Phys102 Lecture 13/14
DC Circuits

Key Points

- EMF and Terminal Voltage
- Resistors in Series and in Parallel
- Circuits Containing Resistor and Capacitor (*RC* Circuits)

References

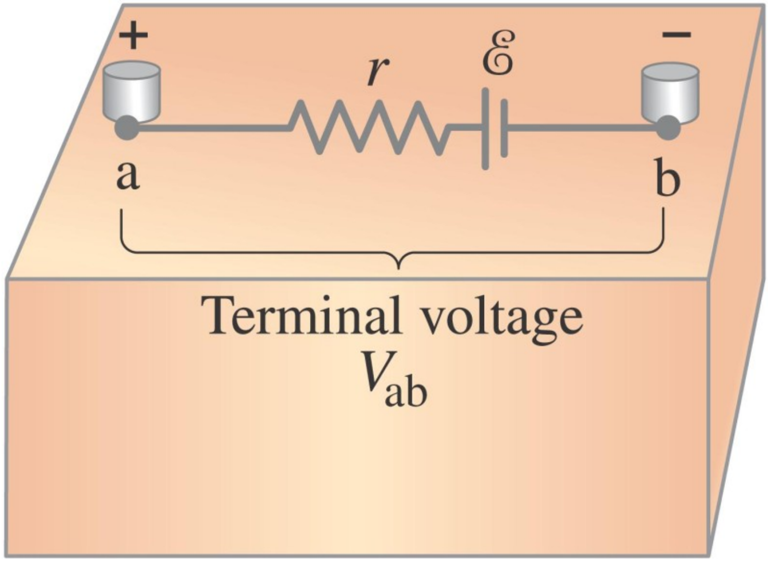
SFU Ed: 26-1,2,5.

6th Ed: 19-1,2,5,6.

26-1 EMF and Terminal Voltage

Electric circuit needs battery or generator to produce current – these are called sources of emf.

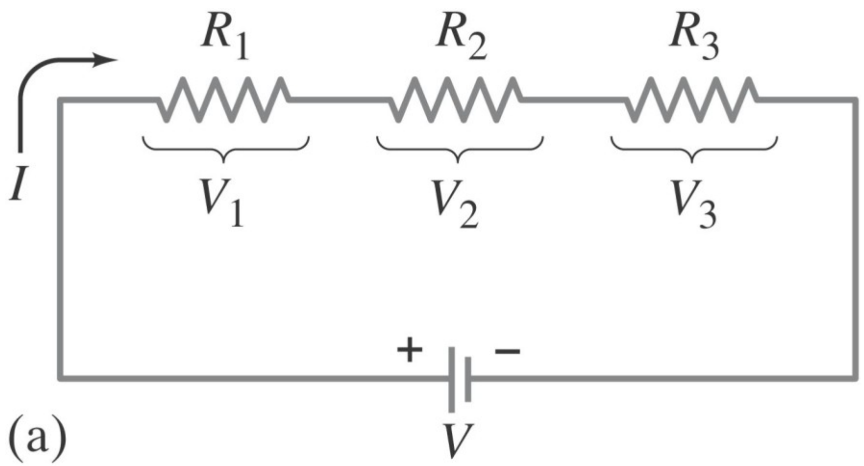
Battery is a nearly constant voltage source, but does have a small internal resistance (in series with the emf) , which reduces the actual voltage from the ideal emf:

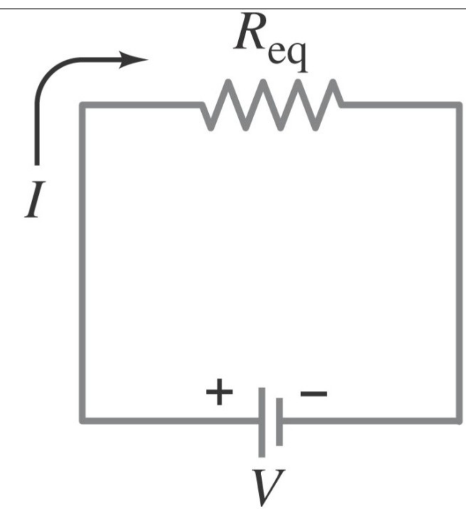


$$V_{ab} = \mathscr{E} - Ir.$$

Resistors in Series

A series connection has a single path with the same current.





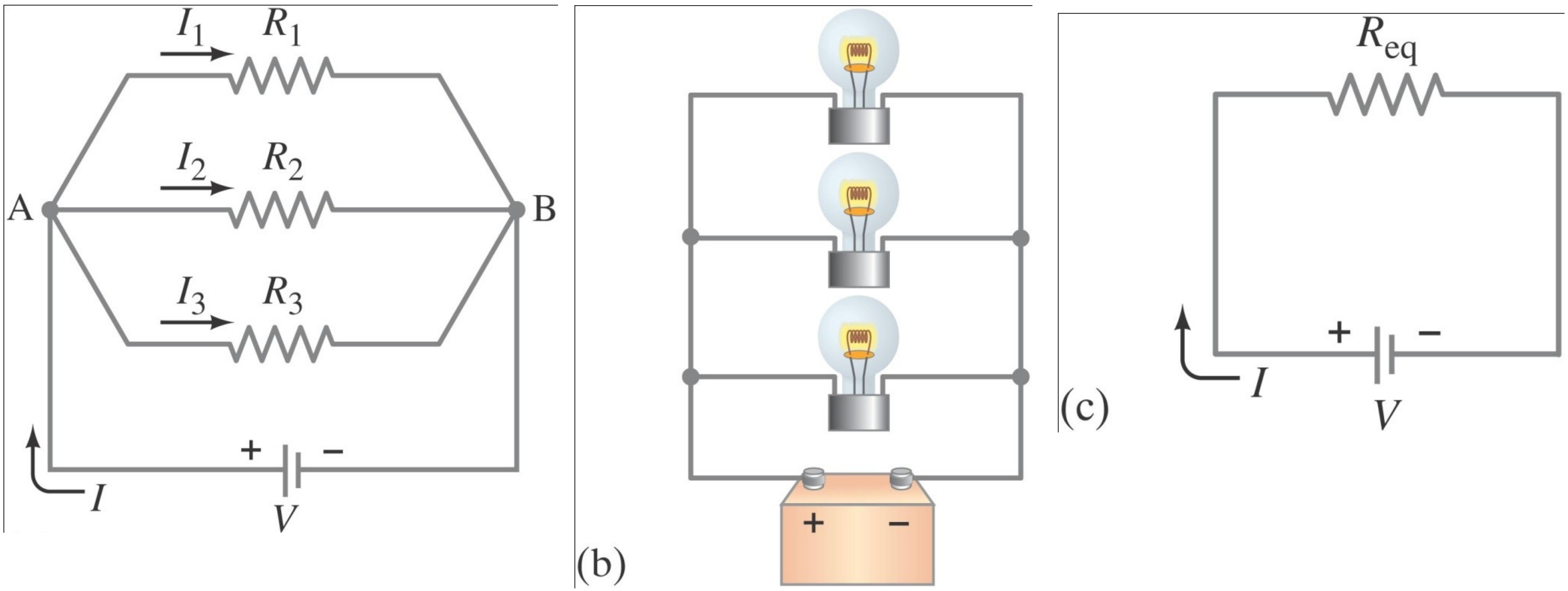
The sum of the voltage drops across the resistors equals the battery voltage

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3. \quad \text{[series]}$$

$$R_{eq} = R_1 + R_2 + R_3. \quad \text{[series]}$$

Resistors in Parallel

A parallel connection splits the current; the voltage across each resistor is the same:



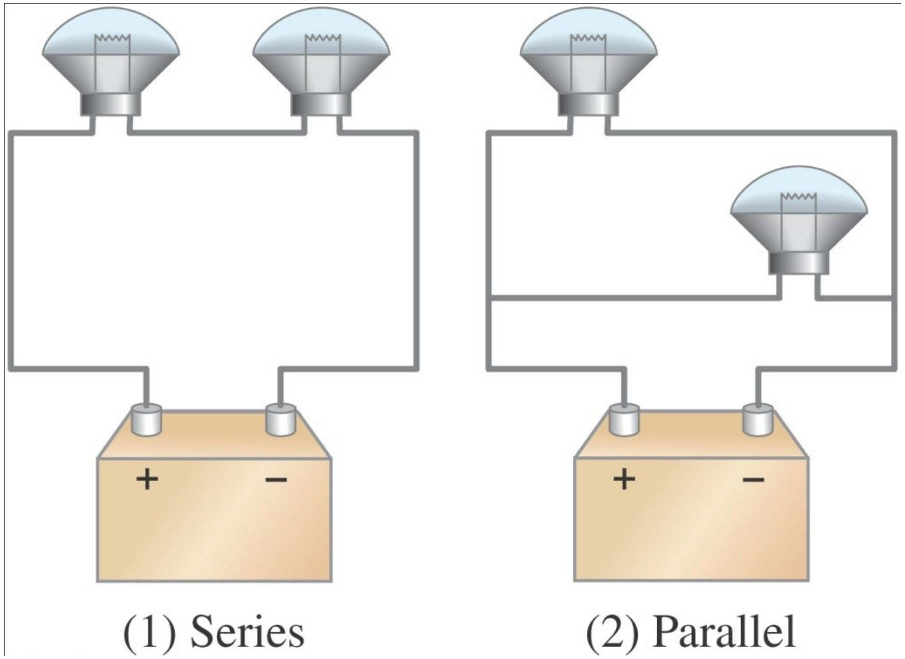
$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}.$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Conceptual Example 26-2: Series or parallel?

(a) The lightbulbs in the figure are identical. Which configuration produces more light? (b) Which way do you think the headlights of a car are wired? Ignore change of filament resistance R with current.

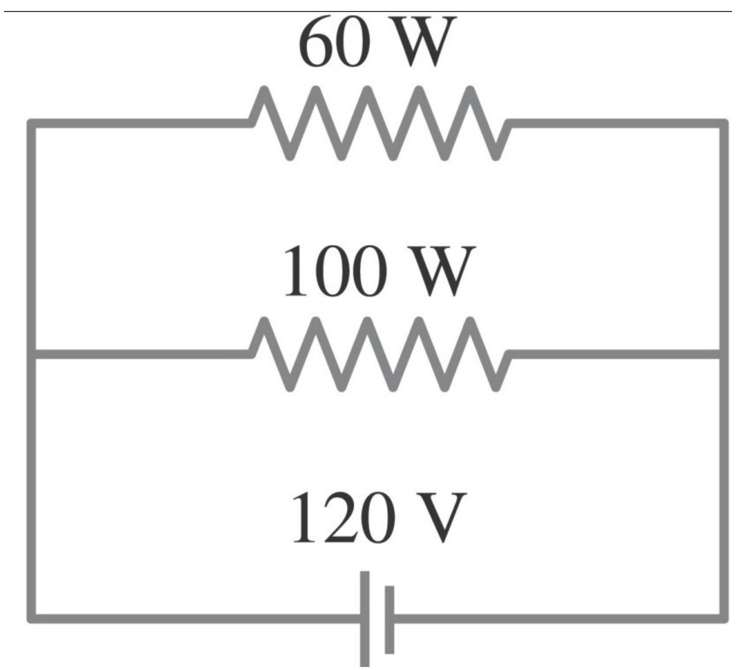


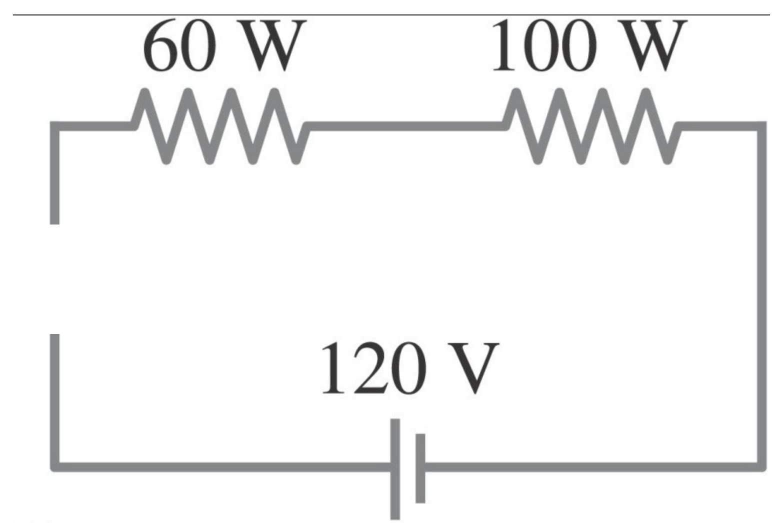
$$P = \frac{V^2}{R}$$

26-2 Resistors in Series and in Parallel

Conceptual Example 26-3: An illuminating surprise.

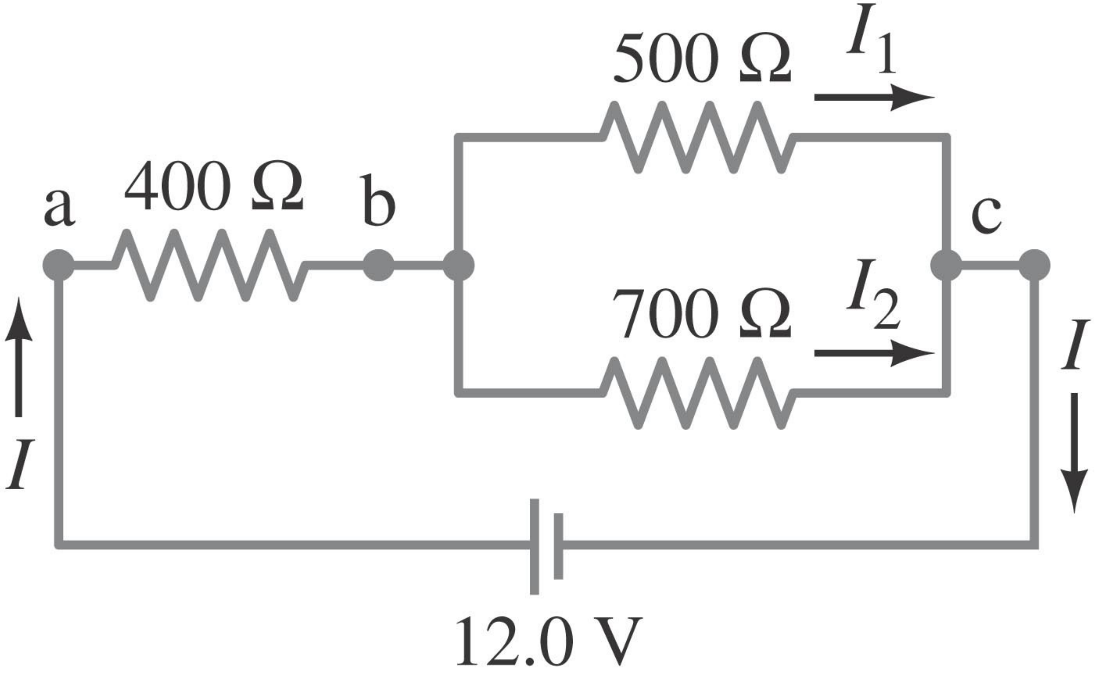
A 100-W, 120-V lightbulb and a 60-W, 120-V lightbulb are connected in two different ways as shown. In each case, which bulb glows more brightly? Ignore change of filament resistance with current (and temperature).





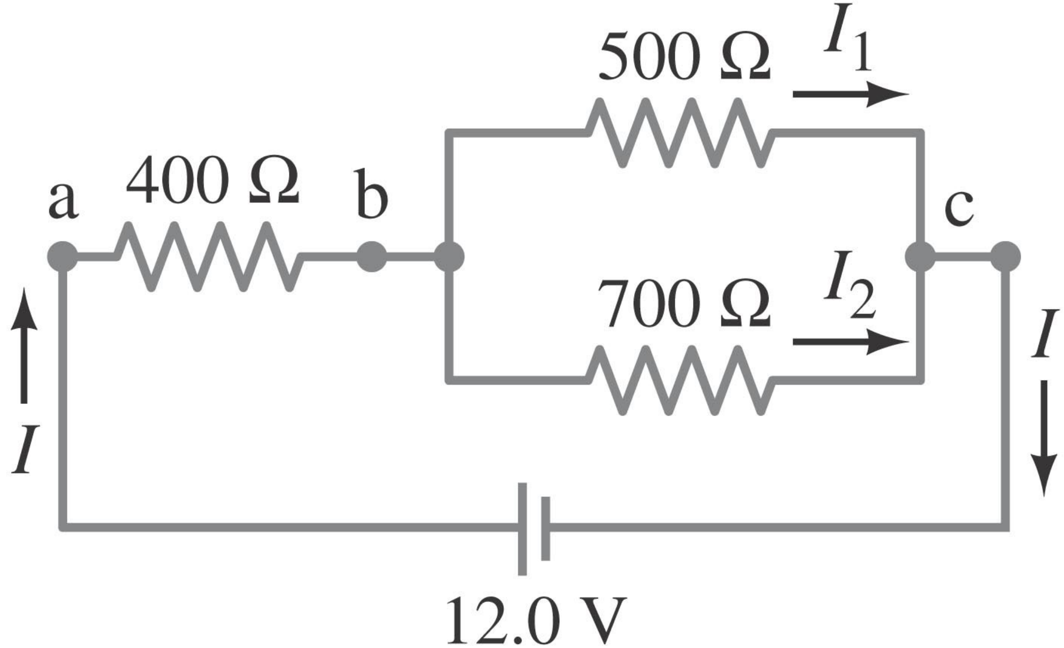
Example 26-4: Circuit with series and parallel resistors.

How much current is drawn from the battery shown?



Example 26-5: Current in one branch.

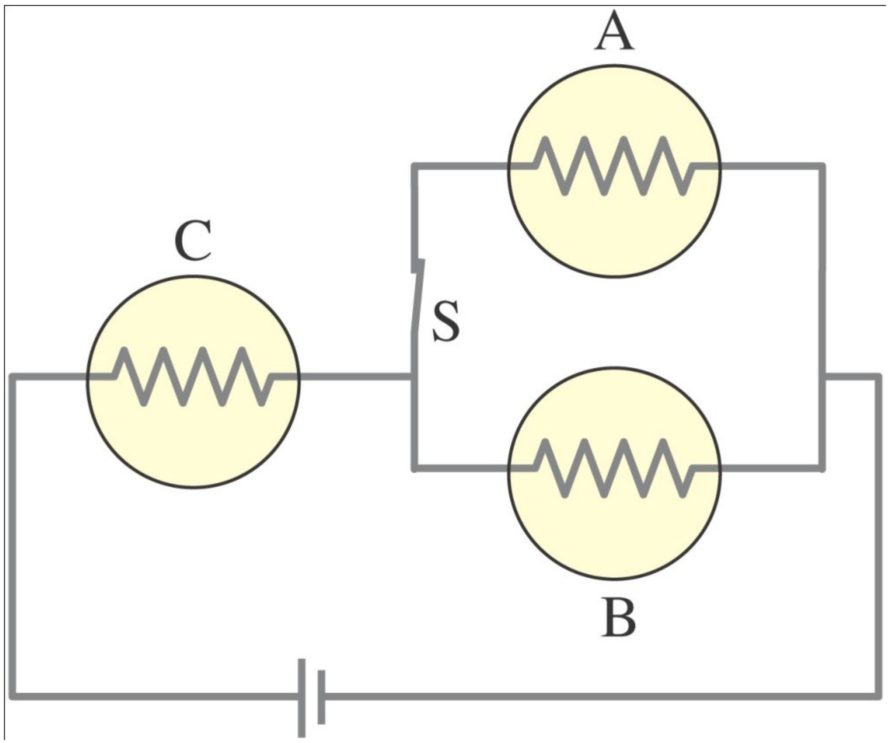
What is the current through the 500-Ω resistor shown? (Note: This is the same circuit as in the previous problem.) The total current in the circuit was found to be 17 mA.



Conceptual Example 26-6: Bulb brightness in a circuit

(i-clicker 13-1).

The circuit shown has three identical lightbulbs, each of resistance R. When switch S is closed,



A) C is brighter than A and B (A and B are equally bright);

B) A and B are brighter than C (A and B are equally bright);

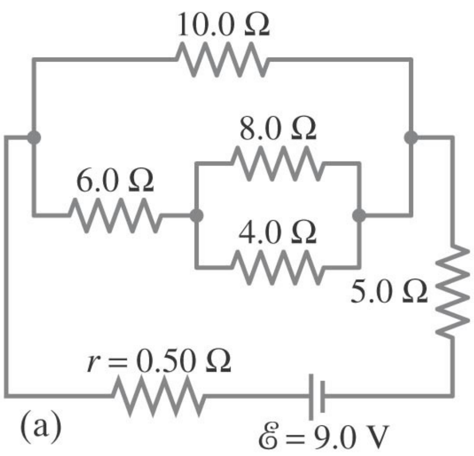
C) A is brighter than C, and C is brighter than B;

D) C is brighter than B, and B is brighter than A;

E) A, B and C are equally bright.

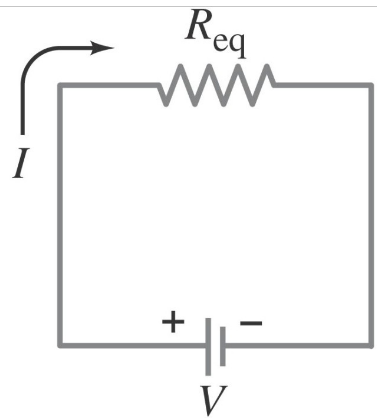
Example 26-8: Analyzing a circuit.

A 9.0-V battery whose internal resistance r is 0.50 Ω is connected in the circuit shown. (a) How much current is drawn from the battery? (b) What is the terminal voltage of the battery?



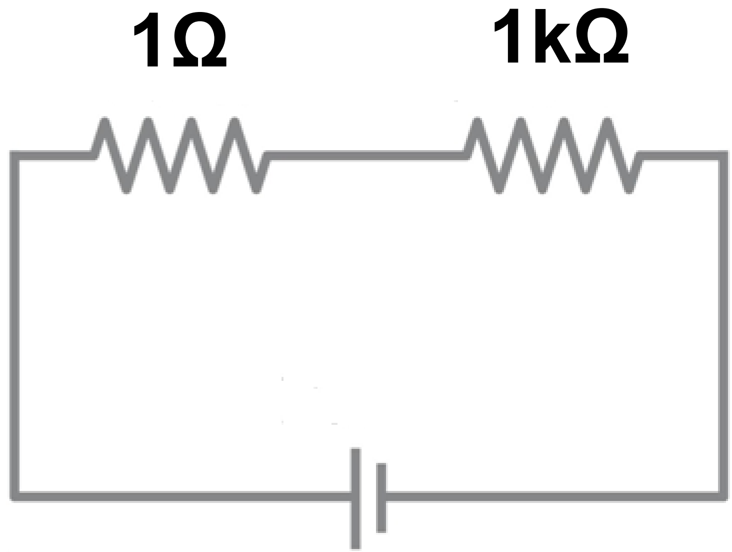
(c) What is the current in the 6.0-Ω resistor?

Ultimately, we want an equivalent circuit like:



i-clicker 13-2.

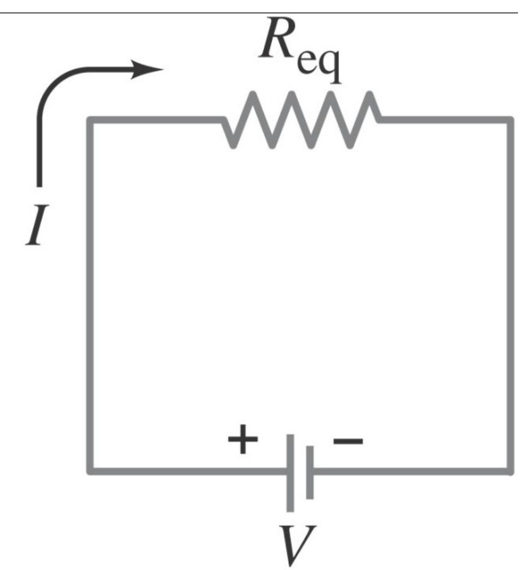
Estimate the equivalent resistance R_{eq}.



A) $R_{eq} \sim 1\Omega$;

B) $R_{eq} \sim 1k\Omega$;

C) $R_{eq} \sim 500\Omega$.



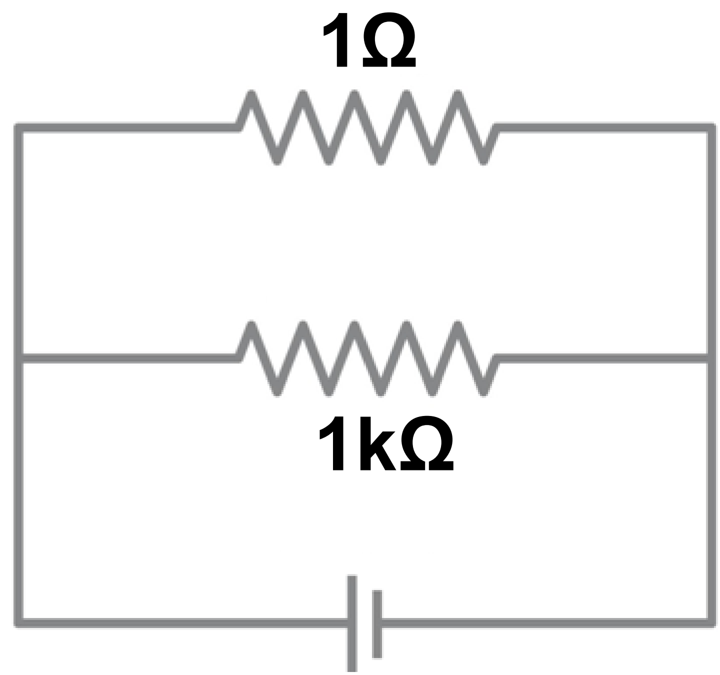
i-clicker 13-3.

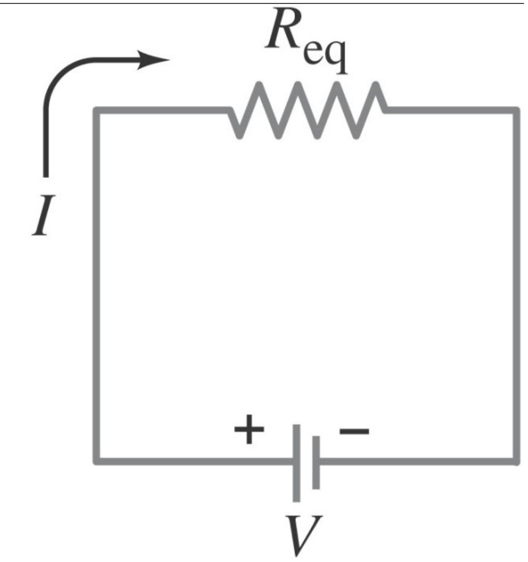
Estimate the equivalent resistance R_{eq}.

A) $R_{eq} \sim 1\Omega$;

B) $R_{eq} \sim 1k\Omega$;

C) $R_{eq} \sim 500\Omega$.





26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

When the switch is closed, the capacitor will begin to charge. As it does, the voltage across it increases, and the current through the resistor decreases.

Charging a capacitor C through a resistor R:

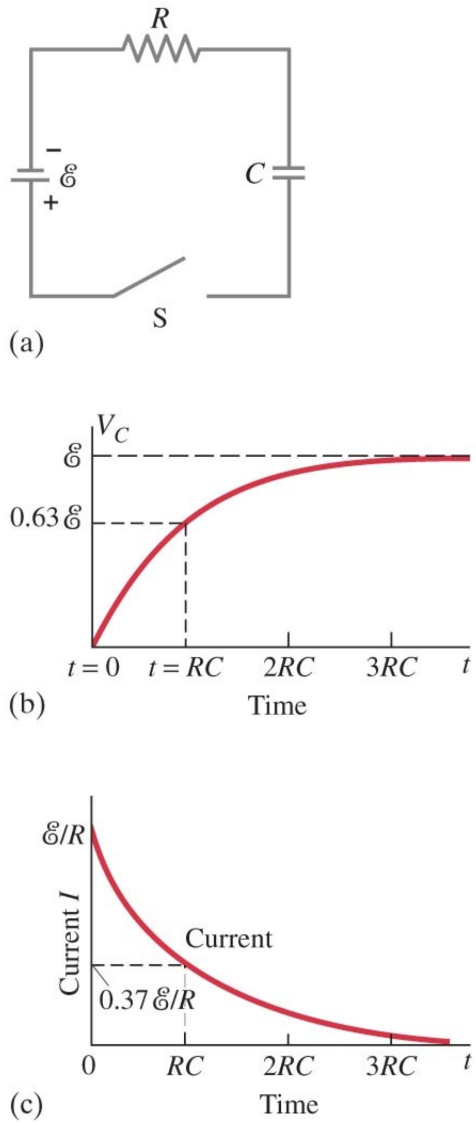
$$I = \frac{\varepsilon}{R} e^{-\frac{t}{RC}} = I_0 e^{-t/\tau}$$

$$I_0 = \frac{\varepsilon}{R}; \quad \tau = RC$$

$$V_C = \mathscr{E}\left(1 - e^{-t/RC}\right).$$

At t=0 (when you just close the switch), the capacitor behaves like being shorted;

At t=∞ (after a long time), the capacitor behaves like being open.



Math Derivation (not required)

To find the voltage as a function of time, we write the equation for the voltage changes around the loop:

$$\mathcal{E} = IR + \frac{Q}{C}.$$

Since $Q = dI/dt$, we can integrate to find the charge as a function of time:

$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right).$$

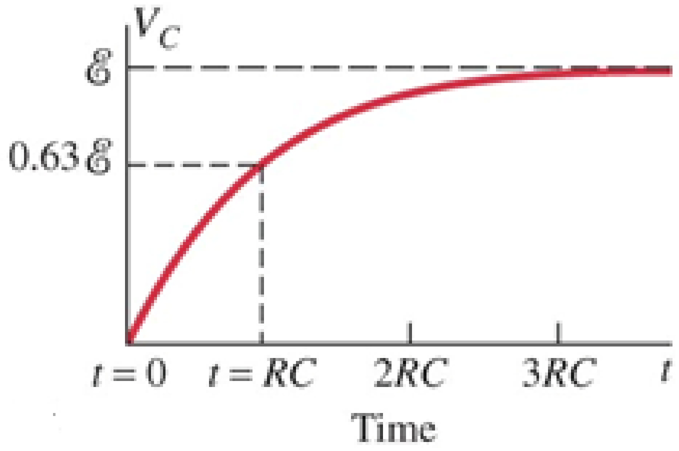
The voltage across the capacitor is $V_C = Q/C$:

$$V_C = \mathscr{E}\left(1 - e^{-t/RC}\right).$$

The quantity RC that appears in the exponent is called the time constant of the circuit:

$$\tau = RC.$$

RC is the time it takes to charge the capacitor to 63%.



The larger the C, the more charge it can store, so the longer time to charge;

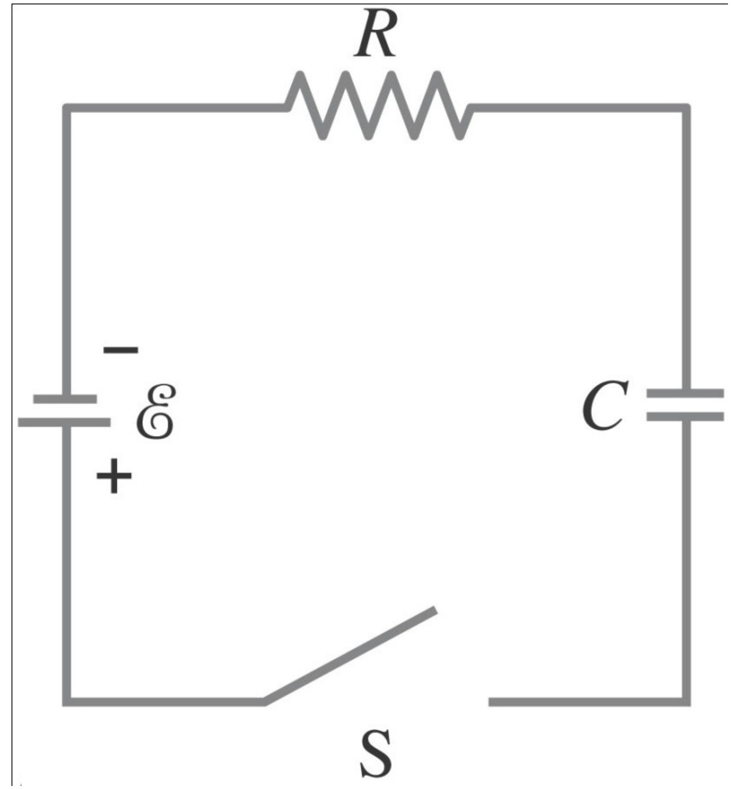
The larger the R, the more resistance, so the longer it takes to charge the capacitor

The current at any time t can be found by differentiating the charge:

$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}\, e^{-t/RC}.$$

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

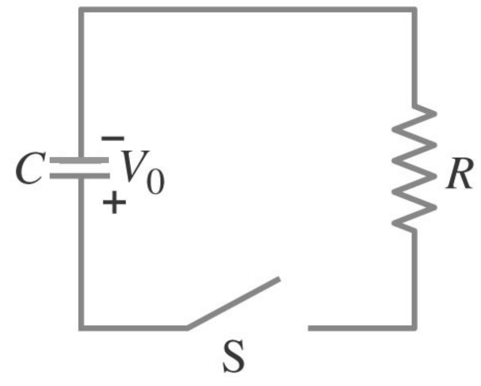
Example 26-11: *RC* circuit, with emf.



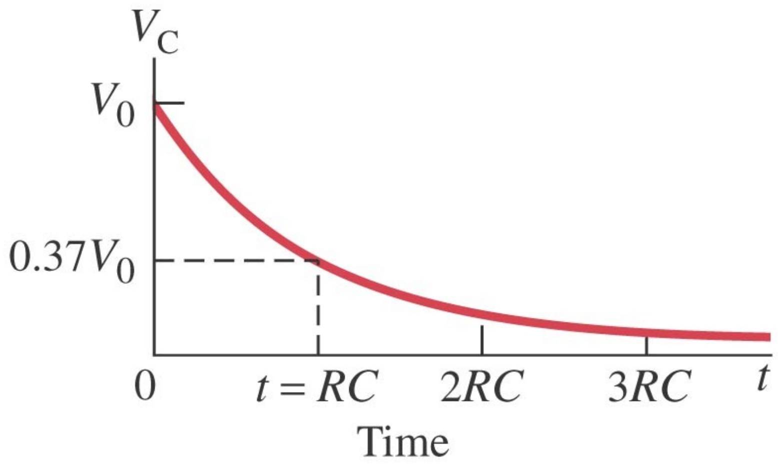
The capacitance in the circuit shown is $C = 0.30\ \mu F$, the total resistance is 20 kΩ, and the battery emf is 12 V. Determine (a) the time constant, (b) the maximum charge the capacitor could acquire, (c) the time it takes for the charge to reach 99% of this value, (d) the current I when the charge Q is half its maximum value, (e) the maximum current, and (f) the charge Q when the current I is 0.20 its maximum value.

26-5 Circuits Containing Resistor and Capacitor (*RC* Circuits)

If an isolated charged capacitor is connected across a resistor, it discharges:



$$Q = Q_0 e^{-t/RC}.$$



Once again, the voltage and current as a function of time can be found from the charge:

$$V_C = V_0 e^{-t/RC}$$

and

$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} = I_0 e^{-t/RC}.$$

Example 26-12: Discharging RC circuit.

In the RC circuit shown, the battery has fully charged the capacitor, so $Q_0 = C\mathcal{E}$. Then at $t = 0$ the switch is thrown from position a to b. The battery emf is 20.0 V, and the capacitance $C = 1.02\ \mu$F. The current I is observed to decrease to 0.50 of its initial value in 40 μs. (a) What is the value of Q, the charge on the capacitor, at $t = 0$? (b) What is the value of R? (c) What is Q at $t = 60\ \mu$s?