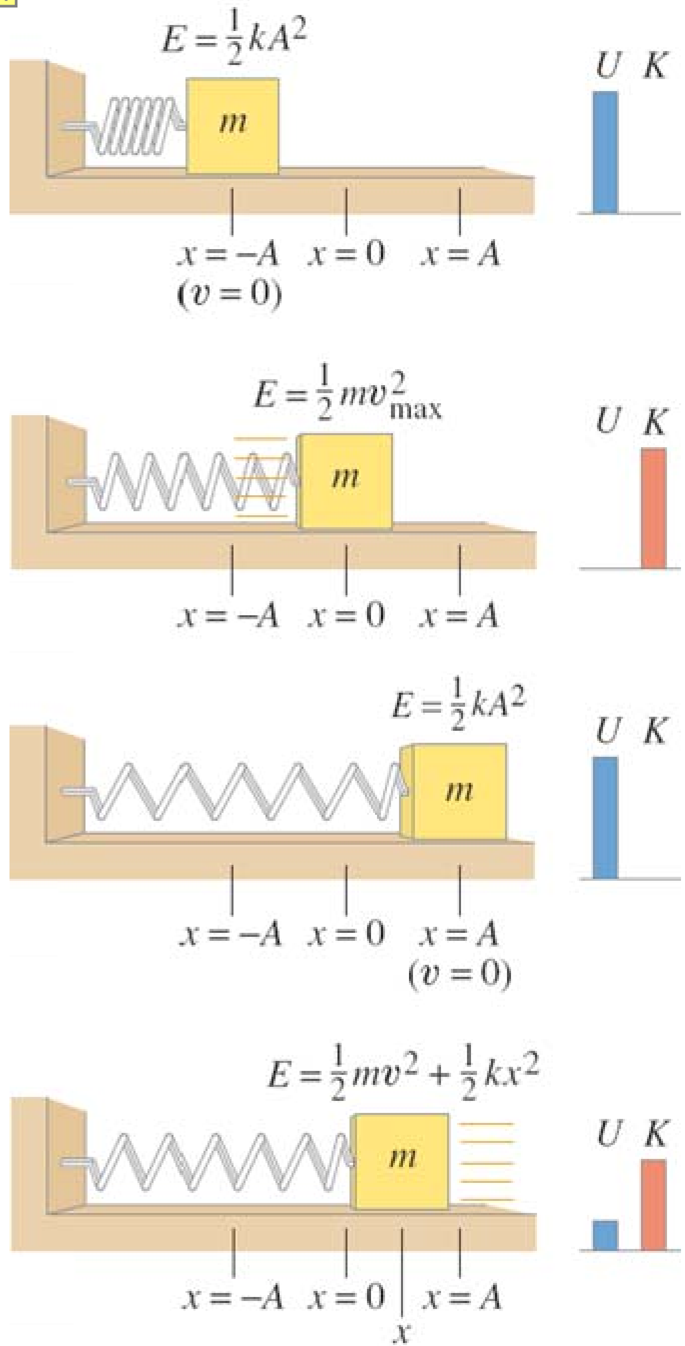
We already know that the **potential energy** of a spring is given by:

$$U = -\int F\,dx = \tfrac{1}{2}kx^2.$$

The **total mechanical energy** is then:

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2.$$

The total mechanical energy will be **conserved**, as we are assuming the system is **frictionless**.



14-3 Energy in the Simple Harmonic Oscillator

If the mass is at the **limits** of its motion, the energy is all **potential**.

If the mass is at the **equilibrium** point, the energy is all **kinetic**.

We know what the potential energy is at the turning points:

$$E = \frac{1}{2}kA^2.$$

14-3 Energy in the Simple Harmonic Oscillator

The **total energy** is, therefore, $\frac{1}{2}kA^2$.

And we can write:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2.$$

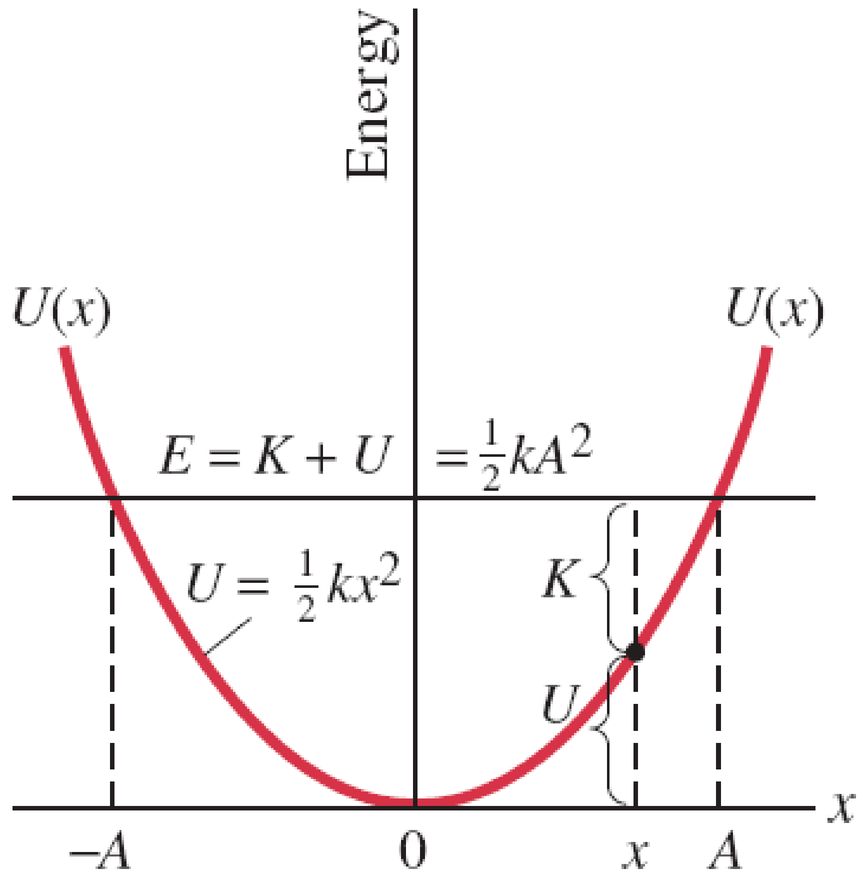
This can be solved for the **velocity** as a function of **position**:

$$v = \pm v_{\max}\sqrt{1 - \frac{x^2}{A^2}},$$

where $$v_{\max}^2 = (k/m)A^2.$$

14-3 Energy in the Simple Harmonic Oscillator

This graph shows the potential energy function of a spring. The total energy is constant.

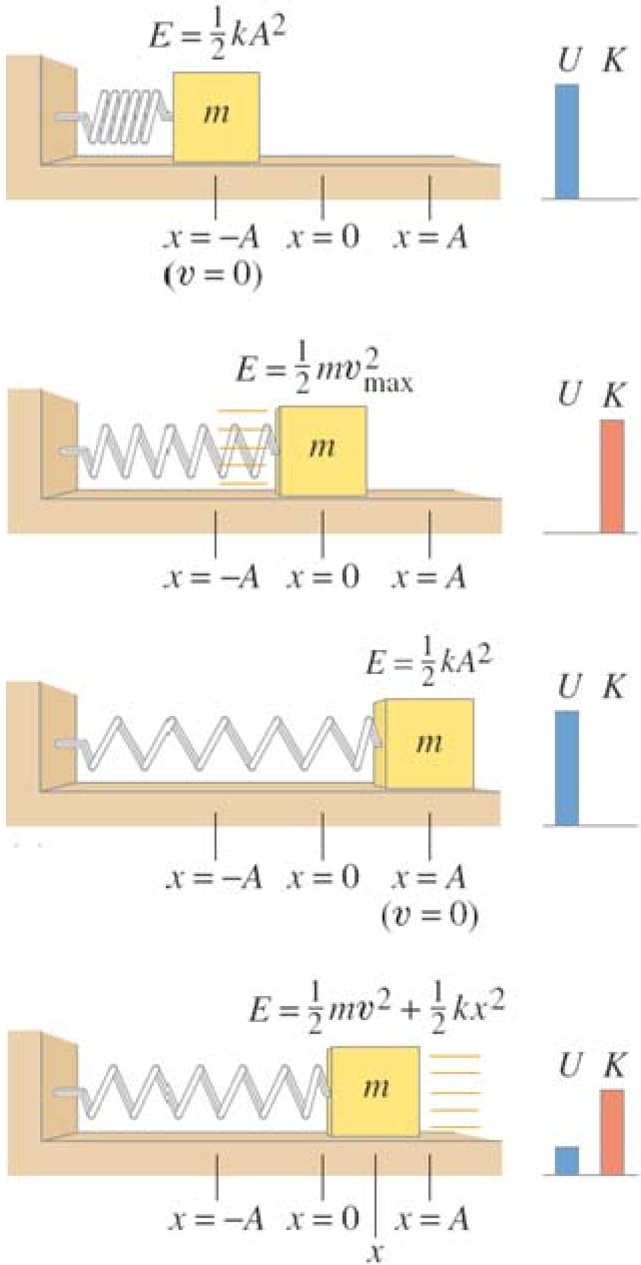


14-3 Energy in the Simple Harmonic Oscillator

Example 14-7: Energy calculations.

For the simple harmonic oscillation of Example 14–5 (where k = 19.6 N/m, A = 0.100 m, x = -(0.100 m) cos 8.08t, and v = (0.808 m/s) sin 8.08t), determine (a) the total energy, (b) the kinetic and potential energies as a function of time, (c) the velocity when the mass is 0.050 m from equilibrium, (d) the kinetic and potential energies at half amplitude ($x = \pm A/2$).

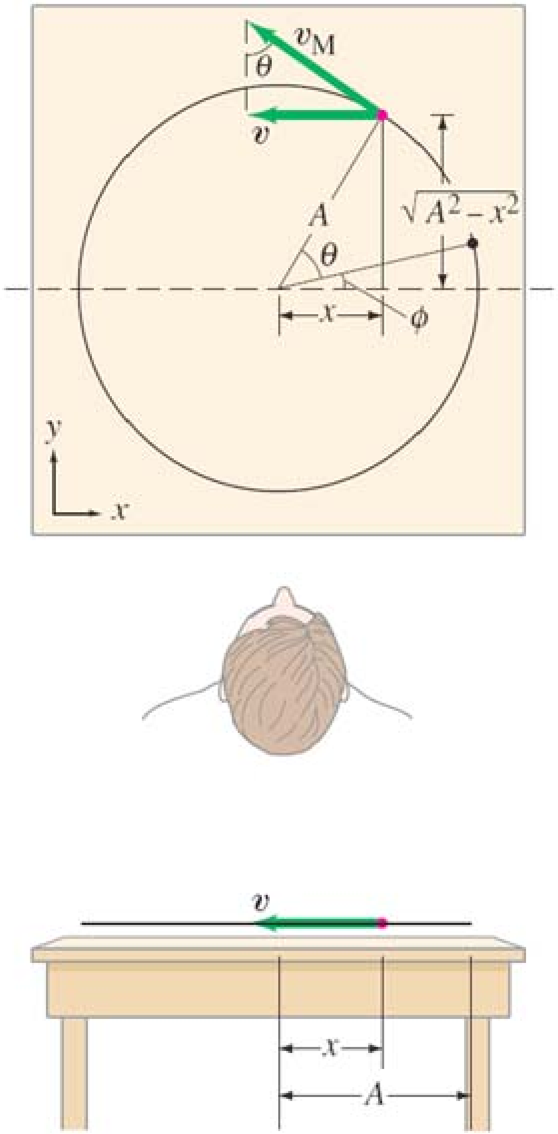
14-3 Energy in the Simple Harmonic Oscillator



Conceptual Example 14-8: Doubling the amplitude.

Suppose this spring is stretched twice as far (to $x = 2A$).What happens to (a) the energy of the system, (b) the maximum velocity of the oscillating mass, (c) the maximum acceleration of the mass?

14-4 Simple Harmonic Motion Related to Uniform Circular Motion

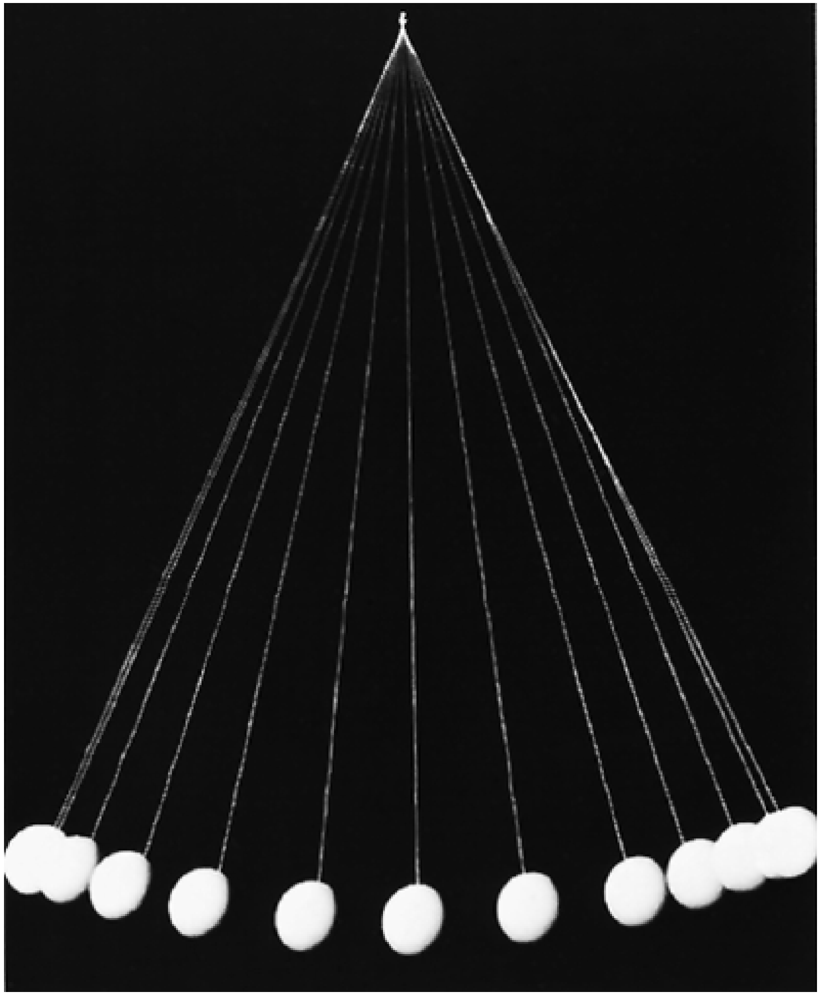


If we look at the **projection** onto the *x* axis of an object moving in a **circle** of radius *A* at a constant speed v_M, we find that the ***x* component** of its velocity varies as:

$$v = v_M \sqrt{1 - \frac{x^2}{A^2}}.$$

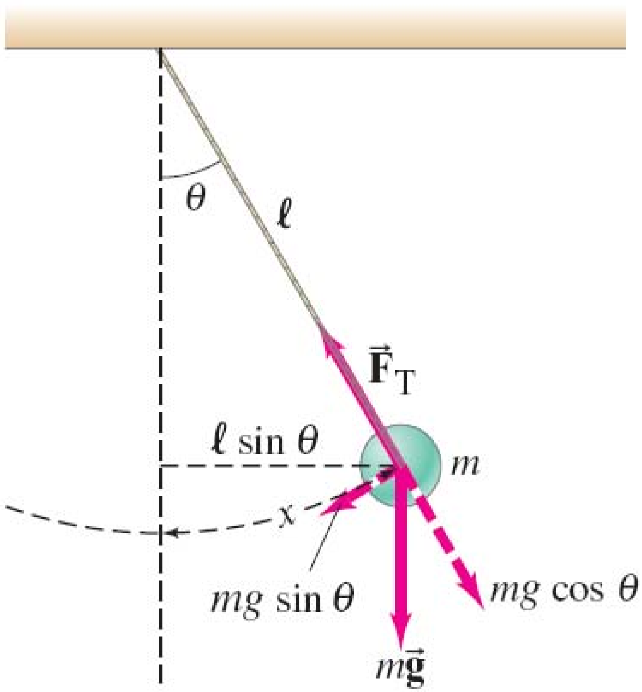
This is **identical** to SHM.

14-5 The Simple Pendulum



A **simple pendulum** consists of a mass at the end of a lightweight cord. We assume that the cord does not **stretch**, and that its mass is **negligible**.

14-5 The Simple Pendulum



In order to be in SHM, the restoring force must be **proportional** to the negative of the **displacement**. Here we have: $F = -mg\sin\theta,$

which is proportional to **sin *θ*** and not to *θ* itself.

However, if the angle is **small**, **sin *θ* ≈ *θ***.

14-5 The Simple Pendulum

Therefore, for small angles, we have:

$$F \approx -\frac{mg}{L}x,$$

where $x = L\theta.$

The period and frequency are:

$$T = 2\pi\sqrt{\frac{L}{g}},$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}.$$

14-5 The Simple Pendulum

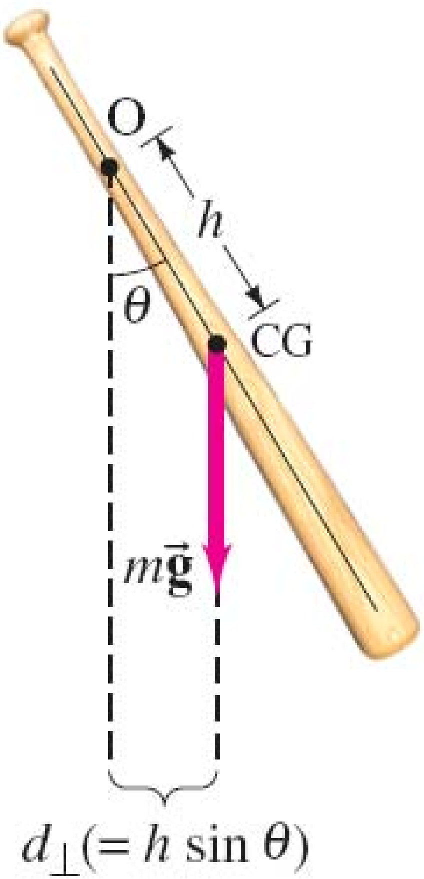
So, as long as the cord can be considered **massless** and the amplitude is **small**, the period does not depend on the **mass**.

14-5 The Simple Pendulum

Example 14-9: Measuring *g*.

A geologist uses a simple pendulum that has a length of 37.10 cm and a frequency of 0.8190 Hz at a particular location on the Earth. What is the acceleration of gravity at this location?

14-6 The Physical Pendulum and the Torsional Pendulum



A physical pendulum is any real extended object that oscillates back and forth.

The torque about point O is:

$$\tau = -mgh\sin\theta.$$

Substituting into Newton's second law gives:

$$I\frac{d^2\theta}{dt^2} = -mgh\sin\theta.$$

14-6 The Physical Pendulum and the Torsional Pendulum

For small angles, this becomes:

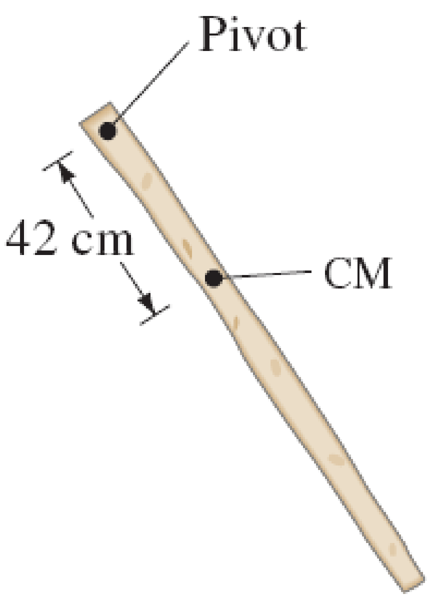
$$\frac{d^2\theta}{dt^2} + \left(\frac{mgh}{I}\right)\theta = 0,$$

which is the equation for SHM, with

$$\theta = \theta_{\max}\cos(\omega t + \phi),$$

$$T = 2\pi\sqrt{\frac{I}{mgh}}.$$

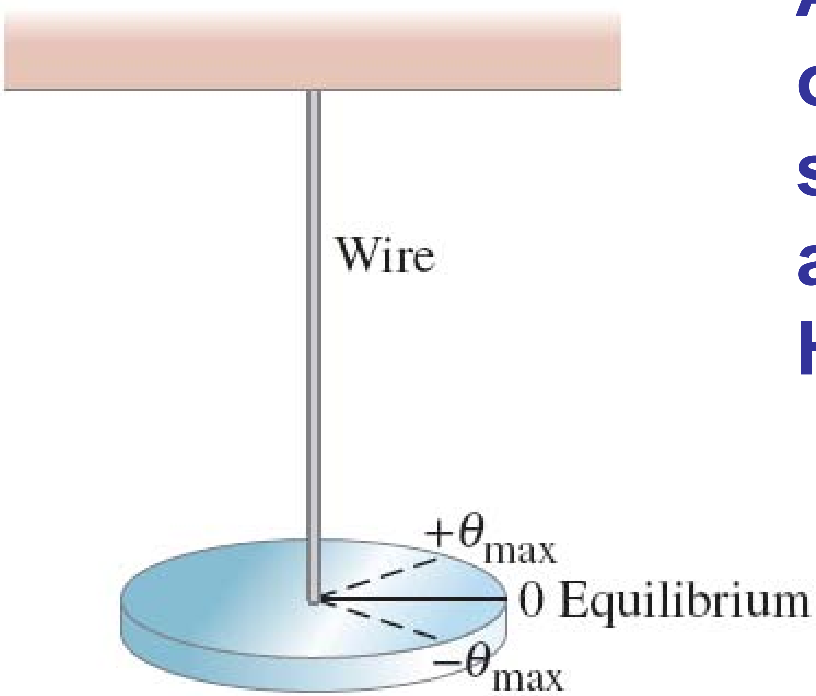
14-6 The Physical Pendulum and the Torsional Pendulum



Example 14-10: Moment of inertia measurement.

An easy way to measure the moment of inertia of an object about any axis is to measure the period of oscillation about that axis. (a) Suppose a nonuniform 1.0-kg stick can be balanced at a point 42 cm from one end. If it is pivoted about that end, it oscillates with a period of 1.6 s. What is its moment of inertia about this end? (b) What is its moment of inertia about an axis perpendicular to the stick through its center of mass?

14-6 The Physical Pendulum and the Torsional Pendulum



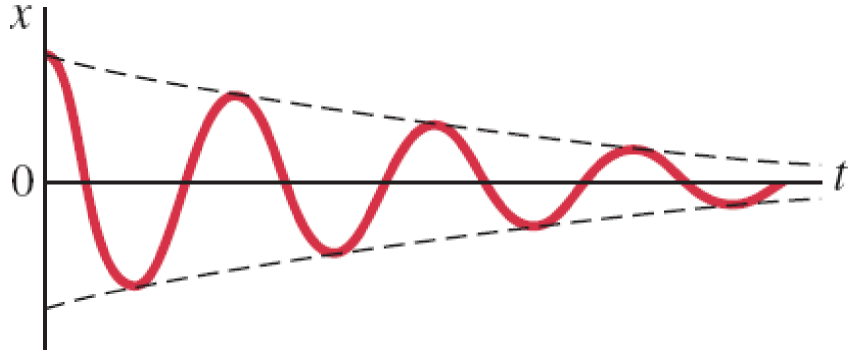
A torsional pendulum is one that twists rather than swings. The motion is SHM as long as the wire obeys Hooke's law, with

$$\omega = \sqrt{K/I}.$$

(K is a constant that depends on the wire.)

14-7 Damped Harmonic Motion

Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an "envelope" that modifies the undamped oscillation.



If $F_{\text{damping}} = -bv,$

then $ma = -kx - bv.$

14-7 Damped Harmonic Motion

This gives $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0.$

If b is small, a solution of the form

$$x = Ae^{-\gamma t}\cos\omega' t$$

will work, with

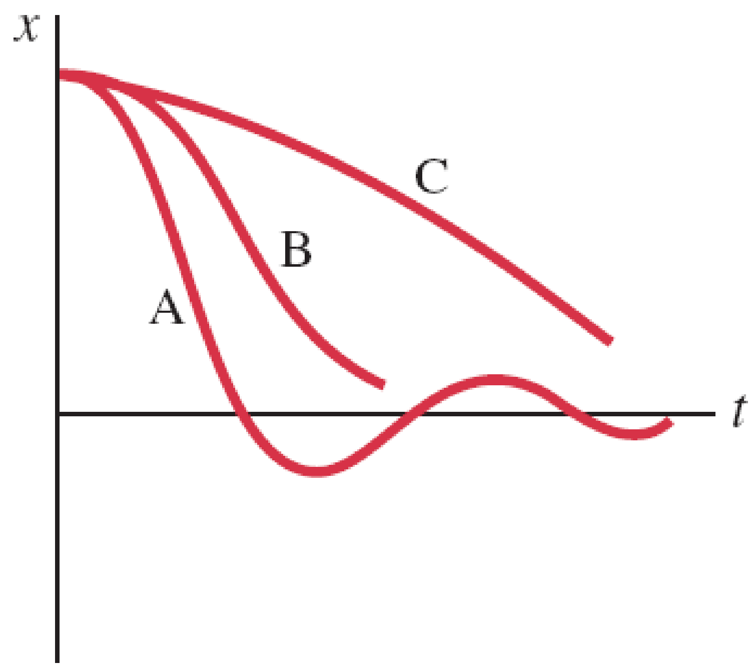
$$\gamma = \frac{b}{2m}$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

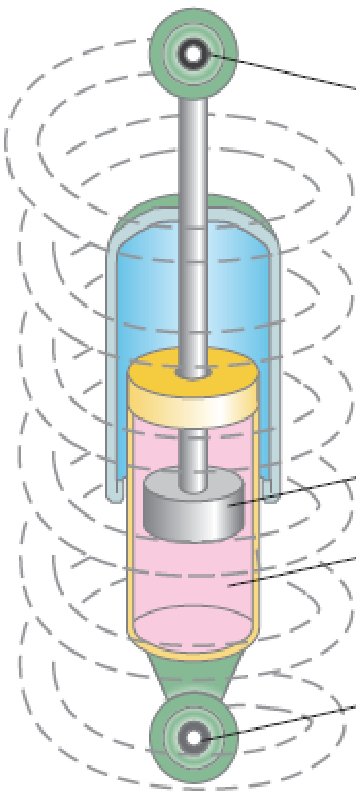
14-7 Damped Harmonic Motion

If $b^2 > 4mk$, ω' becomes imaginary, and the system is overdamped (C).

For $b^2 = 4mk$, the system is critically damped (B)—this is the case in which the system reaches equilibrium in the shortest time.



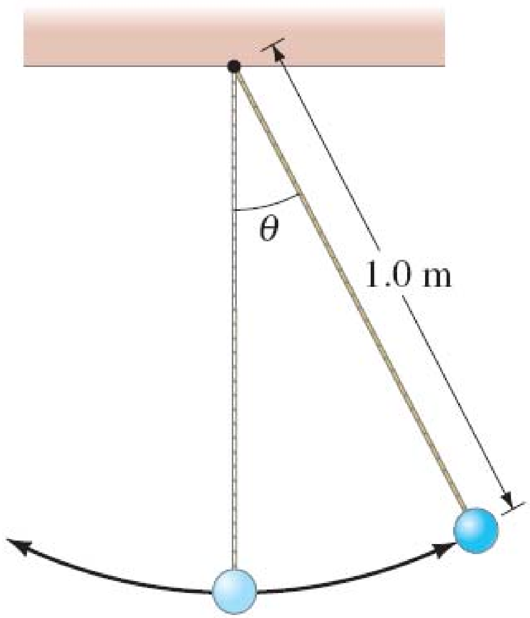
14-7 Damped Harmonic Motion



There are systems in which damping is **unwanted**, such as **clocks** and **watches**.

Then there are systems in which it is **wanted**, and often needs to be as close to critical damping as possible, such as **automobile shock absorbers** and **earthquake protection** for buildings.

14-7 Damped Harmonic Motion



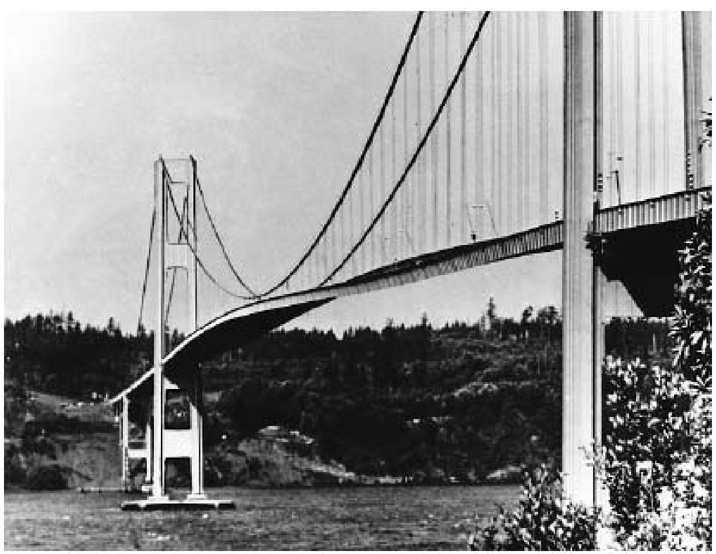
Example 14-11: Simple pendulum with damping.

A simple pendulum has a length of 1.0 m. It is set swinging with small-amplitude oscillations. After 5.0 minutes, the amplitude is only 50% of what it was initially. (a) What is the value of γ for the motion? (b) By what factor does the frequency, f', differ from f, the undamped frequency?

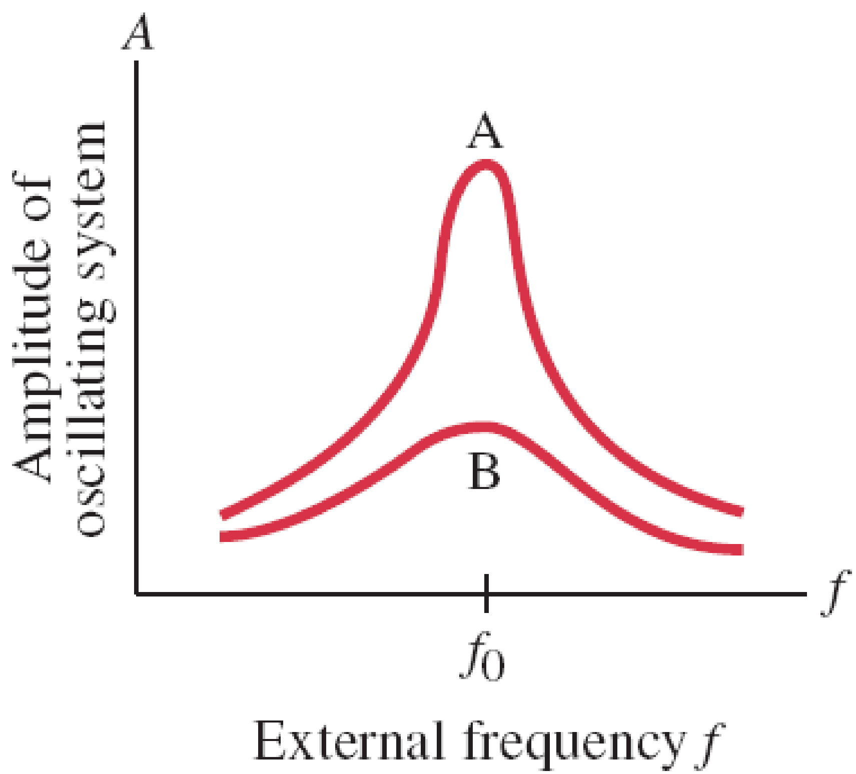
14-8 Forced Oscillations; Resonance

Forced vibrations occur when there is a **periodic** driving force. This force may or may not have the same period as the **natural frequency** of the system.

If the frequency is the same as the natural frequency, the **amplitude** can become quite large. This is called **resonance**.



14-8 Forced Oscillations; Resonance



The **sharpness** of the resonant peak depends on the damping. If the damping is **small** (A) it can be quite sharp; if the damping is **larger** (B) it is less sharp.

Like damping, resonance can be wanted or unwanted. **Musical instruments** and **TV/radio receivers** depend on it.

14-8 Forced Oscillations; Resonance

The equation of motion for a forced oscillator is:

$$ma = -kx - bv + F_0 \cos \omega t.$$

The solution is: $x = A_0 \sin(\omega t + \phi_0),$

where $A_0 = \dfrac{F_0}{m\sqrt{(\omega^2 - \omega_0^2)^2 + b^2\omega^2/m^2}}$

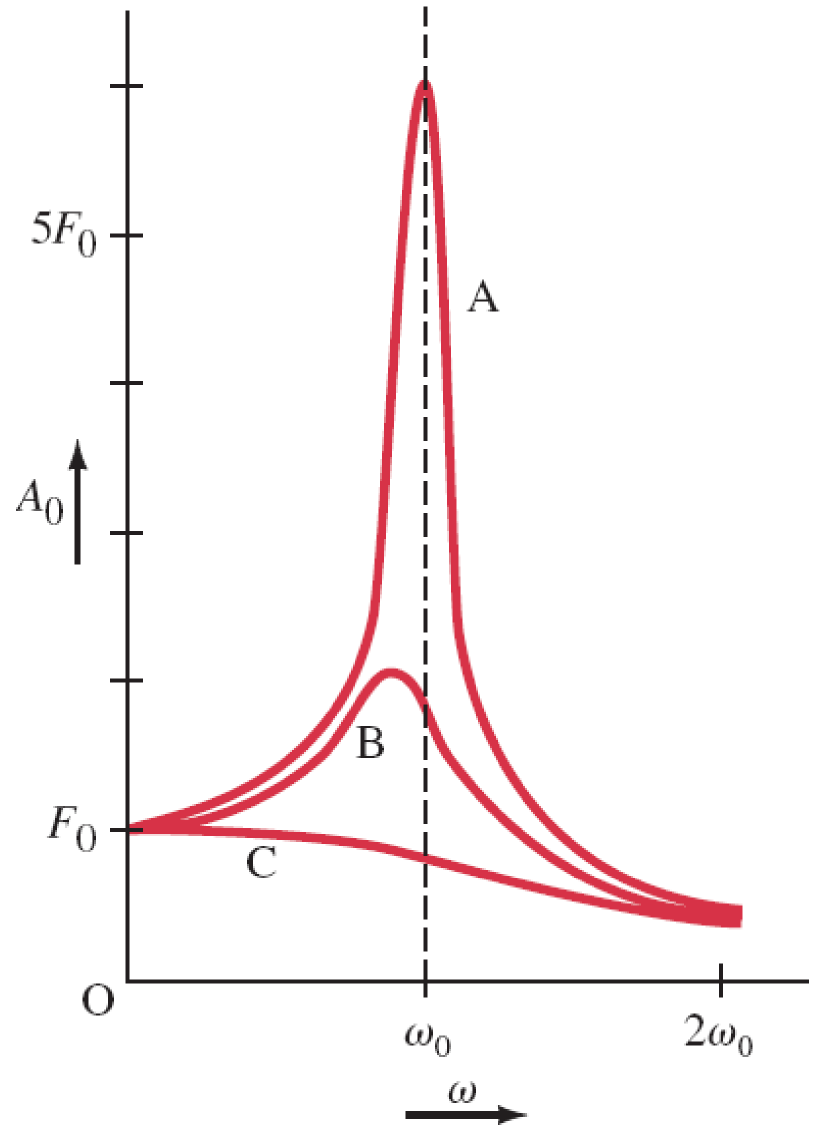
and

$$\phi_0 = \tan^{-1} \frac{\omega_0^2 - \omega^2}{\omega(b/m)}.$$

14-8 Forced Oscillations; Resonance

The width of the resonant peak can be characterized by the Q factor:

$$Q = \frac{m\omega_0}{b}.$$



Summary of Chapter 14

• For SHM, the restoring force is proportional to the displacement.

• The period is the time required for one cycle, and the frequency is the number of cycles per second.

• Period for a mass on a spring: $T = 2\pi\sqrt{\frac{m}{k}}.$

• SHM is sinusoidal.

• During SHM, the total energy is continually changing from kinetic to potential and back.

Summary of Chapter 14

- A simple pendulum approximates SHM if its amplitude is not large. Its period in that case is:

$$T = 2\pi\sqrt{\frac{L}{g}}.$$

- When friction is present, the motion is damped.

- If an oscillating force is applied to a SHO, its amplitude depends on how close to the natural frequency the driving frequency is. If it is close, the amplitude becomes quite large. This is called resonance.