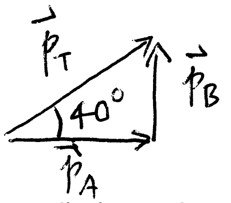
A E B E D

**Part I** (Multiple choice questions. 1 mark each. Please circle ***one*** answer for each question).

A E B E D

1. The area under the curve on a Force versus time (F vs. t) graph represents
A) impulse.
B) momentum.
C) work.
D) kinetic energy.
E) potential energy.

2. Two automobiles traveling at right angles to each other collide and stick together. Car A has a mass of 1200 kg and had a speed of 25 m/s before the collision. Car B has a mass of 1600 kg. The skid marks show that, immediately after the collision, the wreckage was moving in a direction making an angle of 40° with the original direction of car A. What was the speed of car B before the collision?
A) 14 m/s
B) 18 m/s
C) 11 m/s
D) 21 m/s
E) 16 m/s




$\tan 40^\circ = \frac{p_B}{p_A} = \frac{m_B v_B}{m_A v_A}$

$v_B = \frac{m_A v_A \cdot \tan 40^\circ}{m_B} = \frac{1200 \times 25 \times \tan 40^\circ}{1600} = 16$ m/s

3. Two equal forces are applied to a door. The first force is applied at the midpoint of the door; the second force is applied at the doorknob. Both forces are applied perpendicular to the door. Which force exerts the greater torque?
A) the first at the midpoint
B) the second at the doorknob
C) both exert equal non-zero torques
D) both exert zero torques
E) additional information is needed

$\tau = F_\perp \cdot r$

4. A disk, a hoop, and a solid sphere are released at the same time at the top of an inclined plane. They all roll without slipping. In what order do they reach the bottom?
A) disk, hoop, sphere
B) hoop, sphere, disk
C) sphere, hoop, disk
D) hoop, disk, sphere
E) sphere, disk, hoop

$I_{sphere} < I_{disk} < I_{hoop}$

$K_R + K_L = const = mgh$

$K_{RS} < K_{Rd} < K_{Rh}$

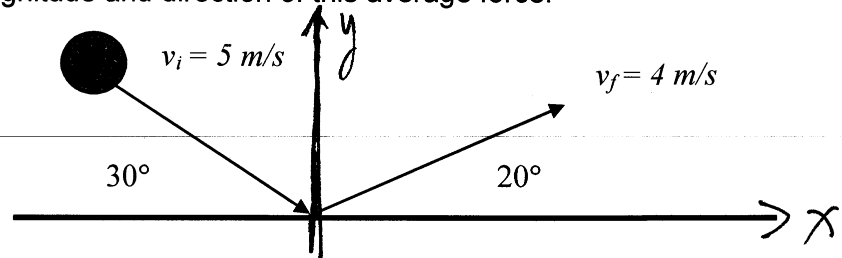
$K_{LS} > K_{Ld} > K_{Lh}$

$K_L \propto v^2$

5. A figure skater is spinning slowly with arms outstretched. She brings her arms in close to her body and her angular speed increases dramatically. The speed increase is a demonstration of
A) conservation of kinetic energy: her moment of inertia is decreased, and so her angular speed must increase to conserve energy.
B) conservation of total energy: her moment of inertia is decreased, and so her angular speed must increase to conserve energy.
C) Newton's second law for rotational motion: she exerts a torque and so her angular speed increases.
D) conservation of angular momentum: her moment of inertia is decreased, and so her angular speed must increase to conserve angular momentum.
E) This has nothing to do with mechanics, it is simply a result of her natural ability to perform.

**Part II** Full solution questions, **SHOW ALL WORK FOR FULL MARKS !**

(6) A 0.2 kg billiard ball bounces off the side of a pool table as shown. If the ball is in contact with the pool table cushion for 0.02s, find the average force exerted by the cushion on the ball. Find the magnitude and direction of this average force.




Pool table cushion

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{F}_{av}\,\Delta t$$

$$\vec{F} = \frac{1}{\Delta t}(\vec{p}_f - \vec{p}_i)$$

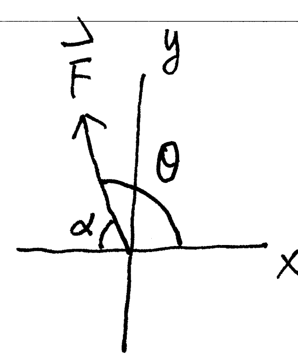
x-comp: $F_x = \frac{1}{\Delta t}\cdot(p_{fx} - p_{ix}) = \frac{m}{\Delta t}(v_f \cos 20 - v_i \cos 30°)$

$$= \frac{0.2}{0.02}(4\cos 20° - 5\cos 30°) = -5.71\ N$$

y-comp: $F_y = \frac{1}{\Delta t}(p_{fy} - p_{iy}) = \frac{m}{\Delta t}[v_f \sin 20° - (-v_i \sin 30°)]$

$$= \frac{0.2}{0.02}[4\sin 20° + 5\sin 30°) = 38.7\ N$$

magnitude: $F = \sqrt{F_x^2 + F_y^2} = 39.1\ N$



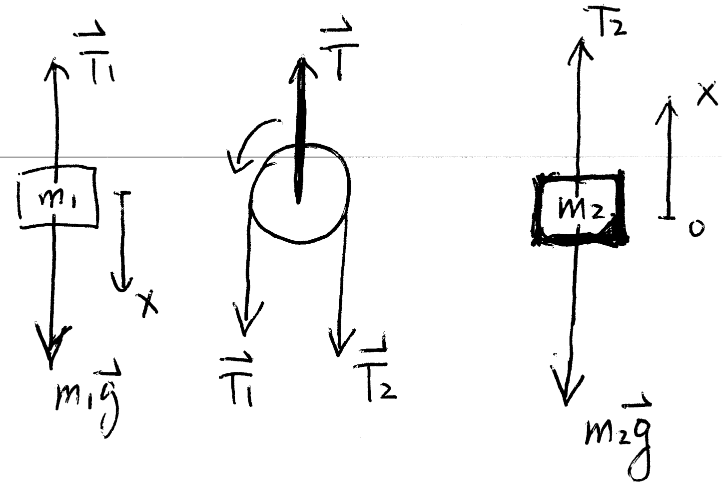

$$\tan\theta = \frac{38.7}{-5.71}$$

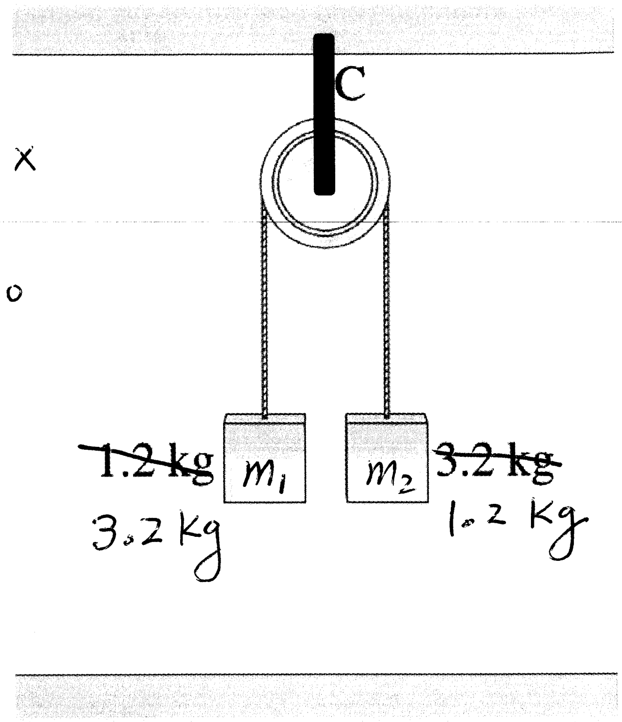
$$\theta = 180° - 81.6° = 98.4°$$

$$\left(\alpha = \tan^{-1}\frac{38.7}{5.71} = 81.6°,\quad \theta = 180° - \alpha\right)$$

(7) Suppose the pulley in the figure has a radius of 0.20m and a moment of inertia of $0.12 \mathrm{kg \cdot m^2}$. When the masses are released, the pulley will turn without slipping. Ignore the mass of the cable. Find the angular acceleration of the pulley.

Free-Body Diagrams:




$$\begin{cases} m_1 g - T_1 = m_1 a & (1) \\ T_2 - m_2 g = m_2 a & (2) \\ T_1 r - T_2 r = I\alpha & (3) \\ a = \alpha r & (4) \end{cases}$$

Solve for $\alpha$: (1)+(2): $T_2 - T_1 + g(m_1 - m_2) = a(m_1 + m_2)$

(3): $T_1 - T_2 = \dfrac{I\alpha}{r}$

+

$$g(m_1 - m_2) = a(m_1 + m_2) + \frac{I\alpha}{r}.$$

sub (4) $\Rightarrow$: $g(m_1 - m_2) = \alpha r(m_1 + m_2) + \dfrac{I\alpha}{r}$.

$$\alpha = \frac{rg(m_1 - m_2)}{I + (m_1 + m_2)r^2}$$

$$= \frac{(0.2)(9.8)(3.2 - 1.2)}{0.12 + (3.2 + 1.2)(0.2)^2}$$

$$= \frac{3.92}{0.296}$$

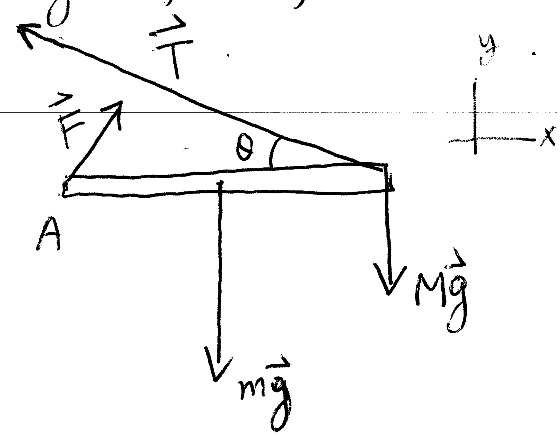
$$= 13.2 \ \mathrm{rad/s^2}$$

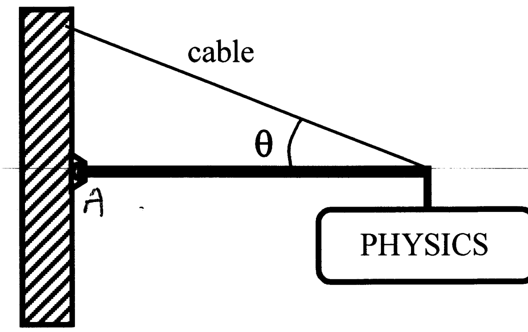
(8) A uniform beam, 1.50 m long with mass $m$ = 4.00 kg, is mounted by a small hinge on a wall. The beam is held in a horizontal position by a cable that makes an angle $\theta$ = 25.0°. The beam supports a sign of mass $M$ = 1.00 kg suspended from its end.

(a) Find the tension in the cable.

(b) Find the magnitude of the force exerted by the hinge on the beam.

Free-body diagram of the beam.




$\sum \vec{F} = 0$ : x: $F_x - T\cos\theta = 0$ ①

y: $F_y - mg - Mg + T\sin\theta = 0$ ②

$\sum \tau = 0$ — about pivot point A : $TL\sin\theta - mg\frac{L}{2} - MgL = 0$ ③

Solve ③ for T: $T = \dfrac{g(\frac{m}{2} + M)}{\sin\theta} = \dfrac{9.8\,(2+1)}{\sin 25°} = 69.6\ N$

①: $F_x = T\cos\theta = 69.6 \cdot \cos 25° = 63.0\ N$

②: $F_y = g\,(m+M) - T\sin\theta$

$= 9.8\,(4+1) - 69.6\ \sin 25°$

$= 19.6\ N$

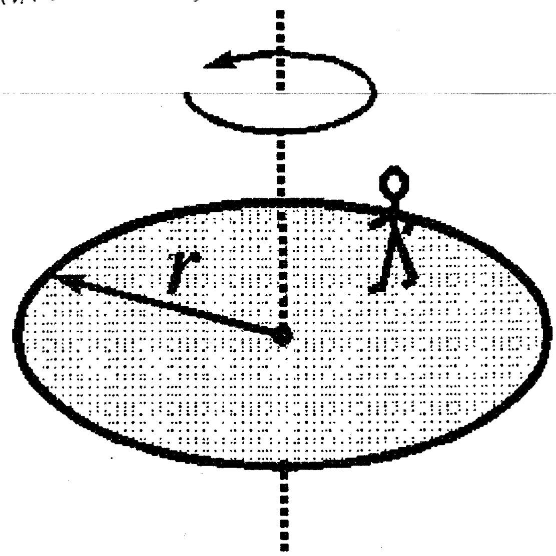
$F = \sqrt{F_x^2 + F_y^2} = 66\ N$

(9) A horizontal platform with a radius of 5.00 m rotates about a frictionless vertical axle. The moment of inertia of the platform about the axle is 1000 kg·m$^2$. A student (m=80.0 kg) walks slowly from the rim of the platform toward the center and stops when he is at the centre. The initial angular velocity ω of the system is 3.00 rad/s when the student is at the rim. (You may treat the person as a point mass)

(a) Find the angular speed when the student is at the center.

(b) Find the work done by the student.

(a). conservation of angular momentum (since $\Sigma\tau = 0$)




$$I_i \omega_i = I_f \omega_f .$$

$$\omega_f = \frac{I_i}{I_f}\omega_i .$$

$$I_i = 1000\ \text{kg}\cdot\text{m}^2 + mr^2$$
$$= 1000 + 80 \times 5^2$$
$$= 3000\ \text{kg}\cdot\text{m}^2$$
$$I_f = 1000\ \text{kg}\cdot\text{m}^2 .$$

$$\therefore\ \omega_f = \frac{I_i}{I_f}\omega_i = \frac{3000}{1000}(3) = 9\ \text{rad/s} .$$

(b). $$W = \Delta K = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i^2$$
$$= \frac{1}{2} \times 1000 \times 9^2 - \frac{1}{2} \times 3000 \times 3^2$$
$$= 27000\ \text{J} .$$