Phys101 Lectures 9

Work and Kinetic Energy

Key points:

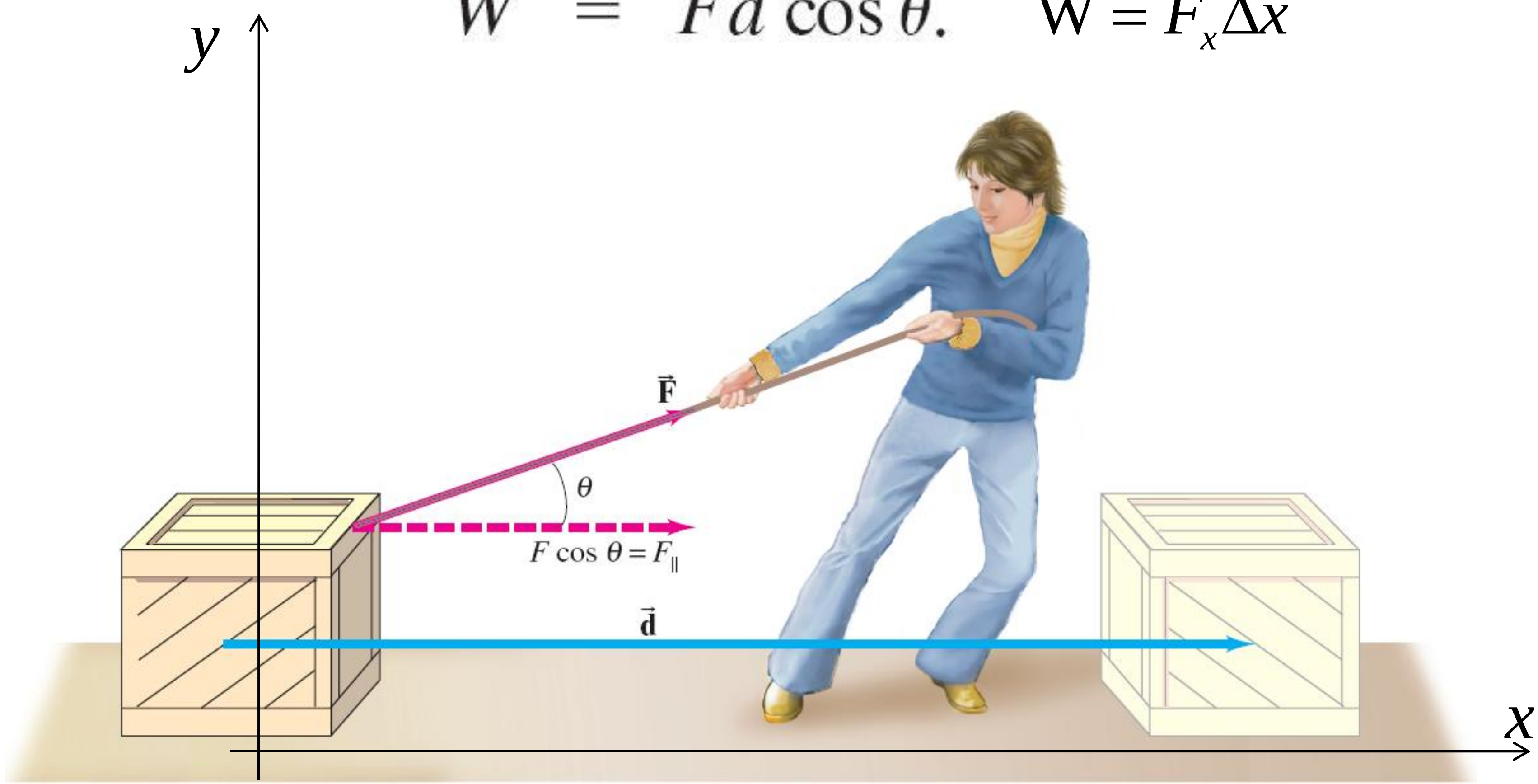
- **Work Done by a Force**
- **Scalar Product of Two Vectors (dot product)**
- **Kinetic Energy and the Work-Energy Principle**

Ref: 6-1,2,3.

Work Done by a Constant Force

The **work** done by a **constant** force is defined as the **distance** moved multiplied by the component of the **force** in the direction of displacement:

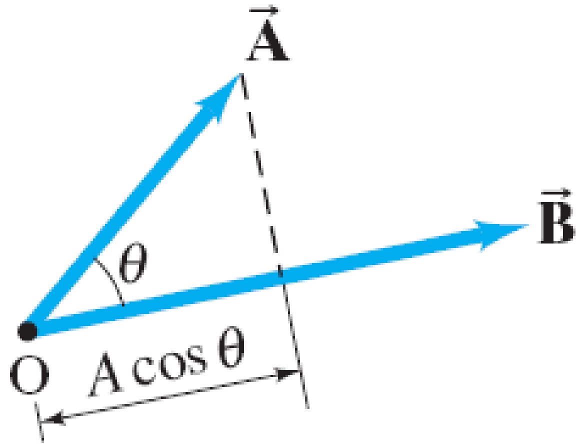
$$W = Fd\cos\theta. \qquad W = F_x \Delta x$$



Scalar Product of Two Vectors

Definition of the scalar product (dot product):

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta$$



Component form:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The work done by a force is the dot product between the force and the displacement

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta.$$

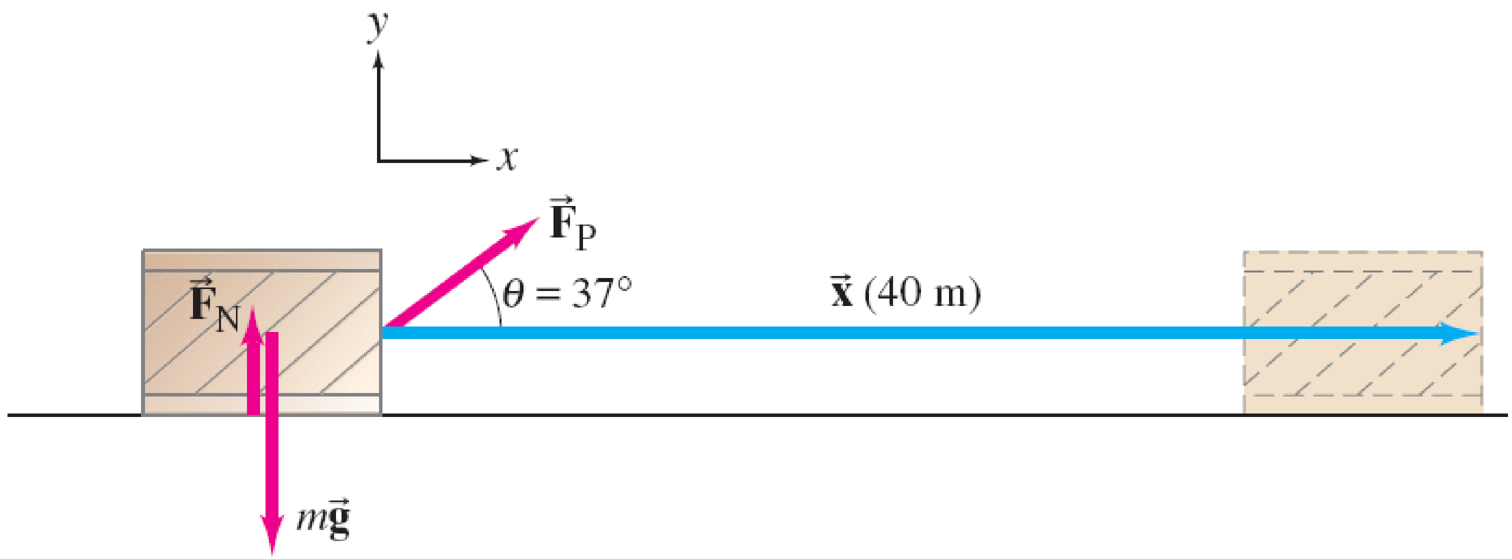
i-clicker question 9-1

Is it possible to do work on an object that remains at rest?

1) **yes**
2) **no**

Example: Work done on a crate.

A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force F_P = 100 N, which acts at a 37° angle as shown. The floor is smooth and exerts no friction force. Determine (a) the work done by each force acting on the crate, and (b) the net work done on the crate.



$$W_P = \vec{F}_P \cdot \vec{x} = F_P x \cos\theta = (100)(40)\cos 37° = 3200\ (J)$$

$$W_N = \vec{F}_N \cdot \vec{x} = F_N x \cos 90° = 0$$

$$W_W = m\vec{g} \cdot \vec{x} = mgx \cos 90° = 0$$

the net work done on the crate:
$W_{NET} = W_P = 3200$ J

i-clicker question 9-2: Friction and Work

A box is being pulled across a rough floor at a constant speed. What can you say about the work done by friction?

1) **friction does no work at all**
2) **friction does negative work**
3) **friction does positive work**

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta.$$

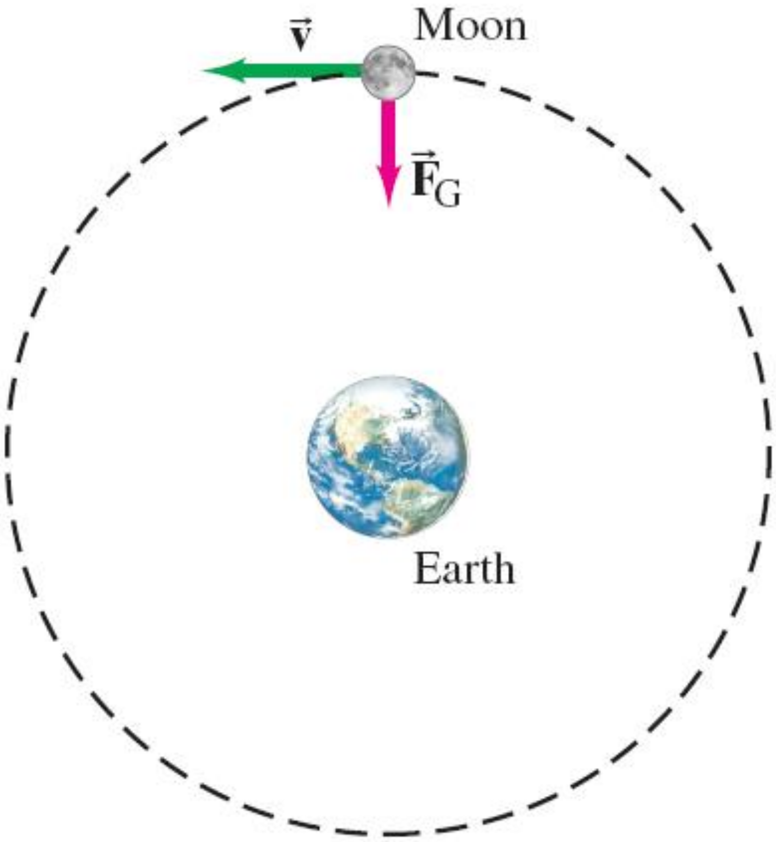
$$\mathrm{W} < 0 \quad \text{when} \quad \theta > 90°$$

Force and displacement are in opposite directions.

i-clicker question 9-3: Does the Earth do work on the Moon?

The Moon revolves around the Earth in a nearly circular orbit, with approximately constant tangential speed, kept there by the gravitational force exerted by the Earth. Does gravity do

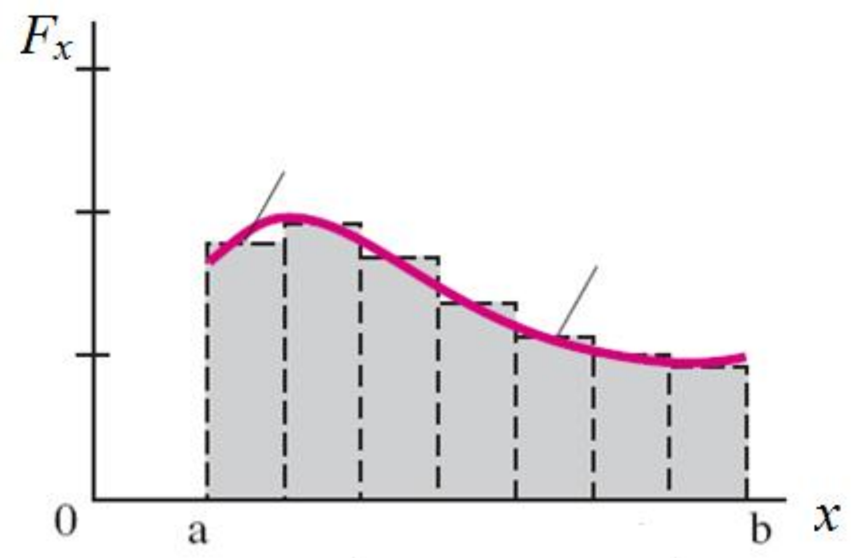
(A) positive work,
(B) negative work, or
(C) no work at all on the Moon?



Work Done by a Varying Force

For a force that varies, the work can be approximated by dividing the distance up into small pieces, finding the work done during each, and adding them up.

$$W \approx \sum F_x \Delta x$$



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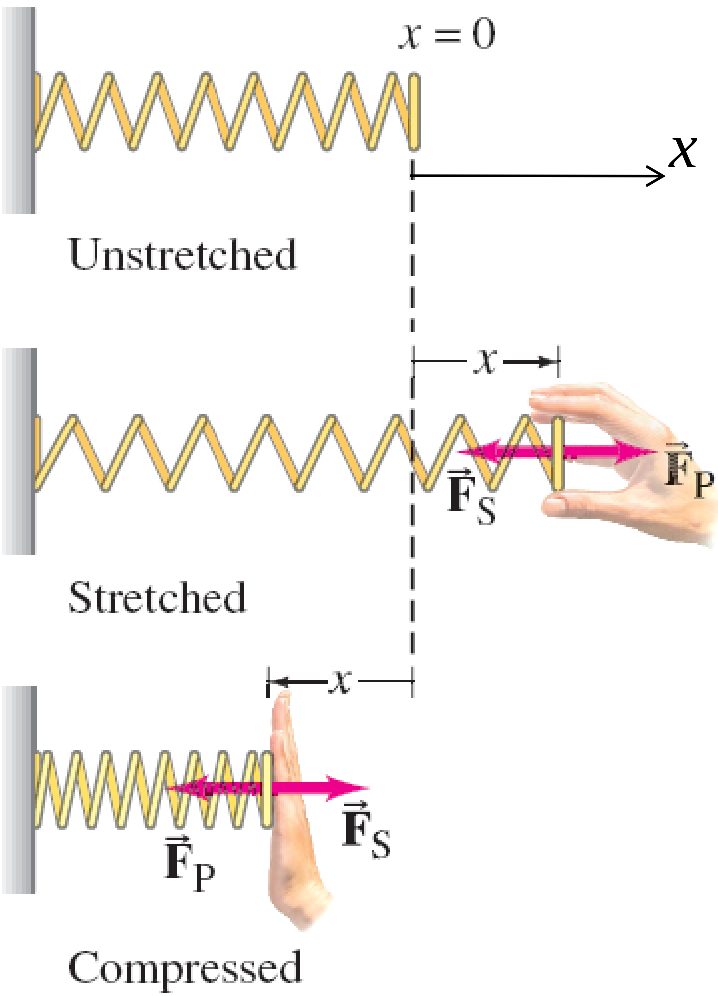
~ area under the F-x curve.

Work Done by a Varying Force

Work done by a spring force:

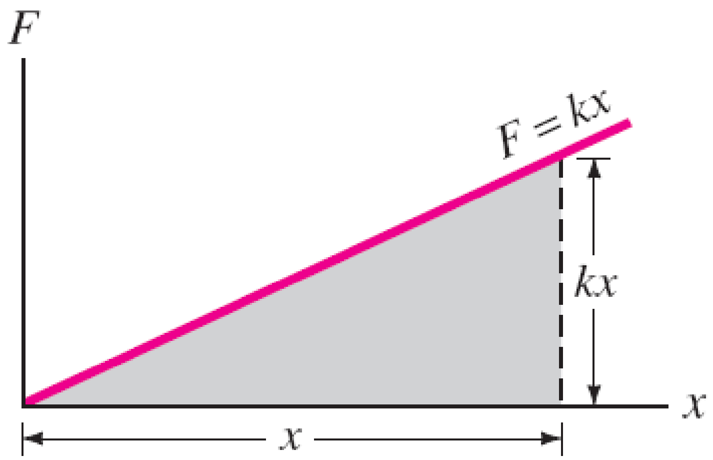


The force exerted by a spring is given by Hooke's law:

$$F_S = -kx$$

Meaning of negative sign: It's a restoring force, always pointing to the equilibrium position.

Work Done by a Varying Force



Plot of F vs. x. Work done by the applied force is equal to the shaded area.

$$W_P = \frac{1}{2}(\,height \cdot base\,)$$

$$= \frac{1}{2}kx \cdot x = \frac{1}{2}kx^2$$

Or, using calculus:

$$W_P = \int_{x_a=0}^{x_b=x} [F_P(x)\,\hat{\mathbf{i}}] \cdot [dx\hat{\mathbf{i}}] = \int_0^x F_P(x)\,dx$$

$$= \int_0^x kx\,dx = \tfrac{1}{2}kx^2\Big|_0^x = \tfrac{1}{2}kx^2$$

Example: Work done on a spring.

(a) A person pulls on a spring, stretching it 3.0 cm, which requires a maximum force of 75 N. How much work does the person do?
(b) If, instead, the person compresses the spring 3.0 cm, how much work does the person do?

(a) Can we do this: W=(75N)(0.030m)=2.25J ?

i-clicker question 9-4: (A) Yes; (B) No.

Why? Or Why not?

$$W_P = \frac{1}{2}kx^2 \qquad \text{But how do we find } k\,?$$

$$F_P = -F_S = -(-kx) = kx, \qquad k = \frac{F_P}{x} = \frac{75}{0.03} = 2500\,N/m$$

$$W_P = \frac{1}{2}kx^2 = \frac{1}{2}(2500)(0.03)^2 = 1.1(J)$$

(b) The same as (a) because the sign of x doesn't change the value of x^2. Physically, compressing a spring is equally hard as stretching.

The Work-Energy Principle

The net work done on an object is equal to the increase in kinetic energy of the object:

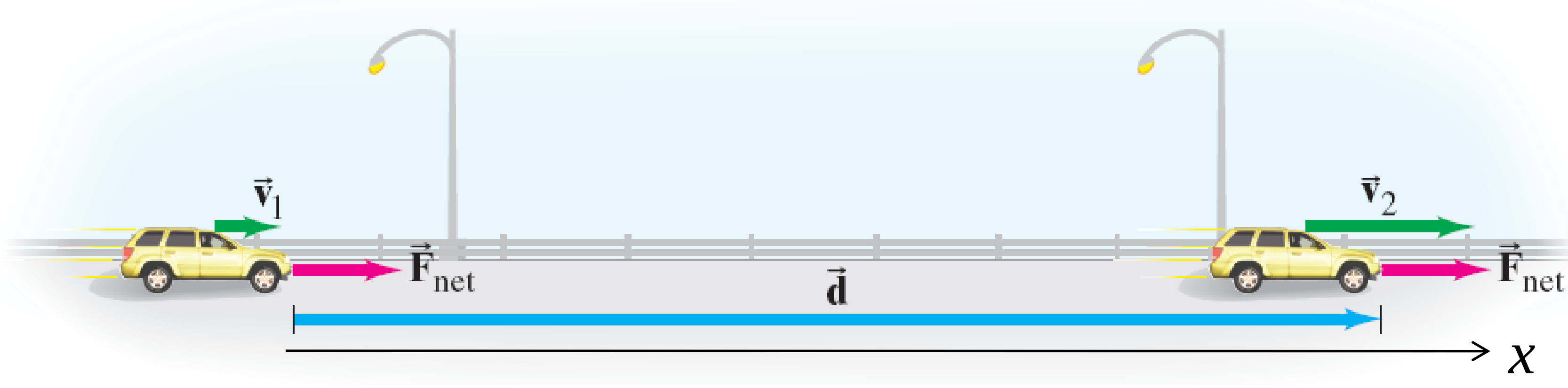
$$W_{net} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2.$$

It's a consequence of Newton's 2nd law:

For example, for 1-d motion with constant acceleration

$$W_{net} = \vec{F}\cdot\vec{d} = m\vec{a}\cdot\vec{d} = ma(x_2 - x_1) \qquad \text{(for 1-d motion, } \theta = 0\text{)}$$

$$= \frac{1}{2}m(v_2^2 - v_1^2) \qquad \left(\text{since } v^2 = v_0^2 + 2a(x - x_0)\right)$$



The Work-Energy Principle

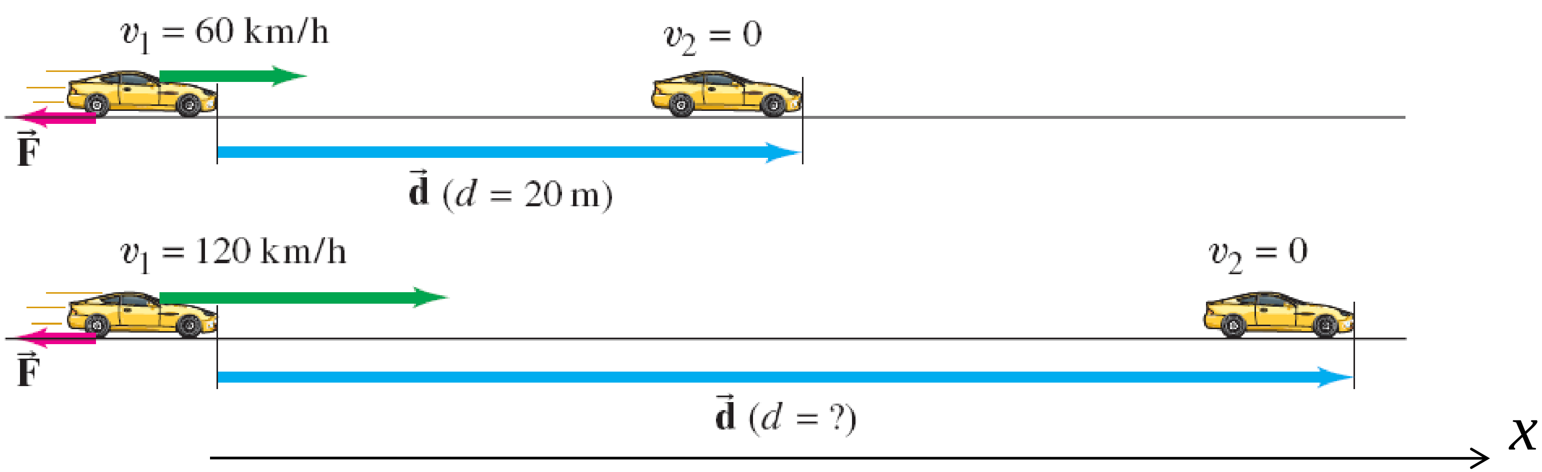
$$W_{\text{net}} = \Delta K = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 .$$

It allows us to solve certain problems without knowing the details of motion and forces between the initial and final states.

On the other hand, Newton's law is an instantaneous relationship between the net force and acceleration. It requires detailed information about the net force throughout the course of motion.

Example: Work to stop a car.

A car traveling 60 km/h can brake to a stop within a distance *d* of 20 m. If the car is going twice as fast, 120 km/h, what is its stopping distance? Assume the maximum braking force is approximately independent of speed.



$$W_{net} = \vec{F} \cdot \vec{d} = -Fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -\frac{1}{2}mv_1^2$$

$$i.e., \quad d = \frac{m}{2F}v_1^2 \propto v_1^2$$

$\therefore$ when v_1 is doubled, d should be quadrupled: $4 \times 20m = 80m$.