Assignment 4

Due: 11:59pm on Friday, May 31, 2013

Note: You will receive no credit for late submissions. To learn more, read your instructor's Grading Policy

± Hooke's Law

Description: ± Includes Math Remediation. Analyze the force of springs on Haitian taptaps as an application of Hooke's law.

Learning Goal:

To understand the use of Hooke's law for a spring.

Hooke's law states that the *restoring force* $\vec{F}$ on a spring when it has been stretched or compressed is proportional to the displacement $\vec{x}$ of the spring from its equilibrium position. The equilibrium position is the position at which the spring is neither stretched nor compressed.

Recall that $\vec{F} \propto \vec{x}$ means that $\vec{F}$ is equal to a constant times $\vec{x}$. For a spring, the proportionality constant is called the *spring constant* and denoted by k. The spring constant is a property of the spring and must be measured experimentally. The larger the value of k, the stiffer the spring.
In equation form, Hooke's law can be written

$$\vec{F} = -k\vec{x}.$$

The minus sign indicates that the force is in the opposite direction to that of the spring's displacement from its equilibrium length and is "trying" to *restore* the spring to its equilibrium position. The magnitude of the force is given by $F = kx$, where x is the magnitude of the displacement.

In Haiti, public transportation is often by taptaps, small pickup trucks with seats along the sides of the pickup bed and railings to which passengers can hang on. Typically they carry two dozen or more passengers plus an assortment of chickens, goats, luggage, etc. Putting this much into the back of a pickup truck puts quite a large load on the truck springs.
A truck has springs for each wheel, but for simplicity assume that the individual springs can be treated as one spring with a spring constant that includes the effect of all the springs. Also for simplicity, assume that all four springs compress equally when weight is added to the truck and that the equilibrium length of the springs is the length they have when they support the load of an empty truck.

Part A

A 68kg driver gets into an empty taptap to start the day's work. The springs compress 2.5×10^{-2}m . What is the effective spring constant of the spring system in the taptap?

Enter the spring constant numerically in newtons per meter using two significant figures.

Hint 1. How to approach the problem

The compression of the springs is governed by Hooke's law. The amount the springs are compressed when the driver climbs into the truck is given in the problem statement. The force that acts to compress the springs is the force caused by the driver getting into the truck.

ANSWER:

$k = \frac{m_d g}{x_d}$ = 2.7×10^{4} $\mathrm{N/m}$

If you need to use the spring constant in subsequent parts, use the full precision value you calculated, only rounding as a final step before submitting your answer.

Part B

After driving a portion of the route, the taptap is fully loaded with a total of 26 people including the driver, with an average mass of 68kg per person. In addition, there are three 15-kg goats, five 3-kg chickens, and a total of 25 kg of bananas on their way to the market. Assume that the springs have somehow not yet compressed to their maximum amount. How much are the springs compressed?

Enter the compression numerically in meters using two significant figures.

Hint 1. How to find the compression of the spring

The spring compression is governed by Hooke's law. Use the spring constant you calculated to full precision in Part A prior to rounding your answer. To find the force add the total weight of the load on the truck. Only round as a final step before submitting your answer.

ANSWER:

x = $x_d\left(p+\frac{85}{m_d}\right)$ = 0.68 m

Also accepted: $\frac{(pm_d+85)\,g}{b}$ = 0.67

Part C

Whenever you work a physics problem you should get into the habit of thinking about whether the answer is physically realistic. Think about how far off the ground a typical small truck is. Is the answer to Part B physically realistic?

Select the best choice below.

ANSWER:

- (selected) No, typical small pickup truck springs are not large enough to compress 0.68m .
- Yes, typical small pickup truck springs can easily compress 0.68m .

The answer to Part B is not physically realistic because the springs of a typical light truck will compress their maximum amount (typically about 10 cm) before the total weight of all the passengers and other cargo given in Part B is added to the truck. When this maximum compression is reached, the springs will bottom out, and the ride will be very rough.

Part D

Now imagine that you are a Haitian taptap driver and want a more comfortable ride. You decide to replace the springs with new springs that can handle the typical heavy load on your vehicle. What spring constant do you want your new spring system to have?

ANSWER:

The new springs should have a spring constant that is

- (selected) substantially larger
- slightly larger
- slightly smaller
- substantially smaller

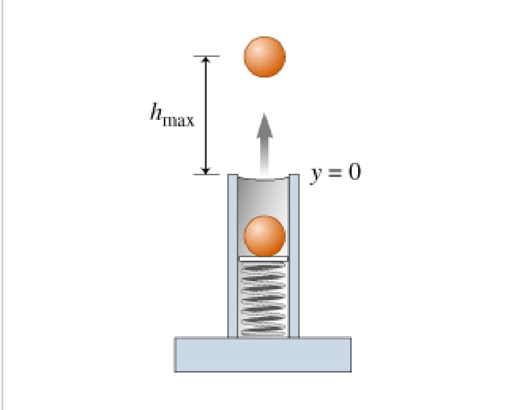
than the spring constant of the old springs.

A spring constant with a large value is a stiff spring. It will take more force to compress (or stretch) a stiff spring. On a taptap, stiffer springs are less likely to bottom out under a heavy load. However, with a lighter load, for most vehicles, very stiff springs will not compress as much for a bump in the road. Hence very stiff springs will give a better ride with a very heavy load, but less-stiff springs (lower spring constant) will give a smoother ride with a light load. This is why larger vehicles need stiffer springs than smaller vehicles.

Fun with a Spring Gun

Description: A ball is launched vertically from a spring gun. Use conservation of energy to compute the velocity of the ball as a function of height, and to compute the maximum height reached by the ball.

A spring-loaded toy gun is used to shoot a ball of mass $m = 1.50\ \mathrm{kg}$ straight up in the air, as shown in the figure. The spring has spring constant $k = 667\ \mathrm{N/m}$. If the spring is compressed a distance of 25.0 centimeters from its equilibrium position $y = 0$ and then released, the ball reaches a maximum height $h_{\max}$ (measured from the equilibrium position of the spring). There is no air resistance, and the ball never touches the inside of the gun. Assume that all movement occurs in a straight line up and down along the *y* axis.



Part A

Which of the following statements are true?

Check all that apply.

Hint 1. Nonconservative forces

Dissipative, or *nonconservative*, forces are those that always oppose the motion of the object on which they act. Forces such as friction and drag are dissipative forces.

Hint 2. Forces acting on the ball

The ball is acted on by the spring force only when the two are in contact. The force of tension in the spring is a conservative force. Also, the ball is *always* acted on by gravity, which is also a conservative, or nondissipative, force.

ANSWER:

- [x] Mechanical energy is conserved because no dissipative forces perform work on the ball.
- [x] The forces of gravity and the spring have potential energies associated with them.
- [ ] No conservative forces act in this problem after the ball is released from the spring gun.

Part B

Find v_m the muzzle velocity of the ball (i.e., the velocity of the ball at the spring's equilibrium position $y = 0$).

Hint 1. Determine how to approach the problem

What physical relationship can you use to solve this problem? Choose the *best* answer.

ANSWER:

- ○ kinematics equations
- ○ Newton's second law
- ◉ law of conservation of energy
- ○ conservation of momentum

Note that the law of conservation of energy applies to *closed* systems. In this case, such a closed system consists of the ball and the spring (and, technically, the Earth, but we will follow the traditional, somewhat imprecise, language and will assume that it is the *ball* that has gravitational potential energy, not the system "ball-Earth.")

Hint 2. Energy equations

Recall that kinetic energy is given by the equation

$$\frac{1}{2}mv^2,$$

where v is the speed of the object and m is the object's mass.

Gravitational potential energy is given by

$$mgy,$$

where y is the object's height measured from $y = 0$.

The elastic potential energy of a spring is given by

$$\frac{1}{2}kx^2,$$

where k is the spring constant and x is the spring's displacement from equilibrium.

Hint 3. Determine which two locations you should examine

Pick the two points along the ball's path that would be most useful to compare in order to find the solution to this problem. Choose from among the following three points:

Check all that apply.

ANSWER:

- [x] $y = -25\ \mathrm{cm}$, the location of the ball when the spring is compressed.
- [x] $y = 0$, the equilibrium position of the spring.
- [ ] $y = h_{\mathrm{max}}$, the maximum height that the ball reaches above the point $y = 0$.

Because you do not know enough information about the ball at $y = h_{\mathrm{max}}$, you need to compare the energy at $y = -25\ \mathrm{cm}$ to the energy at $y = 0$ to find v_{m}.

Hint 4. Find the initial energy of the system

A useful statement of mechanical energy conservation relating the initial and final kinetic (K) and potential (U) energies is

$$K_{\mathrm{initial}} + U_{\mathrm{initial}} = K_{\mathrm{final}} + U_{\mathrm{final}}.$$

In this situation, the initial position is $y = -25.0\ \mathrm{cm}$ and the final position is $y = 0$, which is the equilibrium position of the spring. What kind(s) of energy does the system "spring-ball" have at the initial position?

ANSWER:

- ( ) kinetic only
- ( ) elastic potential only
- ( ) gravitational potential only
- ( ) kinetic and gravitational potential
- ( ) kinetic and elastic potential
- (•) elastic and gravitational potentials

Keep in mind that$y = 0$ at the equilibrium position of the spring. The inital position defined at $y = -25\mathrm{cm}$ will have negative gravitational potential energy.

Hint 5. Determine the final energy

A useful statement of mechanical energy conservation relating the initial and final kinetic (K) and potential (U) energies is

$$K_{\mathrm{initial}} + U_{\mathrm{initial}} = K_{\mathrm{final}} + U_{\mathrm{final}}.$$

In this situation, the initial position is $y = -25.0\ \mathrm{cm}$ and the final position is $y = 0$, which is the equilibrium position of the spring. What kind(s) of energy does the system "spring-ball" have at the final position?

ANSWER:

- (selected) kinetic only
- elastic potential only
- gravitational potential only
- kinetic and gravitational potential
- kinetic and elastic potential
- elastic and gravitational potentials

Hint 6. Creating an equation

From the hints you now know what kinds of energy are present at the initial and final positions chosen for the ball in this part of the problem. You also know that

$$K_{\rm initial} + U_{\rm initial} = K_{\rm final} + U_{\rm final}.$$

It has been determined that $K_{\rm initial}$ is zero and $U_{\rm initial}$ consists of two terms: gravitational potential energy and elastic potential energy. In addition, $U_{\rm final}$ is zero.

ANSWER:

$v_{\rm m}$ = 4.78 $\rm m/s$

Part C

Find the maximum height $h_{\rm max}$ of the ball.

Express your answer numerically, in meters.

Hint 1. Choose two locations to examine

Pick the two points along the ball's movement that would be most useful to compare in order to find a solution to this problem. Choose from among the following three points:

Check all that apply.

ANSWER:

- [x] $y = -25 \ {\rm cm}$, the location of the ball when the spring is compressed.
- [ ] $y = 0$, the equilibrium position of the spring.
- [x] $y = h_{\rm max}$, the maximum height that the ball reaches measured from $y = 0$.

Also accepted: $y = 0$, the equilibrium position of the spring. (with) $y = h_{\rm max}$, the maximum height that the ball reaches measured from $y = 0$.

You could compare $y = h_{\rm max}$ to either $y = -25 \ {\rm cm}$ or $y = 0$. It is probably most convenient to use $y = -25 \ {\rm cm}$ for comparison because using $y = 0$ requires that you know the energy at the equilibrium position of the spring. Of course, you do know it, as long as you got that part of the problem correct. For the remainder of the problem, we will use $y = -25 \ {\rm cm}$ and $y = h_{\rm max}$.

Hint 2. Find the initial energy

A useful statement of mechanical energy conservation is

$$K_{\rm inital} + U_{\rm initial} = K_{\rm final} + U_{\rm final}.$$

Recall that in the problem statement, $y = 0$ is set to correspond to the equilibrium position of the spring. Therefore, in this situation, the initial location is at $y = -25 \ {\rm cm}$ and the final position should be taken as $y = h_{\rm max}$.

What kind(s) of energy does the ball have at the initial location?

ANSWER:

- kinetic only
- elastic potential only
- gravitational potential only
- kinetic and gravitational potential
- kinetic and elastic potential
- (selected) elastic and gravitational potentials

Hint 3. Determine the final energy

A useful statement of mechanical energy conservation is

$$K_{\mathrm{initial}} + U_{\mathrm{initial}} = K_{\mathrm{final}} + U_{\mathrm{final}}.$$

In this situation, the initial location is at $y = -25\ \mathrm{cm}$, and the final position should be taken as $y = h_{\mathrm{max}}$. What kind(s) of energy does the ball have at $y = h_{\mathrm{max}}$?

Hint 1. Find the speed of the ball at the top of its trajectory

What is the speed v_{top} of the ball at the top of its trajectory?

Express your answer numerically, in meters per second.

Hint 1. Motion in the vertical direction

Recall from kinematics that a ball travels upward until its speed decreases to zero, at which point it starts falling back to Earth.

ANSWER:

v_{top} = 0 $\mathrm{m/s}$

Recall that $K = \frac{1}{2}mv^2$. Because the ball has zero velocity at the peak of its trajectory, it has no kinetic energy.

ANSWER:

- kinetic only
- elastic potential only
- (selected) gravitational potential only
- kinetic and gravitational potential
- kinetic and elastic potential
- elastic and gravitational potentials

Hint 4. Creating an equation

From the above hints, you now know what kind of energy is present at the inital and final positions chosen for the ball in this part of the problem. You know that

$$K_{\mathrm{initial}} + U_{\mathrm{initial}} = K_{\mathrm{final}} + U_{\mathrm{final}}.$$

It was determined that K_{initial} is zero and that U_{initial} consists of two terms: gravitational potential energy and elastic potential energy. In addition, K_{final} is zero.

ANSWER:

h_{max} = 1.17 m

In this problem you practiced applying the law of conservation of mechanical energy to a physical situation to find the muzzle velocity and the maximum height reached by the ball.

Part D

Which of the following actions, if done independently, would increase the maximum height reached by the ball?

Check all that apply.

ANSWER:

- [ ] reducing the spring constant k
- [x] increasing the spring constant k
- [ ] decreasing the distance the spring is compressed
- [x] increasing the distance the spring is compressed
- [x] decreasing the mass of the ball
- [ ] increasing the mass of the ball
- [ ] tilting the spring gun so that it is at an angle $\theta < 90$ degrees from the horizontal

Work and Kinetic Energy

Description: Short conceptual problem involving two different masses pushed by forces of equal magnitudes. Students must relate force and distance to the kinetic energy, mass and speed of each object. Requires that students use proportional reasoning. This problem is based on Young/Geller Conceptual Analysis 7.2.

Two blocks of ice, one four times as heavy as the other, are at rest on a frozen lake. A person pushes each block the same distance d. Ignore friction and assume that an equal force $\vec{F}$ is exerted on each block.

Part A

Which of the following statements is true about the kinetic energy of the heavier block after the push?

Hint 1. How to approach the problem

The work-energy theorem states that the change in kinetic energy of an object equals the net work done on that object:

$$W_{\rm total} = \Delta K.$$

The work done on an object can also be related to the distance d that the object moves while being acted on by a force $\vec{F}$:

$$W = F_{||}d,$$

where $F_{||}$ is the component of $\vec{F}$ parallel to the direction of displacement.

Hint 2. Find the work done on each block

What can be said about the net work done on the heavier block?

ANSWER:

- ( ) It is greater than the work done on the lighter block.
- (●) It is equal to the work done on the lighter block.
- ( ) It is less than the work done on the lighter block.

ANSWER:

- It is smaller than the kinetic energy of the lighter block.
- (selected) It is equal to the kinetic energy of the lighter block.
- It is larger than the kinetic energy of the lighter block.
- It cannot be determined without knowing the force and the mass of each block.

The work-energy theorem states that the change in kinetic energy of an object equals the net work done on that object. The only force doing work on the blocks is the force from the person, which is the same in both cases. Since the initial kinetic energy of each block is zero, both blocks have the same final kinetic energy.

Part B

Compared to the speed of the heavier block, what is the speed of the light block after both blocks move the same distance d?

Hint 1. How to approach the problem

In Part A, you determined that the kinetic energy of the heavier block was the same as that of the lighter block. Relate this to the speed of the blocks.

Hint 2. Proportional reasoning

Proportional reasoning becomes easier with practice. First relate the kinetic energies of the blocks to each other. To accomplish this, let the subscript h refer to the heavier block and the subscript ℓ to the lighter block. Now

$$K_{\mathrm{h}} = K_{\ell}$$

can be written as

$$\frac{1}{2}m_{\mathrm{h}}(v_{\mathrm{h}})^2 = \frac{1}{2}m_{\ell}(v_{\ell})^2.$$

The problem states that the heavier block is four time as massive as the lighter block. This can be represented by the expression

$$m_{\mathrm{h}} = 4m_{\ell}.$$

Substituting this expression into the expression for kinetic energy yields

$$\frac{1}{2}(4m_{\ell})(v_{\mathrm{h}})^2 = \frac{1}{2}m_{\ell}(v_{\ell})^2.$$

How many times larger than v_{h}^2 is v_{ℓ}^2?

ANSWER:

v_{ℓ}^2 = 4 v_{h}^2

Now use this information to relate v_{ℓ} to v_{h}.

ANSWER:

- one quarter as fast
- half as fast
- the same speed
- (selected) twice as fast
- four times as fast

Since the kinetic energy of the lighter block is equal to the kinetic energy of the heavier block, the lighter block must be moving faster than the heavier block.

Part C

Now assume that both blocks have the same speed after being pushed with the same force $\vec{F}$. What can be said about the distances the two blocks are pushed?

Hint 1. How to approach the problem

The work-energy theorem states that the change in kinetic energy of an object equals the net work done on that object:

$$W_{\rm total} = \Delta K.$$

The work done on an object can also be related to the distance d that the object moves while being acted on by a force $\vec{F}$:

$$W = F_{||} d,$$

where $F_{||}$ is the component of $\vec{F}$ parallel to the direction of displacement.

Hint 2. Relate the kinetic energies of the blocks

Let the subscript $\rm h$ refer to the heavier block and the subscript ℓ to the lighter block. What is the ratio

$\frac{K_{\rm h}}{K_{\ell}}$?

Hint 1. The kinetic energies

To relate the kinetic energies of the blocks to each other, recall that

$$v_{\rm h} = v_{\ell}$$

and

$$m_{\rm h} = 4m_{\ell}.$$

ANSWER:

$\frac{K_{\rm h}}{K_{\ell}}$ = 4

Hint 3. Compare the amount of work done on each block

In the previous hint, you found that $K_{\rm h} = 4K_{\ell}$. What is the ratio of the work done on the heavy block to the work done on the lighter block,

$\frac{W_{\rm h}}{W_{\ell}}$?

ANSWER:

$\frac{W_{\rm h}}{W_{\ell}}$ = 4

Now relate the amount of work done on each block to the distance each block must be pushed. Keep in mind that the force acting on each block is the same.

ANSWER:

- ○ The heavy block must be pushed 16 times farther than the light block.
- ◉ The heavy block must be pushed 4 times farther than the light block.
- ○ The heavy block must be pushed 2 times farther than the light block.
- ○ The heavy block must be pushed the same distance as the light block.
- ○ The heavy block must be pushed half as far as the light block.

Because the heavier block has four times the mass of the lighter block, when the two blocks travel with the same speed, the heavier block will have four times as much kinetic energy. The work-energy theorem implies that four times more work must be done on the heavier block than on the lighter block. Since the same force is applied to both blocks, the heavier block must be pushed through four times the distance as the lighter block.

Fat: The Fuel of Migrating Birds

Description: Find the maximum distance a bird can fly without feeding, given the amount of fat consumed in the flight and the bird's average speed and power consumption. Also, find how many grams of fat a hummingbird needs to fly across the Gulf of Mexico without feeding.

Small birds can migrate over long distances without feeding, storing energy mostly as fat rather than carbohydrate. Fat is a good form of energy storage because it provides the most energy per unit mass: 1 gram of fat provides about 9.4 (food) Calories, compared to 4.2 (food) Calories per 1 gram of carbohydrate. Remember that Calories associated with food, which are always capitalized, are not exactly the same as calories used in physics or chemistry, even though they have the same name. More specifically, one food Calorie is equal to 1000 calories of mechanical work or 4186 joules. Therefore, in this problem use the conversion factor $1\ \mathrm{Cal} = 4186\ \mathrm{J}$.

Part A

Consider a bird that flies at an average speed of 10.7$\mathrm{m/s}$ and releases energy from its body fat reserves at an average rate of 3.70W (this rate represents the power consumption of the bird). Assume that the bird consumes $4\ \mathrm{g}$ of fat to fly over a distance d_b without stopping for feeding. How far will the bird fly before feeding again?

Express your answer in kilometers.

Hint 1. How to approach the problem

From the average speed of the bird, you can calculate how far the bird can fly without stopping if you know the duration of the flight. To determine the duration of the flight, first find the amount of energy available from converting 4 grams of fat, and then use the definition of power.

Hint 2. Find the energy used during the flight

How much energy E_b does the bird have available when it converts 4 grams of fat?

Express your answer in kilojoules.

Hint 1. Converting fat into energy

As stated in the introduction of this problem,1 gram of fat provides about 9.4 (food) Calories. Also keep in mind that $1\ \mathrm{Calorie} = 1000\ \mathrm{calories} = 4186\ \mathrm{J}$.

ANSWER:

E_b = 157 kJ

Hint 3. Find the duration of the flight

If the bird consumes energy at a rate of 3.70W , how many hours t_b can it fly using the energy supply provided by 4 grams of fat?

Express your answer in hours.

Hint 1. Definition of power

The average power P (measured in watts) is the ratio of the energy ΔE transformed in the time interval Δt: $P = \frac{\Delta E}{\Delta t}$. Note that power measures either the rate at which energy is transferred (or transformed) or the rate at which work is performed.

Hint 2. Power: units

Power is measured in watts. One watt is equal to 1 joule per second (i.e., $1\ \mathrm{W} = 1\ \mathrm{J/s}$).

ANSWER:

t_b = $\frac{\frac{4 \cdot 9.4 \cdot 4186}{P_b}}{3600}$ = 11.8 hr

Hint 4. Find the distance in terms of average velocity

Which of the following expressions gives the distance d traveled in the time interval t at an average speed v?

ANSWER:

$d =$

- ○ v/t
- ○ vt^2
- ◉ vt
- ○ t/v

Now use this expression to find the distance traveled by the bird. Make sure that the units are consistent!

ANSWER:

d_b = $\frac{\frac{4 \cdot 9.4 \cdot 4186}{P_b}}{3600} v_b$ = 455 km

Part B

How many grams of carbohydrate m_{carb} would the bird have to consume to travel the same distance d_b?

Express your answer in grams

Hint 1. How to approach the problem

As stated in the introduction of this problem,1 gram of fat provides about 9.4 Calories, while 1 gram of carbohydrate provides 4.2 Calories.

ANSWER:

m_{carb} = 8.95 g

This is more than twice the amount of fat that was needed! In addition, to store 1 gram of carbohydrate (in the form of glycogen, the most common form of animal carbohydrate) about 3 grams of water are needed. Therefore, if energy were stored as carbohydrates, the bird would need to carry more than eight times the fuel mass to perform the same migratory flight!

Part C

Field observations suggest that a migrating ruby-throated hummingbird can fly across the Gulf of Mexico on a nonstop flight traveling a distance of about 800km . Assuming that the bird has an average speed of 40.0$\mathrm{km/hr}$ and an average power consumption of 1.70W , how many grams of fat m_{fat} does a ruby-throated hummingbird need to accomplish the nonstop flight across the Gulf of Mexico?

Express your answer in grams.

Hint 1. How to approach the problem

In Part A you were given the amount of fat consumed over the entire flight and were asked to calculate the distance traveled by the migrating bird. Now you need to solve the reverse problem. That is, given the distance traveled, calculate the amount of energy required to perform the flight. Thus, apply the same method as the one used in part A, only in reverse. From the information on distance and average speed, calculate the duration of the nonstop flight. Then use your result and the given power consumption to determine the amount of energy required for the flight. Finally, calculate how many grams are needed to provide that amount of energy.

Hint 2. Find the duration of the flight

How many hours $t_{\rm h}$ will the ruby-throated hummingbird fly to travel a distance of 800$\rm km$ at an average speed of 40.0$\rm km/hr$?

Express your answer in hours

ANSWER:

$t_{\rm h} = \frac{d_h}{v_h}$ = 20.0 $\rm hr$

Hint 3. Find the energy required for the nonstop flight

Given that the hummingbird consumes energy at an average rate of 1.70$\rm W$, how much energy $E_{\rm h}$ will it require during the nonstop flight over the Gulf of Mexico?

Express your answer in joules.

Hint 1. How to use power and time

Remember that power is energy transferred (or transformed) per unit time. Make sure you are using the correct unit for time.

ANSWER:

$E_{\rm h} = P_h \frac{d_h}{v_h} \cdot 3600$ = 1.22×10^5 $\rm J$

Now calculate how many grams of fat are required to provide this amount of energy. Recall that 1 gram of fat provides 9.4 Calories of energy.

ANSWER:

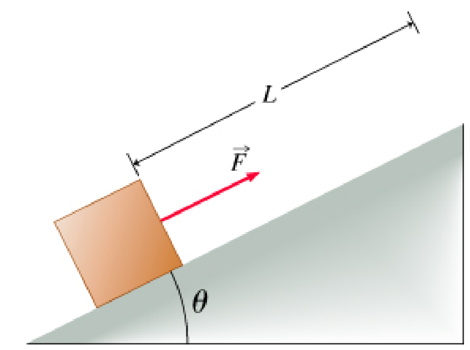
$m_{\rm fat} = \frac{P_h \frac{d_h}{v_h} \cdot 3600}{9.4 \cdot 4186}$ = 3.11 $\rm g$

Considering that in normal conditions the mass of a ruby-throated hummingbird is only 3 or 4 grams, the bird will need to almost double its body mass to store enough fat to perform the nonstop flight.

Work on a Block Sliding Up a Frictionless Incline

Description: A box is pulled up a frictionless incline. Find the work done by gravity, the pulling force, and the normal force. (version for algebra-based courses)

A block of weight w = 40.0$\rm N$ sits on a frictionless inclined plane, which makes an angle θ = 24.0° with respect to the horizontal, as shown in the figure. A force of magnitude F = 16.3$\rm N$, applied parallel to the incline, is just sufficient to pull the block up the plane at *constant speed*.



Part A

The block moves up an incline with constant speed. What is the total work W_{total} done on the block by all forces as the block moves a distance L = 2.50m up the incline? Include only the work done after the block has started moving at constant speed, not the work needed to start the block moving from rest.

Express your answer numerically in joules.

Hint 1. What physical principle to use

To find the total work done on the block, use conservation of energy, which relates the total work done to the initial and final kinetic energies:

$$W_{\text{total}} = K_{\text{final}} - K_{\text{initial}}.$$

Hint 2. Find the change in kinetic energy

What is the change in the kinetic energy of the block during this process? Keep in mind that that the block moves at constant speed.

Express your answer numerically in joules.

Hint 1. A formula for kinetic energy

The kinetic energy of an object is related to its mass and velocity by the formula

$$K = \frac{1}{2}mv^2.$$

ANSWER:

$K_{\text{final}} - K_{\text{initial}}$ = 0 J

ANSWER:

W_{total} = 0 J

Part B

What is W_{g}, the work done on the block by the force of gravity $\vec{w}$ as the block moves a distance L = 2.50m up the incline?

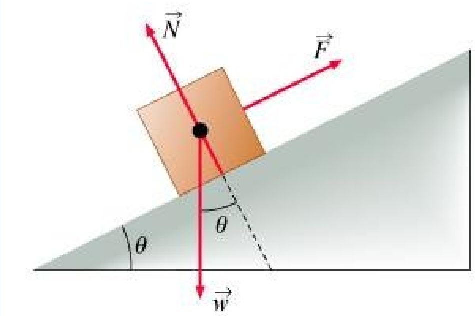
Express your answer numerically in joules.

Hint 1. An equation for work done by a constant force

Recall that the equation for the work done by a constant force is given as $W = Fd\cos\phi$, where ϕ is the angle between the force vector $\vec{F}$ and the displacement vector $\vec{d}$. Another way to express this relationship is through the equation $W = F_{||}d$, where $F_{||}$ is the component of the force vector that lies in the same direction as the displacement. When using this expression, you must be careful to determine the sign of the work done by the force. If $F_{||}$ points in the same direction as $\vec{d}$, then $F_{||}$ is positive. If $F_{||}$ points in the opposite direction from $\vec{d}$, then $F_{||}$ is negative. The next two hints will help you find $w_{||}$, the parallel component of the gravitational force. Then you can find the work done by gravity using the general formula $W = F_{||}d$.

Hint 2. Force diagram

The figure shows a diagram of the forces acting on the block.



Hint 3. Find the component of the gravitational force parallel to the plane

What is $w_{||}$, the magnitude of the component of the force of gravity along the inclined plane?

Express your answer numerically in newtons.

Hint 1. Vector components of the force of gravity

The force due to gravity, often called the weight, has components both parallel and perpendicular to the inclined plane. Based on the force diagram in Hint B.2, the component of $\vec{w}$ parallel to the inclined plane has magnitude given by $w_{||} = w\sin\theta$.

ANSWER:

$w_{||}$ = $w\sin(\theta)$ = 16.3 N

Hint 4. Relative directions of force and motion

Keep in mind that work done by a force is positive if a force acts in the direction of motion, and is negative if the force acts against the direction of motion.

ANSWER:

W_g = $-w(\sin(\theta))L$ = -40.7 J

Part C

What is W_F, the work done on the block by the applied force $\vec{F}$ as the block moves a distance L = 2.50 m up the incline?

Express your answer numerically in joules.

Hint 1. Equation for work done by a constant force

Use the same procedure as that presented in Hint B.1. The displacement $\vec{d}$ is the same, but the force vector is different.

ANSWER:

W_F = FL = 40.7 J

You may have noticed that $W_g = -W_F$. This is not a coincidence, of course. Can you see why? If yes, the next part will be easy.

Part D

What is W_N, the work done on the block by the normal force as the block moves a distance L = 2.50m up the inclined plane?

Express your answer numerically in joules.

Hint 1. The parallel component of the normal force

The normal force and the block's displacement vector are perpendicular.

ANSWER:

W_N = 0 J

Now consider this: $W_{\rm total} = W_{\rm g} + W_F + W_N$. Also, $W_{\rm total} = 0$ and $W_N = 0$. Therefore, $W_{\rm g} + W_F = 0$, or $W_{\rm g} = -W_F$. It may have been easier to solve Part C first. Perhaps that is what you did.

Projectile Motion and Conservation of Energy Ranking Task

Description: Conceptual question on projectile motion using conservation of energy. (ranking task)

Part A

Six baseball throws are shown below. In each case the baseball is thrown at the same initial speed and from the same height H above the ground. Assume that the effects of air resistance are negligible. Rank these throws according to the speed of the baseball the instant before it hits the ground.

Rank from largest to smallest. To rank items as equivalent, overlap them.

Hint 1. How to approach the problem

Although this situation can be investigated using the concepts of projectile motion, the conservation of mechanical energy is a better approach. Consider the initial total energy (kinetic plus gravitational potential). By conservation of energy, the final total energy must be equal to the initial total energy. You can use the final energy to determine the final speed when it reaches the ground. Note that the launch angle does not affect the initial kinetic or initial gravitational potential energy of the ball.

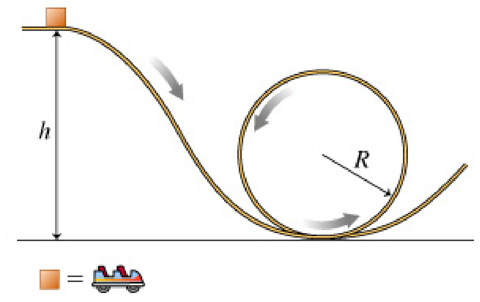
ANSWER:

This answer is best understood in terms of conservation of energy. The initial energy of the ball is independent of the direction in which it is thrown. The initial and final potential energies of the ball are the same regardless of the trajectory. Therefore, the final kinetic energy, and therefore the final speed, of the ball must be the same no matter in what direction it is thrown.

Loop the Loop

Description: Use conservation of energy to find the kinetic energy of a roller-coaster car at the top of a loop. Then analyze forces to find the minimum initial height the car needs in order to make it around the loop. (version for algebra-based courses)

A roller-coaster car may be represented by a block of mass 50.0kg . The car is released from rest at a height h = 45.0m above the ground and slides along a frictionless track. The car encounters a loop of radius R = 15.0m at ground level, as shown. As you will learn in the course of this problem, the initial height 45.0m is great enough so that the car never loses contact with the track.



Part A

Find an expression for the kinetic energy K of the car at the top of the loop.

Express the kinetic energy numerically, in joules.

Hint 1. Find the potential energy at the top of the loop

What is the potential energy U_{top} of the car when it is at the top of the loop? Define the gravitational potential energy to be zero at ground level.

Express your answer numerically, in joules.

ANSWER:

U_{top} = $mg(2R)$ = 1.47×10^4 J

ANSWER:

K = $mg(h-2R)$ = 7350 J

Part B

Find the minimum initial height h_{min} at which the car can be released that still allows the car to stay in contact with the track at the top of the loop.

Express your answer numerically, in meters.

Hint 1. The meaning of "just staying on the track"

For the car to *just* stay in contact through the loop, without falling, the magnitude n of the normal force $\vec{n}$ acting on the car at the top of the loop must be just *barely greater* than zero. In your calculations, you can use $n = 0$.

Hint 2. How to approach the problem

Find the velocity at the top of the loop such that the only force acting on the car (its weight) provides exactly the necessary centripetal acceleration. This will give you the answer. Consider this: If the velocity were any greater, it would take some force from the track to provide the necessary centripetal force. If the velocity were any smaller, the car would simply fall off the track.

Use the described condition to find the velocity and then conservation of mechanical energy to find the required height.

Hint 3. Find the velocity at the top of the loop

What is the minimum velocity $v_{\rm min}$ that the car can have at the top of the loop?

Express your answer numerically, in meters per second.

Hint 1. Equation for the centripetal acceleration

Recall that the centripetal acceleration $a_{\rm c}$ is given by $a_{\rm c} = v^2/R$ for an object traveling with velocity v along a circle of radius R.

Hint 2. Weight and acceleration

At this minimum velocity, the only force acting on the car at the top of the loop is its weight. This force, then, is the net force acting on the car. Use Newton's second law to link the weight and the centripetal acceleration.

ANSWER:

$v_{\rm min}$ = $\sqrt{Rg}$ = 12.1 $\rm m/s$

ANSWER:

$h_{\rm min}$ = $\frac{5R}{2}$ = 37.5 $\rm m$

If you solve the problem using variables instead of numbers, you will find that the minimum height required is given by $h_{\rm min} = 2.5R$.

For $h > 2.5R$ the car can still complete the loop, of course. In this case, the normal force will be greater than zero even at the top of the loop.

For $h < R$ the car would oscillate in the bottom part of the loop. Could you predict this?

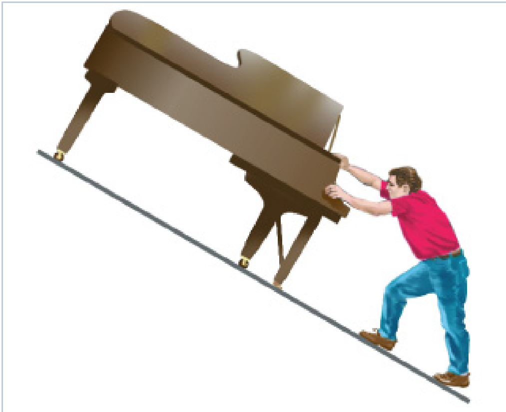
For $R < h < 2.5R$, the car would lose contact with the track before reaching the top. That is why roller coasters must have a *lot* of safety features. If you like, you can check that the angle at which the car loses contact with the track is given by $\theta = \arcsin \frac{2}{3}\left(\frac{h}{R} - 1\right)$.

Interestingly, if you try to check your result in a school or a univeristy lab with a steel or glass marble, you will see that the necessary minimum height is *greater* than $2.5R$. This is because a marble also has *rotational* kinetic energy in addition to *translational* kinetic energy.

Problem 6.8

Description: A m piano slides d down a theta incline and is kept from accelerating by a man who is pushing back on it parallel to the incline (see the figure). The effective coefficient of kinetic friction is mu_k. (a) Calculate the force exerted by the man. (b) ...

A 350$\rm kg$ piano slides 3.7$\rm m$ down a 28° incline and is kept from accelerating by a man who is pushing back on it *parallel to the incline* (see the figure). The effective coefficient of kinetic friction is 0.40.



Part A

Calculate the force exerted by the man.

Express your answer using two significant figures.

ANSWER:

F_p = $m \cdot 9.80\left(\sin(\theta) - \mu_k \cos(\theta)\right)$ = 4.0×10^{2} N

Part B

Calculate the work done by the man on the piano.

Express your answer using two significant figures.

ANSWER:

W_p = $-m \cdot 9.80\left(\sin(\theta) - \mu_k \cos(\theta)\right) d$ = -1.5×10^{3} J

Part C

Calculate the work done by the friction force.

Express your answer using two significant figures.

ANSWER:

W_{fr} = $-\mu_k m \cdot 9.80 d \cos(\theta)$ = -4.5×10^{3} J

Part D

Calculate the work done by the force of gravity.

Express your answer using two significant figures.

ANSWER:

W_G = $m \cdot 9.80 d \cos\left(\frac{\pi}{2} - \theta\right)$ = 6.0×10^{3} J

Part E

Calculate the net work done on the piano.

ANSWER:

W_{Net} = 0 J

Problem 6.19

Description: A m arrow is fired from a bow whose string exerts an average force of F on the arrow over a distance of d. (a) What is the speed of the arrow as it leaves the bow?

A 83g arrow is fired from a bow whose string exerts an average force of 115N on the arrow over a distance of 78cm .

Part A

What is the speed of the arrow as it leaves the bow?

Express your answer using two significant figures.

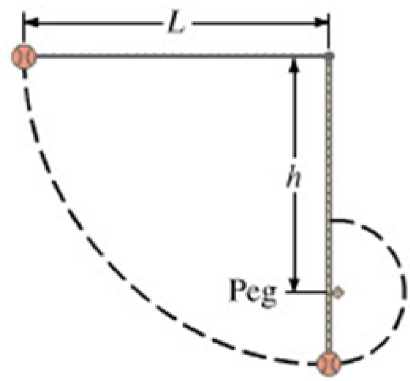
ANSWER:

$v = \sqrt{\frac{2Fd}{m}}$ = 46 $\mathrm{m/s}$

Problem 6.77

Description: A ball is attached to a horizontal cord of length L, whose other end is fixed. (a) If the ball is released, what will its speed be at the lowest point of its path? (b) A peg is located at a distance h directly below the point of attachment of the...

A ball is attached to a horizontal cord of length L, whose other end is fixed.



Part A

If the ball is released, what will its speed be at the lowest point of its path?

Express your answer in terms of g and L.

ANSWER:

v_{bottom} = $\sqrt{2gL}$

Part B

A peg is located at a distance h directly below the point of attachment of the cord. If $h = 0.8L$, what will be the speed of the ball when it reaches the top of its circular path about the peg?

Express your answer in terms of g and L.

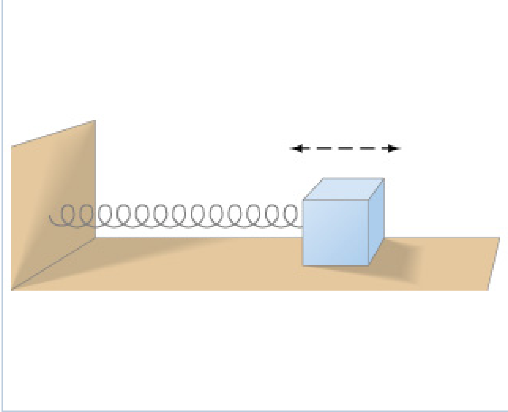
ANSWER:

v_{top} = $\sqrt{1.2gL}$

Problem 6.55

Description: A m-kg wood block is firmly attached to a very light horizontal spring k as shown in the figure . It is noted that the block-spring system, when compressed x_1 and released, stretches out x_2 beyond the equilibrium position before stopping and turning ...

A 0.670-kg wood block is firmly attached to a very light horizontal spring $(k = 210 \mathrm{N/m})$ as shown in the figure . It is noted that the block-spring system, when compressed 5.2cm and released, stretches out 2.5cm beyond the equilibrium position before stopping and turning back.



Part A

What is the coefficient of kinetic friction between the block and the table?

Express your answer using two significant figures.

ANSWER:

$\mu_k = \frac{k\left(x_1{}^2 - x_2{}^2\right)}{2m \cdot 9.80\left(x_2 + x_1\right)}$ = 0.43