Phys101 Lectures 26-27

Fluids II

Key points:

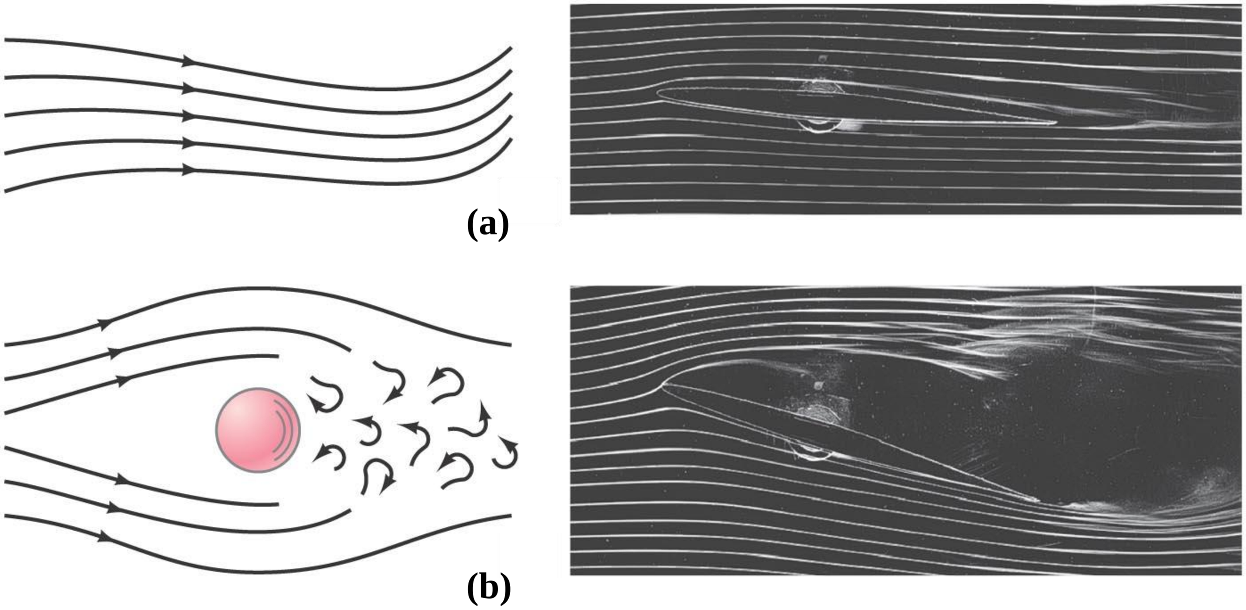
- **Bernoulli's Equation**
- **Poiseuille's Law**

Ref: 10-8,9,10,11,12.

10-8 Fluids in Motion; Flow Rate and the Equation of Continuity

If the flow of a fluid is smooth, it is called **streamline** or **laminar** flow (a).

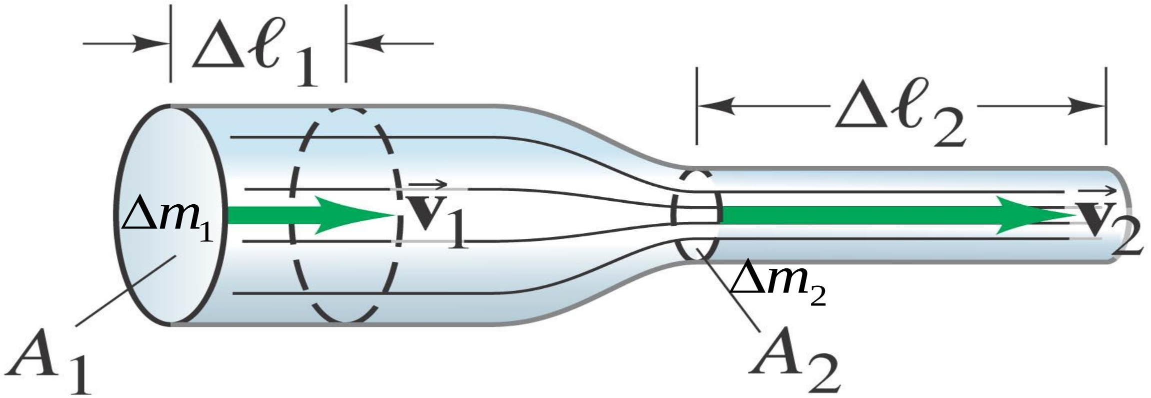
Above a certain speed, the flow becomes **turbulent** (b). Turbulent flow has **eddies**; the **viscosity** of the fluid is much greater when eddies are present.



Flow Rate and the Equation of Continuity

We will deal with **laminar** flow.

The **mass flow rate** is the mass that passes a given point per unit time. The flow rates at any two points must be **equal**, as long as no fluid is being added or taken away.

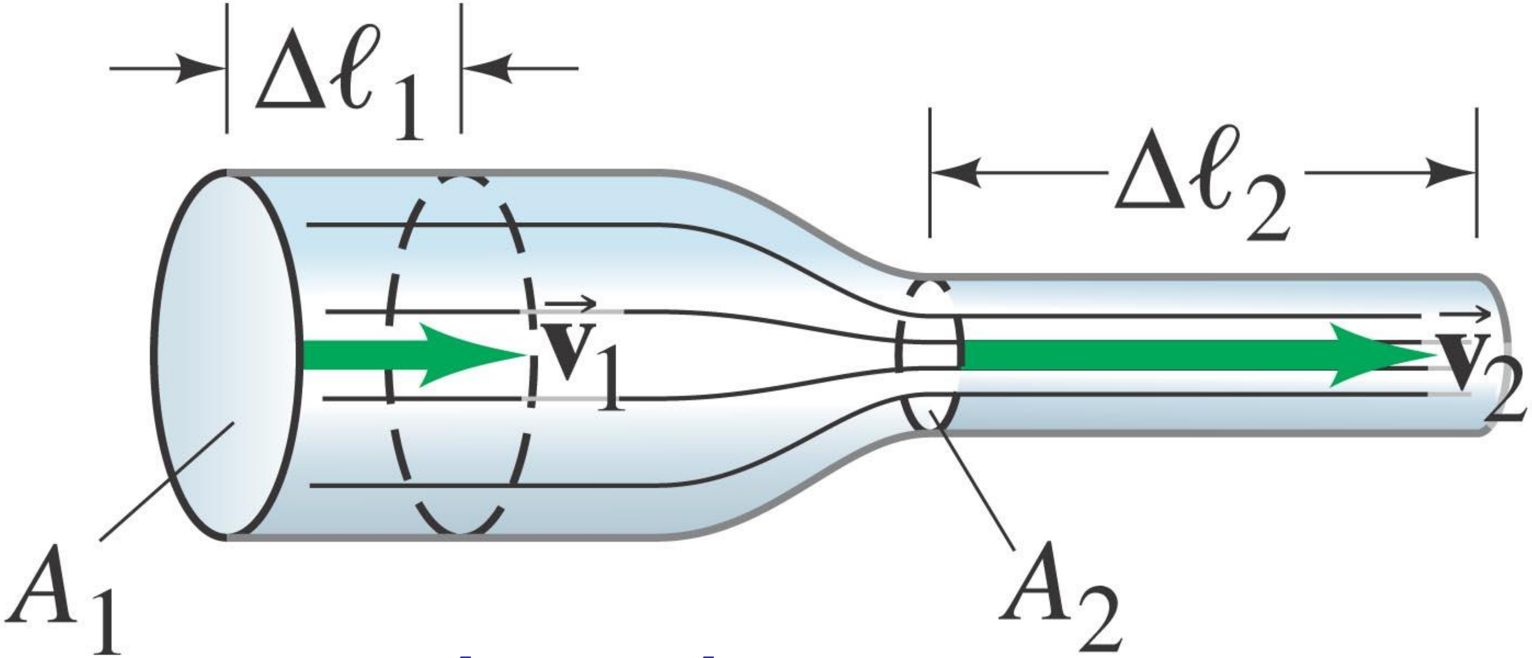


This gives us the **equation of continuity**:

Since $$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t},$$ (in = out, $\Delta m_1 = \Delta m_2$)

then $$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 .$$

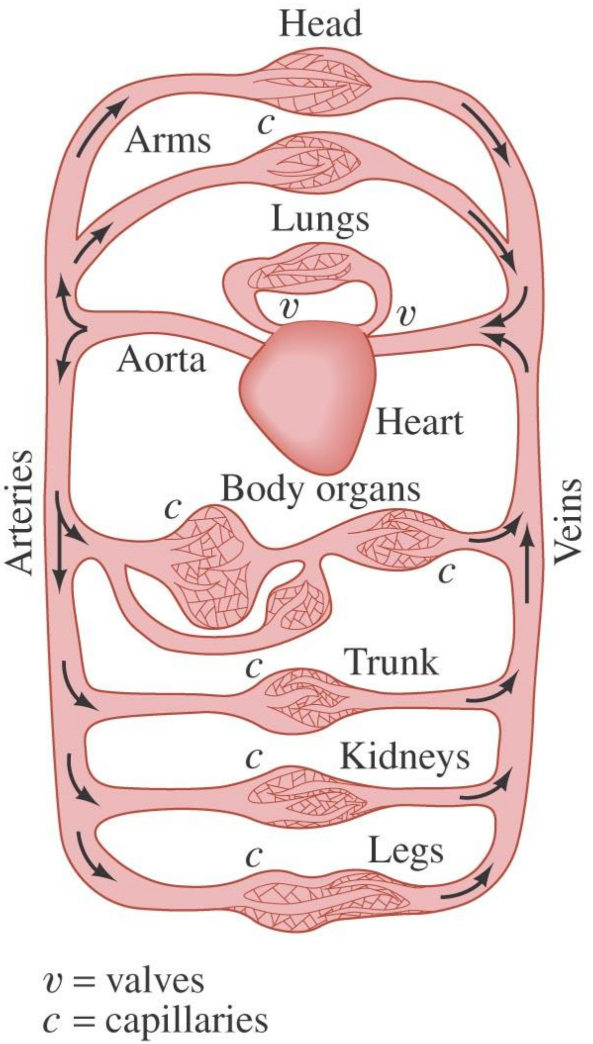
If the density doesn't change—typical for liquids—this simplifies to $A_1v_1 = A_2v_2$. Where the pipe is wider, the flow is slower.



$$A_1v_1 = A_2v_2$$

Example: Blood flow.

In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4} cm, and blood flows through it at a speed of about 5×10^{-4} m/s. Estimate the number of capillaries that are in the body.



Bernoulli's Equation

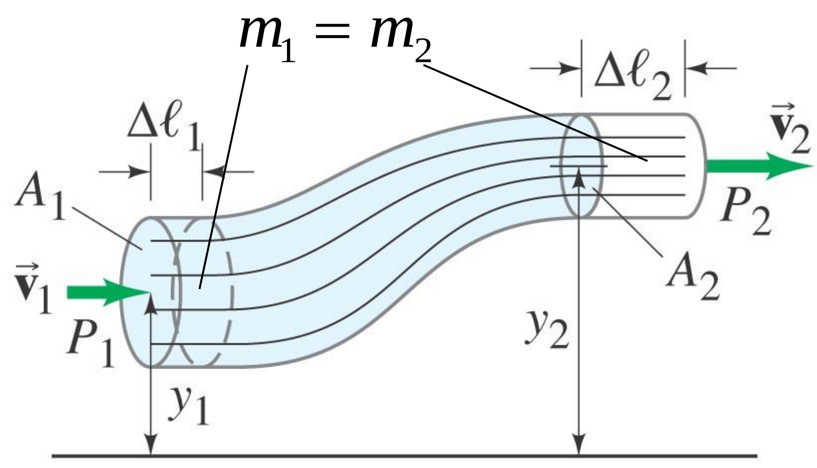
In time interval Δt, m_1 moves in and m_2 moves out. Continuity requires

$$m_1 = m_2 = m = \rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

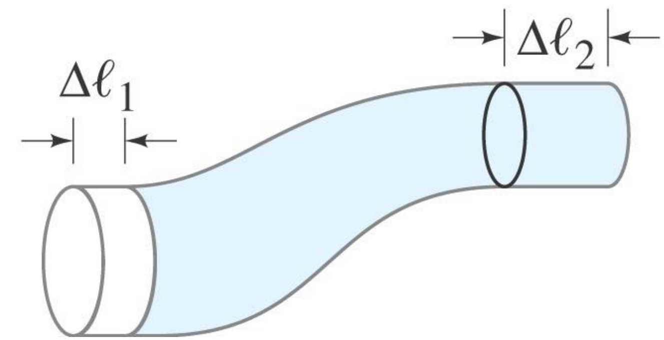
Work done by pressure:

$$W_P = P_1 A_1 v_1 \Delta t - P_2 A_2 v_1 \Delta t = \frac{m}{\rho}\left(P_1 - P_2\right)$$

Ideally, **when there is no drag**, W_p should be equal to the gain in mechanical energy:



$$\frac{m}{\rho}\left(P_1 - P_2\right) = \Delta E = mgy_2 + \frac{1}{2}mv_2^2 - mgy_1 - \frac{1}{2}mv_1^2$$



$$P_1 - P_2 = \rho g y_2 + \frac{1}{2}\rho v_2^2 - \rho g y_1 - \frac{1}{2}\rho v_1^2$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\text{OR:} \quad P + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

This is known as Bernoulli's equation, which is a consequence of conservation of energy.

Bernoulli's principle:

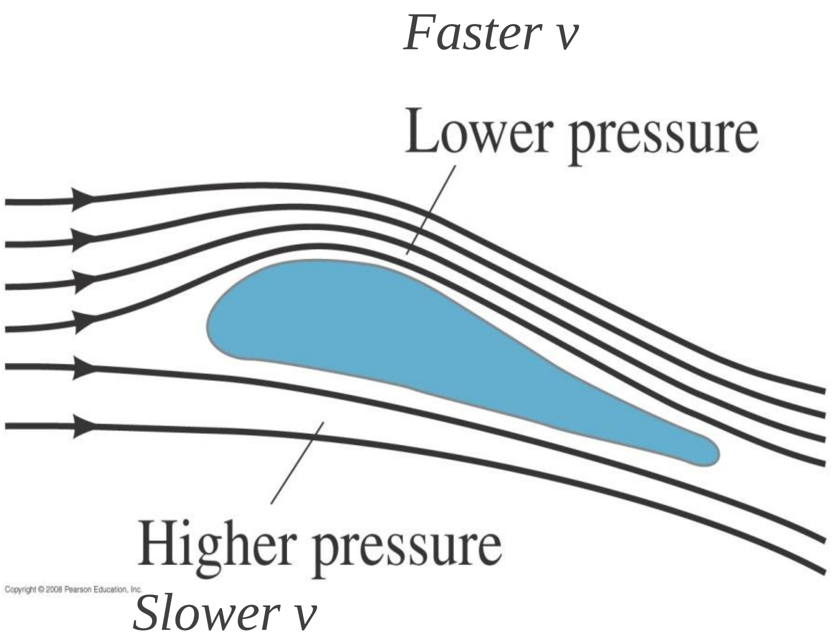
When the height y doesn't change much, Bernoulli's equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\text{OR}: \quad P + \frac{1}{2}\rho v^2 = \text{constant}$$

Where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high.

Lift on an airplane wing is due to the different air speeds and pressures on the two surfaces of the wing.

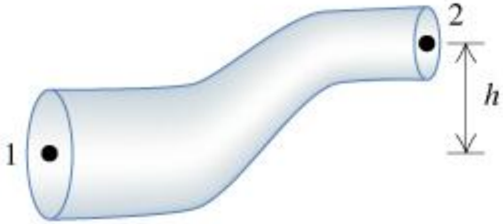


Demo

Example: Flow and pressure in a hot-water heating system.

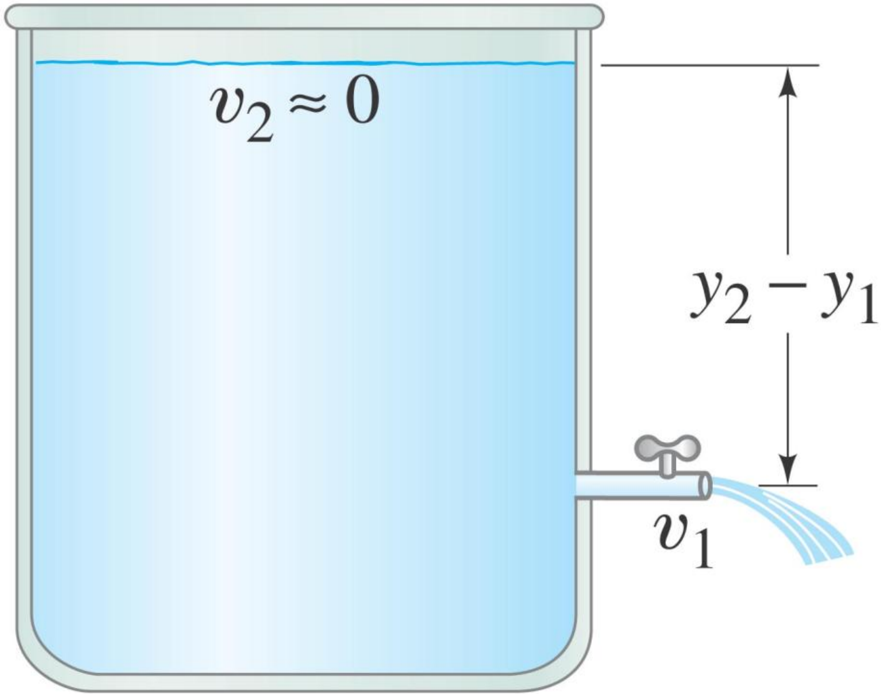
Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5 m/s through a 4.0-cm-diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6-cm-diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

[Solution]



Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, TIA

Using Bernoulli's principle, we find that the speed of fluid coming from a spigot on an open tank is:



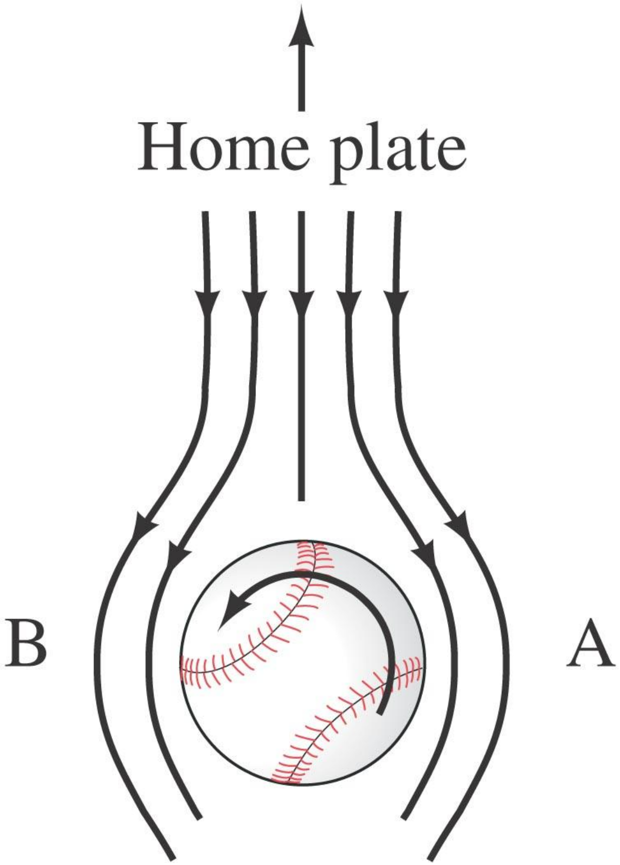
$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

or

$$v_1 = \sqrt{2g(y_2 - y_1)}.$$

This is called Torricelli's theorem.

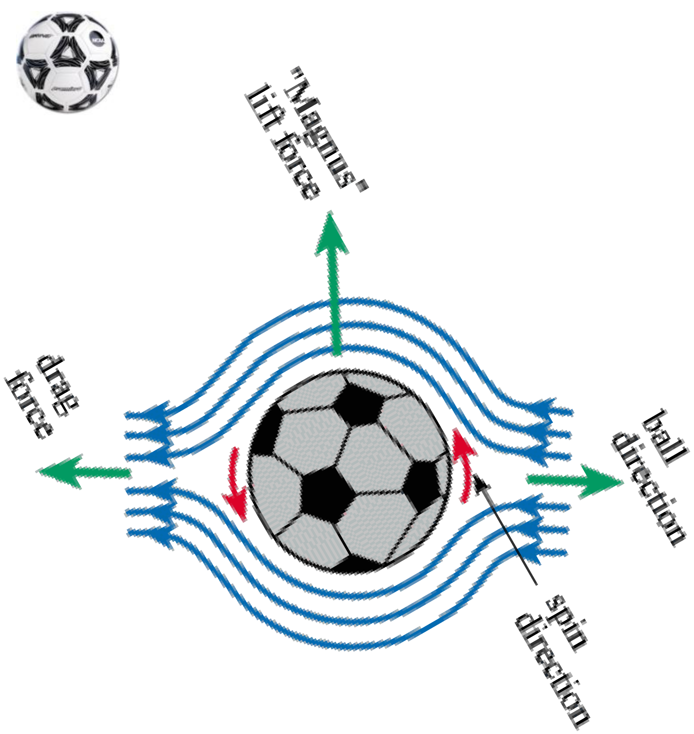
Applications of Bernoulli's Principle



The air travels faster relative to the center of the ball where the periphery of the ball is moving in the same direction as the airflow.

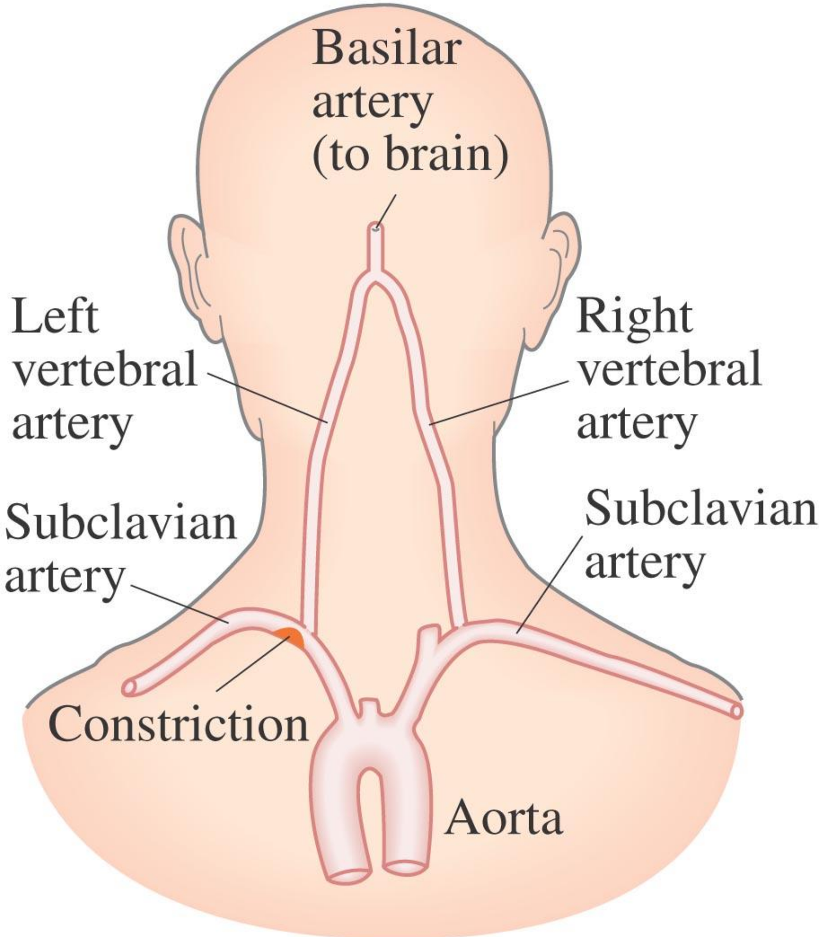
A ball's path will curve due to its spin, which results in the air speeds on the two sides of the ball not being equal; therefore there is a pressure difference.

Free kick – a curving soccer ball.



Video

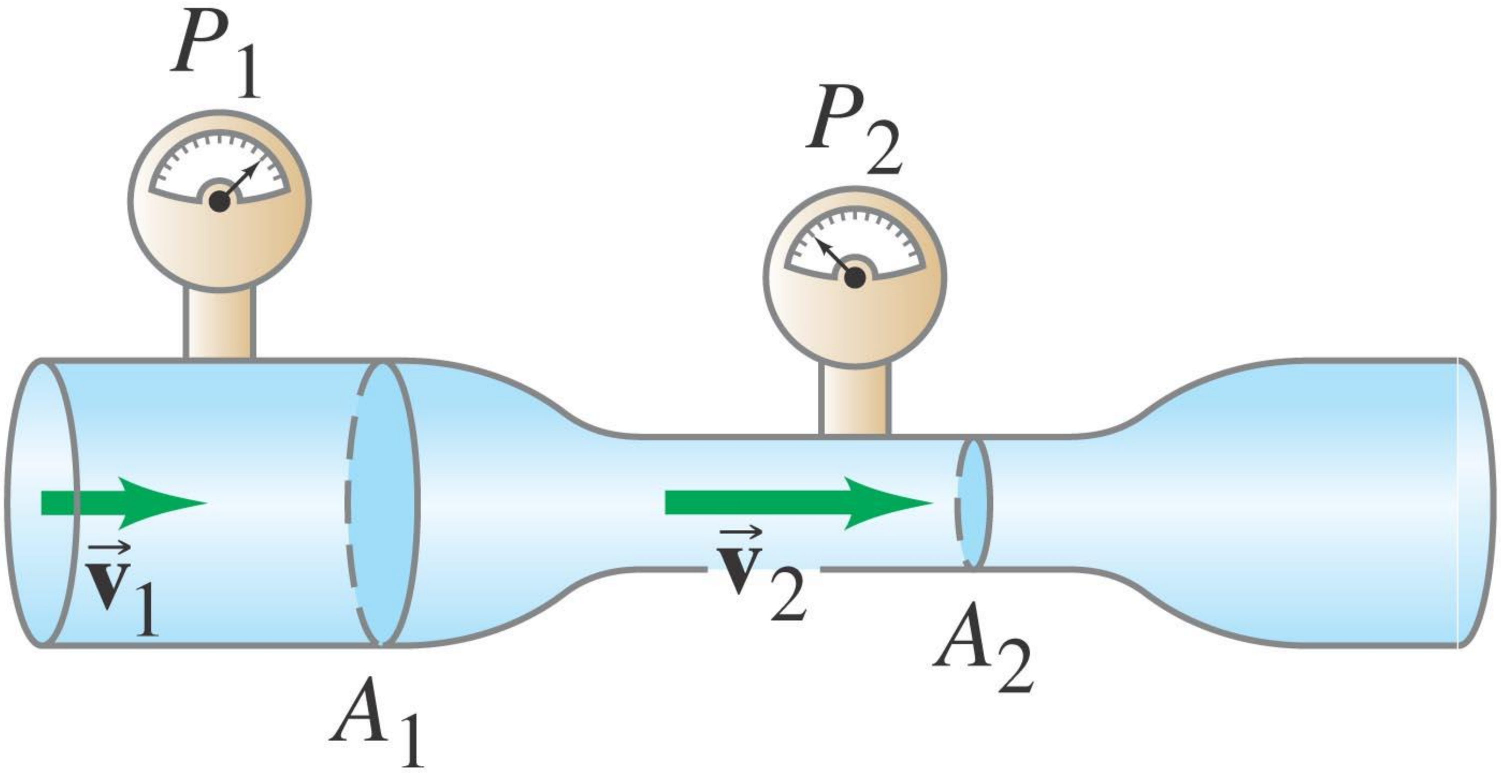
Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, TIA



A person with constricted arteries may experience a temporary lack of blood to the brain (TIA) as blood speeds up to get past the constriction, thereby reducing the pressure.

Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, TIA

A **venturi meter** can be used to measure fluid **flow** by measuring **pressure** differences.

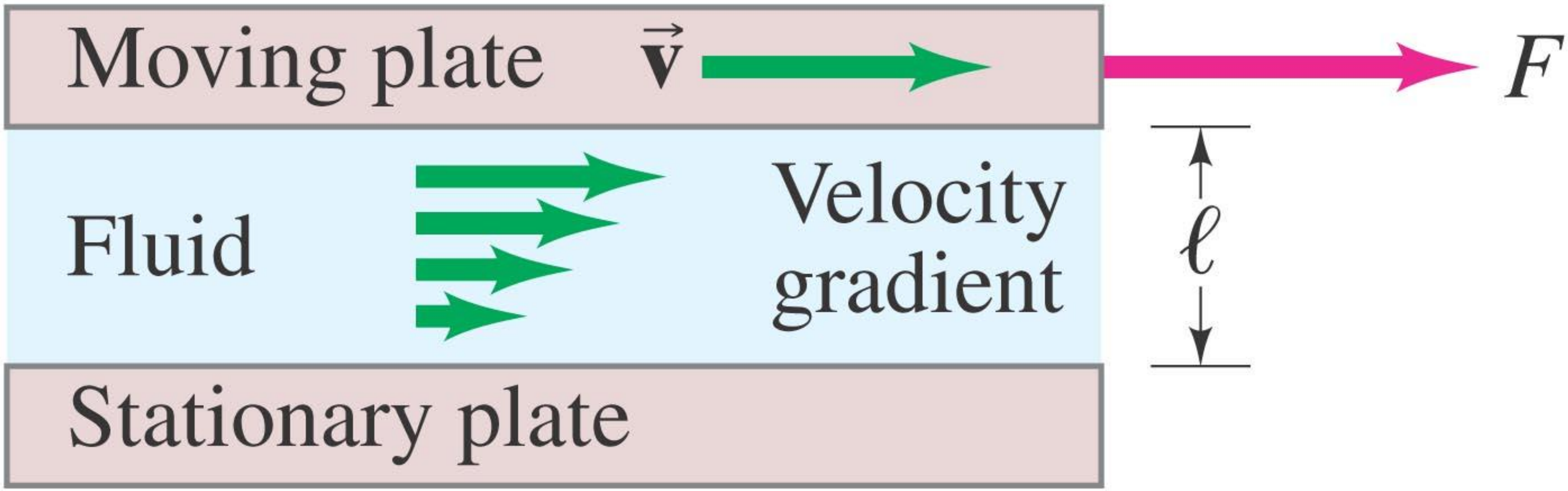


Viscosity

Real fluids have some **internal friction**, called **viscosity**.

The **viscosity** can be measured; it is found from the relation

$$F = \eta A \frac{v}{\ell}.$$



Flow in Tubes; Poiseuille's Equation, Blood Flow

The **rate of flow** in a fluid in a round tube depends on the **viscosity** of the fluid, the **pressure** difference, and the **dimensions** of the tube.

The volume flow rate is proportional to the **pressure difference**, inversely proportional to the **length** of the tube and to the **pressure difference**, and proportional to the **fourth** power of the radius of the tube.

$$Q = \frac{\pi R^4 \left(P_1 - P_2\right)}{8\eta l}$$