Phys101 Lecture 7,8
Circular Motion, Gravity

Key points:

• Centripetal acceleration
• Uniform Circular Motion – dynamics
• Gravity

*Ref: 5-1,2,3,5,6,7,8.*
Highway Curves: Banked and Unbanked

When a car goes around a **curve**, there must be a net force toward the center of the circle of which the curve is an arc. If the road is flat, that force is supplied by **friction**.
Highway Curves: Banked and Unbanked

If the frictional force is insufficient, the car will tend to move more nearly in a straight line, as the skid marks show.
Banking the curve can help keep cars from skidding. When the curve is banked, the centripetal force can be supplied by the horizontal component of the normal force. In fact, for every banked curve, there is one speed at which the entire centripetal force is supplied by the horizontal component of the normal force, and no friction is required.
Example: Banking angle.

(a) For a car traveling with speed $v$ around a curve of radius $r$, determine a formula for the angle at which a road should be banked so that no friction is required.

(b) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

(a) $F_N \sin \theta = m \frac{v^2}{R}$

$F_N \cos \theta - mg = 0$

$tan \theta = \frac{v^2}{Rg}$

(b) $R = 50 \text{ m}$, $v = 50 \text{ km/h} = 13.89 \text{ m/s}$

$\theta = tan^{-1} \frac{v^2}{Rg} = tan^{-1} \frac{13.89^2}{50g} = 22^\circ$
A ping pong ball is shot into a circular tube that is lying flat (horizontal) on a tabletop. When the ping pong ball leaves the track, which path will it follow?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5
You drive your dad’s car too fast around a curve and the car starts to skid. What is the correct description of this situation?

(A) car’s engine is not strong enough to keep the car from being pushed out

(B) friction between tires and road is not strong enough to keep car in a circle

(C) car is too heavy to make the turn

(D) none of the above
Two equal-mass rocks tied to strings are whirled in horizontal circles. The radius of circle 2 is twice that of circle 1. If the period of motion is the same for both rocks, what is the tension in cord 2 compared to cord 1?

A) \( T_2 = \frac{1}{4} T_1 \)
B) \( T_2 = \frac{1}{2} T_1 \)
C) \( T_2 = T_1 \)
D) \( T_2 = 2 T_1 \)
E) \( T_2 = 4 T_1 \)
Example: Revolving ball (vertical circle)

A 0.150-kg ball on the end of a 1.10-m long cord (negligible mass) is swung in a vertical circle. Determine the minimum speed the ball must have at the top of its arc so that the ball continues moving in a circle.

\[
\begin{align*}
\text{\textbf{Solution:}} \\
& \text{At the top of the arc, the centripetal force is provided by the tension in the cord.} \\
& \text{Centripetal force:} \quad F_{\text{c}} = m \frac{v^2}{r} \\
& \text{Tension at top:} \quad F_{\text{T}} = m \frac{v^2}{r} \\
& \text{Since the tension is maximum at the top, we use:} \\
& F_{\text{T}} = mg \\
& \text{Thus, the minimum speed at the top is:} \\
& v = \sqrt{gr} \\
& v = \sqrt{1.10 \times 9.81} \\
& v \approx 3.30 \text{ m/s}
\end{align*}
\]
Newton’s Universal Gravitation

\[ F = G \frac{m_1 m_2}{r^2}, \]

\[ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2. \]

In vector form:

\[ \vec{F}_{12} = -G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21}. \]

Direction: attractive
Gravity Near the Earth’s Surface

On the surface of the Earth, an object with mass \( m \) experiences the gravitational force due to the earth

\[
G \frac{M_E m}{R_E^2} = mg
\]

\[
g = G \frac{M_E}{R_E^2} = 9.80 \, m / s^2
\]
Satellites are routinely put into orbit around the Earth. The tangential speed must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth’s gravity altogether. If a satellite can be considered in uniform circular motion around the earth, the gravity due to the earth must equal to the centripetal force.

\[ G \frac{M_E m}{R^2} = m \frac{v^2}{R} \]

See example 5-14 on page 123