Phys102 Lecture 31-33
Diffraction of Light

Key Points

- Diffraction by a Single Slit
- Diffraction in the Double-Slit Experiment
- Limits of Resolution
- Diffraction Grating and Spectroscopy
- Polarization

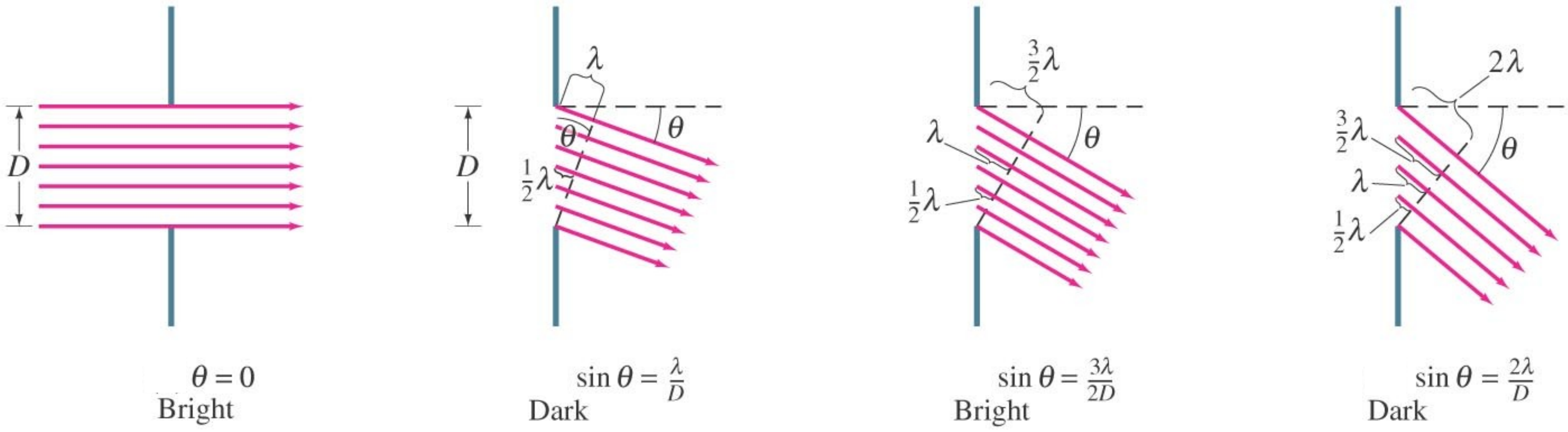
References

SFU Ed: 35-1,2,3,4,5,6,7,8,11.

6th Ed: 24-5,6,7,10; 25-7,8,9.

Diffraction by a Single Slit

Different points along a slit create wavelets that interfere with each other.

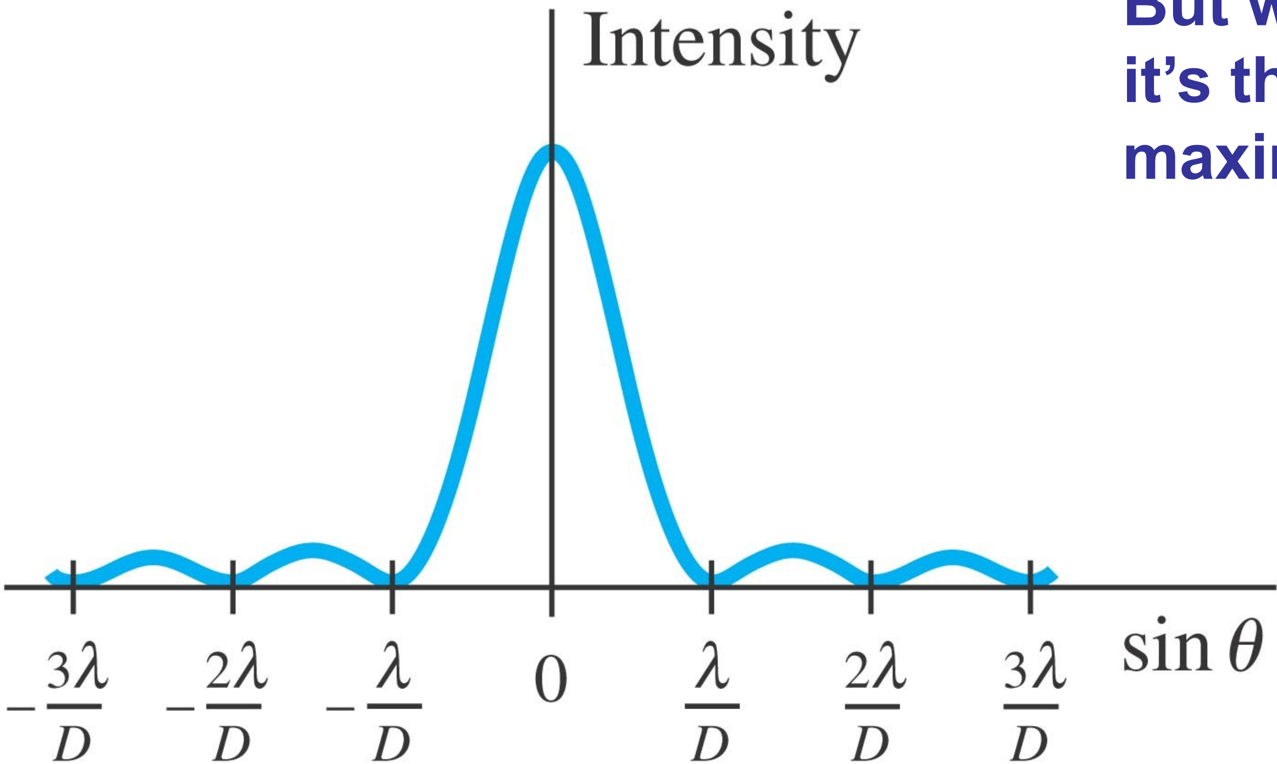


Diffraction by a Single Slit

The minima of the single-slit diffraction pattern occur when

$$D \sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \cdots. \qquad \text{[minima]}$$

But when m=0, it's the central maximum.



Diffraction by a Single Slit

Example 35-1: Single-slit diffraction maximum.

Light of wavelength 750 nm passes through a slit 1.0 x 10^{-3} mm wide. How wide is the central maximum (a) in degrees, and (b) in centimeters, on a screen 20 cm away?

$$\text{First minimum}: \; D \sin\theta = \lambda$$

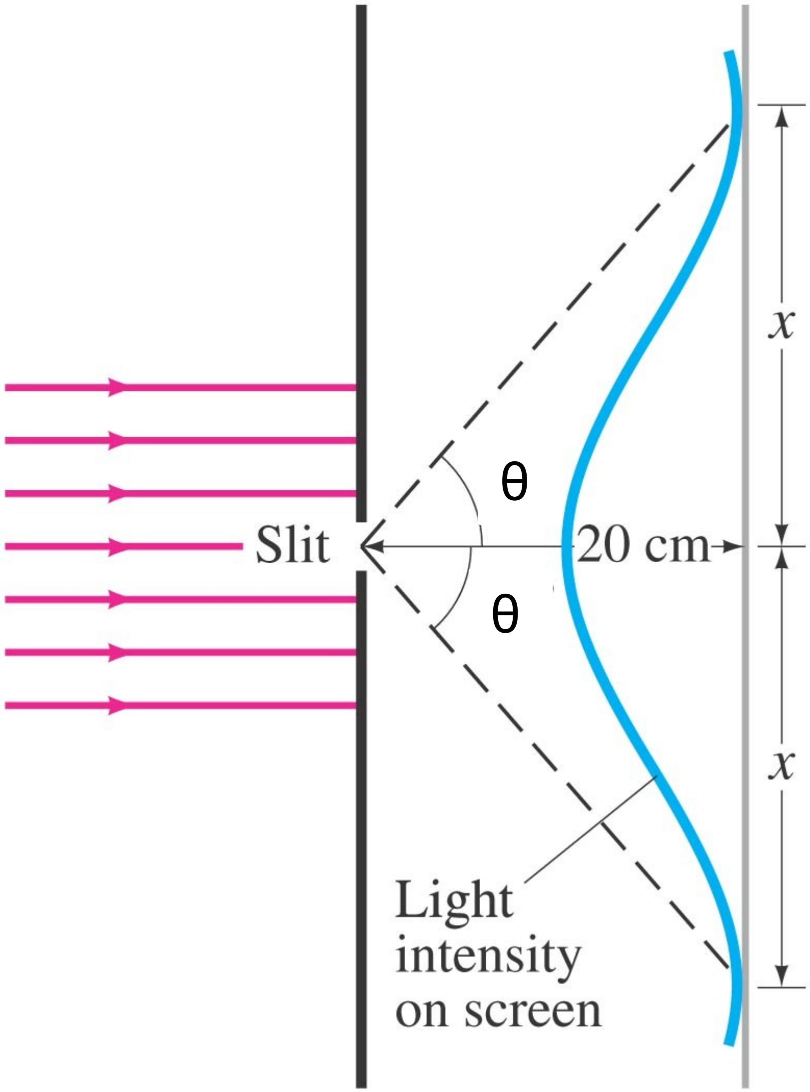
$$\theta = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{7.5\times10^{-7}}{1.0\times10^{-6}}\right) = 48.6^\circ$$

$$x = 0.20 \times \tan 48.6^\circ = 0.227 m$$

$$\text{Width of central maximum}:$$

$$2\theta = 97.2^\circ$$

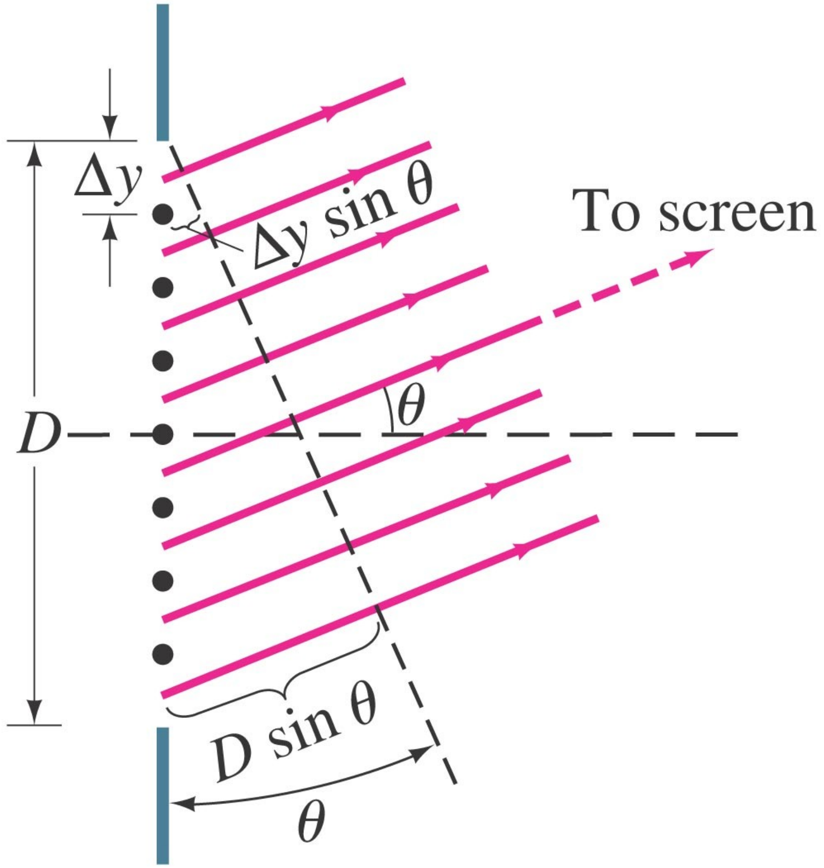
$$2x = 0.454 m$$



Intensity in Single-Slit Diffraction Pattern

Light passing through a single slit can be divided into a series of narrower strips; each contributes the same amplitude to the total intensity on the screen, but the phases differ due to the differing path lengths:

$$\Delta\beta = \frac{2\pi}{\lambda}\Delta y \sin\theta$$



Path difference between the two edges:

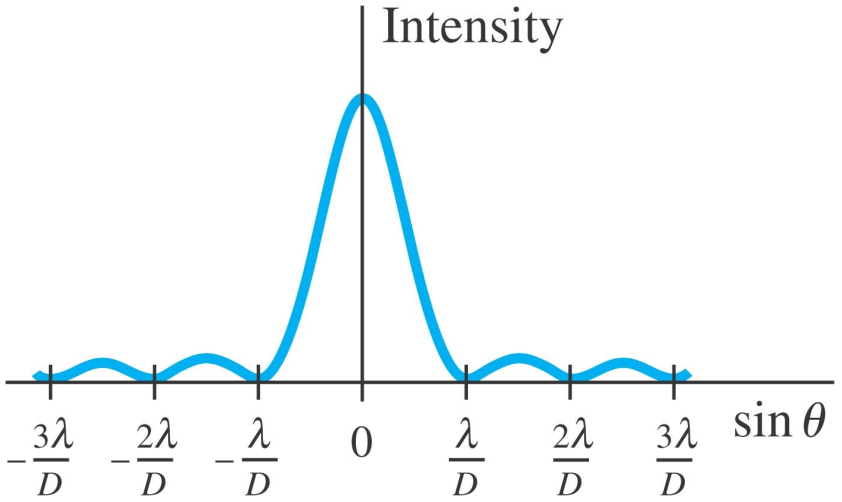
$$\beta = \frac{2\pi}{\lambda} D \sin\theta.$$

Intensity in Single-Slit Diffraction Pattern

Finally, we have the intensity as a function of angle:

$$I_\theta = I_0\left(\frac{\sin \beta/2}{\beta/2}\right)^2.$$

$$I_\theta = I_0\left[\frac{\sin\left(\frac{\pi D \sin\theta}{\lambda}\right)}{\left(\frac{\pi D \sin\theta}{\lambda}\right)}\right]^2$$

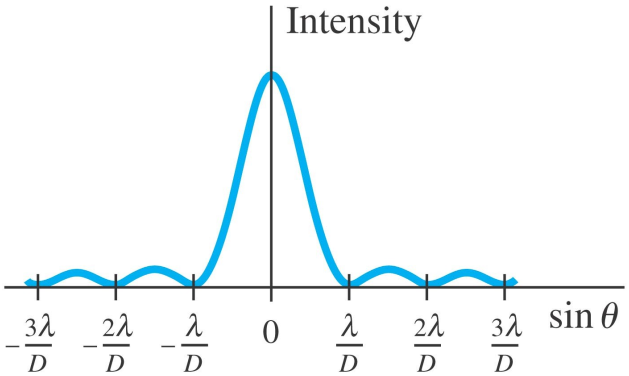


Condition for minima:

$$D\sin\theta = m\lambda, \qquad m = \pm 1, \pm 2, \pm 3, \cdots. \qquad \text{[minima]}$$

Example 35-3: Intensity at secondary maxima.

Estimate the intensities of the first two secondary maxima to either side of the central maximum.



[Solution] The secondary maxima occur approximately halfway between the minima:

$$D\sin\theta = \left(m+\frac{1}{2}\right)\lambda \quad \text{(secondary maxima)}$$

For the first secondary maximum: $D\sin\theta = \frac{3}{2}\lambda$

$$I = I_0\left[\frac{\sin\left(\frac{\pi D\sin\theta}{\lambda}\right)}{\left(\frac{\pi D\sin\theta}{\lambda}\right)}\right]^2 = I_0\left[\frac{\sin\left(\frac{3\pi}{2}\right)}{\left(\frac{3\pi}{2}\right)}\right]^2 = 0.045 I_0$$

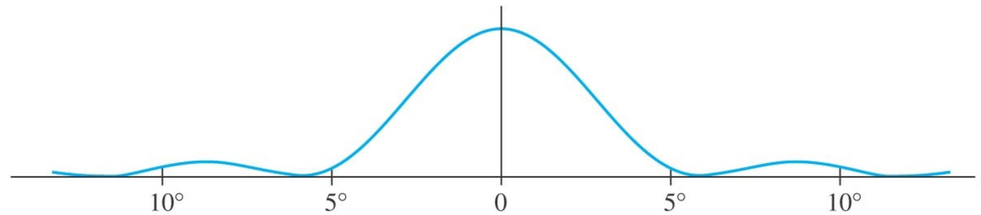
For the second secondary maximum: $D\sin\theta = \frac{5}{2}\lambda \quad (m=2)$

$$I = I_0\left[\frac{\sin\left(\frac{\pi D\sin\theta}{\lambda}\right)}{\left(\frac{\pi D\sin\theta}{\lambda}\right)}\right]^2 = I_0\left[\frac{\sin\left(\frac{5\pi}{2}\right)}{\left(\frac{5\pi}{2}\right)}\right]^2 = 0.016 I_0$$

Diffraction in the Double-Slit Experiment

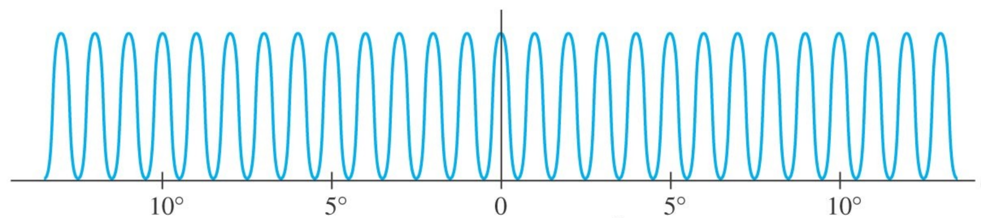
The double-slit experiment also exhibits diffraction effects, as the slits have a finite width.

Single slit diffraction with slit width D.



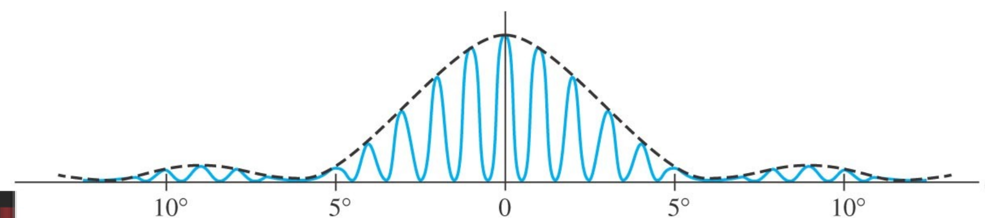
(a) Diffraction factor, $(\sin^2\beta/2)/(\beta/2)^2$ vs. θ

Double slit interference with extremely small slit width.

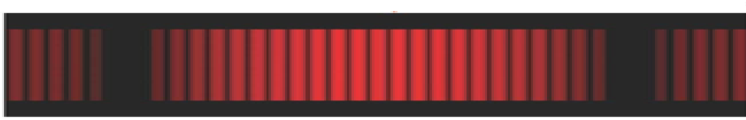


(b) Interference factor, $\cos^2 \frac{\delta}{2}$ vs. θ

Double slit interference with finite slit width D.



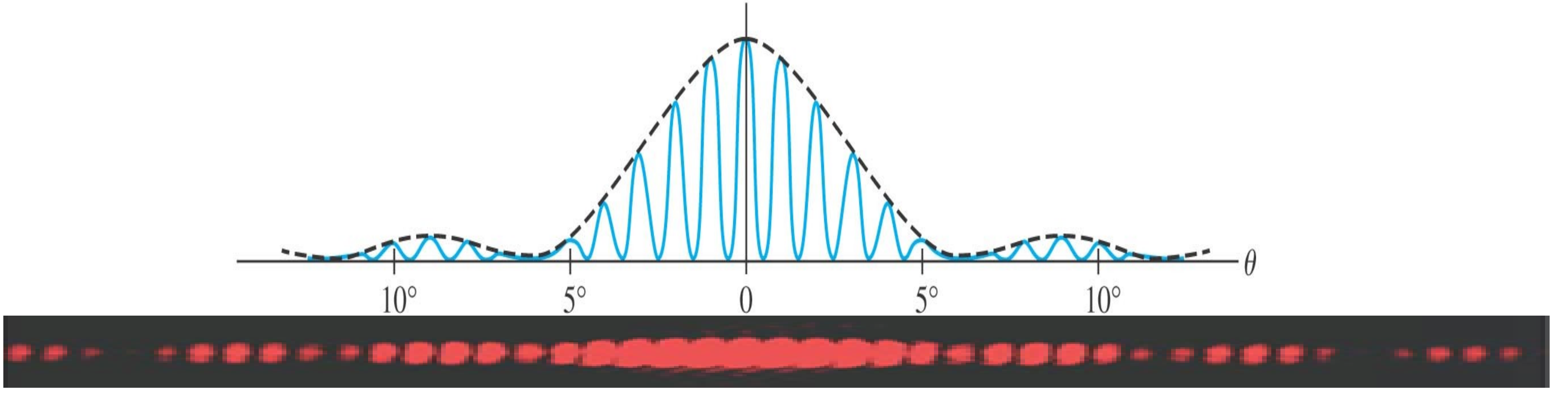
(c) Intensity, I_θ vs. θ



The diffraction factor appears as an "envelope" modifying the more rapidly varying interference factor.

Example 35-4: Diffraction plus interference.

Show why the central diffraction peak shown, plotted for the case where $d = 6D = 60\lambda$, contains 11 interference fringes.



The first minimum for the single slit diffraction with slit width D:

$$\theta \approx \sin\theta = \frac{\lambda}{D}, \quad \text{since } D\sin\theta = \lambda.$$

The 6th maximum for double-slit interference with slit spacing d:

$$\theta \approx \sin\theta = \frac{6\lambda}{d} = \frac{\lambda}{D}, \quad \text{since } d\sin\theta = 6\lambda.$$

The two angle are the same. Therefore, the central diffraction peak contains 11 interference fringes (5+1+5=11).

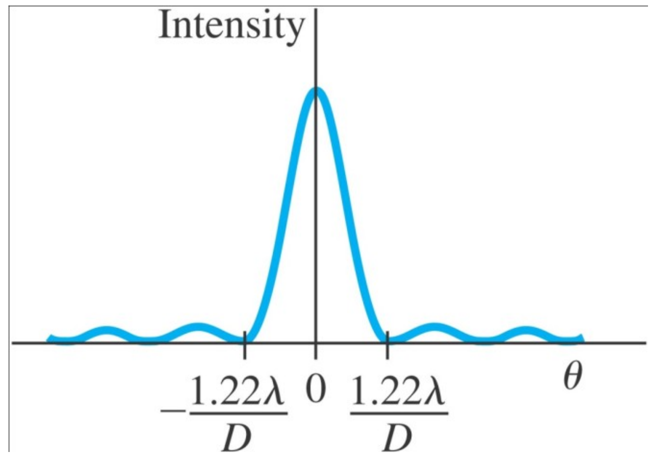
Limits of Resolution; Circular Apertures

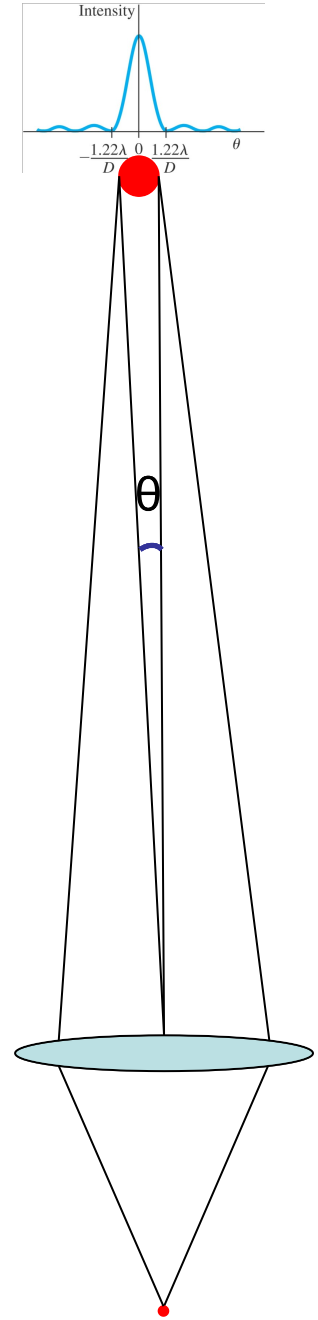
Resolution is the distance at which a lens can barely distinguish two separate objects.

Resolution is limited by aberrations and by diffraction. Aberrations can be minimized, but diffraction is unavoidable; it is due to the size of the lens compared to the wavelength of the light.

Limits of Resolution; Circular Apertures

When a lens forms the image of a point object, the image in fact is diffraction pattern. For a circular aperture of diameter *D*, the central maximum has an angular width:

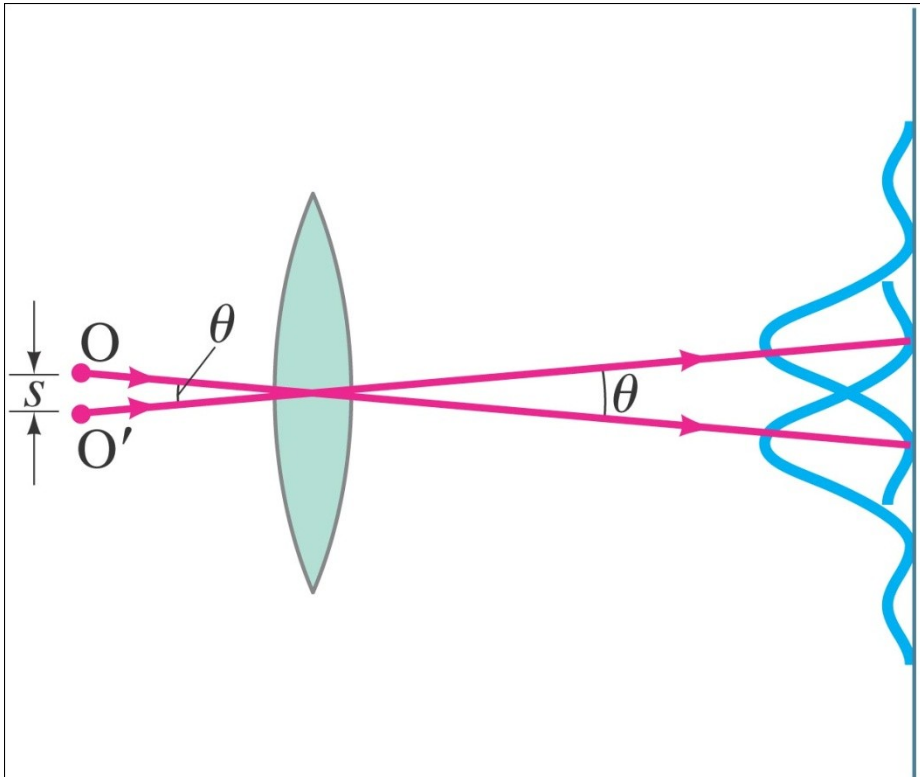




$$\theta = \frac{1.22\lambda}{D} \quad \text{(rad)}$$

Limits of Resolution; Circular Apertures

The Rayleigh criterion states that two images are just resolvable when the center of one peak is over the first minimum of the other.



Example 35-5: Hubble Space Telescope.

The Hubble Space Telescope (HST) is a reflecting telescope that was placed in orbit above the Earth's atmosphere, so its resolution would not be limited by turbulence in the atmosphere. Its objective diameter is 2.4 m. For visible light, say λ = 550 nm, estimate the improvement in resolution the Hubble offers over Earth-bound telescopes, which are limited in resolution by movement of the Earth's atmosphere to about half an arc second. (Each degree is divided into 60 minutes each containing 60 seconds, so 1° = 3600 arc seconds.)

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 5.50 \times 10^{-7}}{2.4} = 2.8 \times 10^{-7}\,(\text{rad}) = 5.77 \times 10^{-2}\ (\text{arc sec})$$

Almost 10 times better.

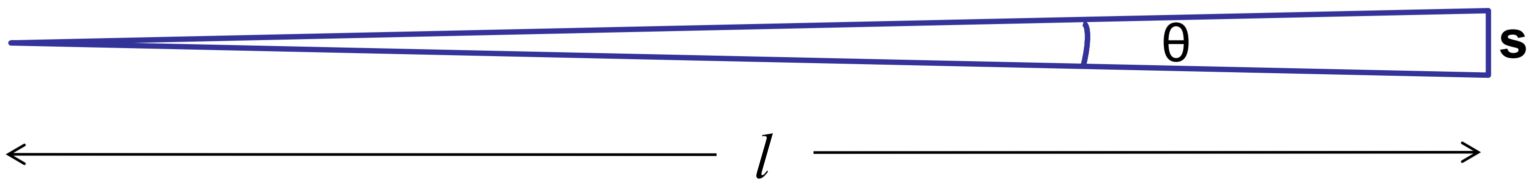
Example 35-6: Eye resolution.

You are in an airplane at an altitude of 10,000 m. If you look down at the ground, estimate the minimum separation s between objects that you could distinguish. Could you count cars in a parking lot? Consider only diffraction, and assume your pupil is about 3.0 mm in diameter and λ = 550 nm.

Eye's resolution:

$$\theta = \frac{1.22\lambda}{D} = \frac{1.22\times 5.50\times 10^{-7}}{3.0\times 10^{-3}} = 2.24\times 10^{-4}\,(\text{rad})$$

Distinguishable separation s:



$$s = l\theta = 10000\times 2.24\times 10^{-4} = 2.24\ \text{m}$$

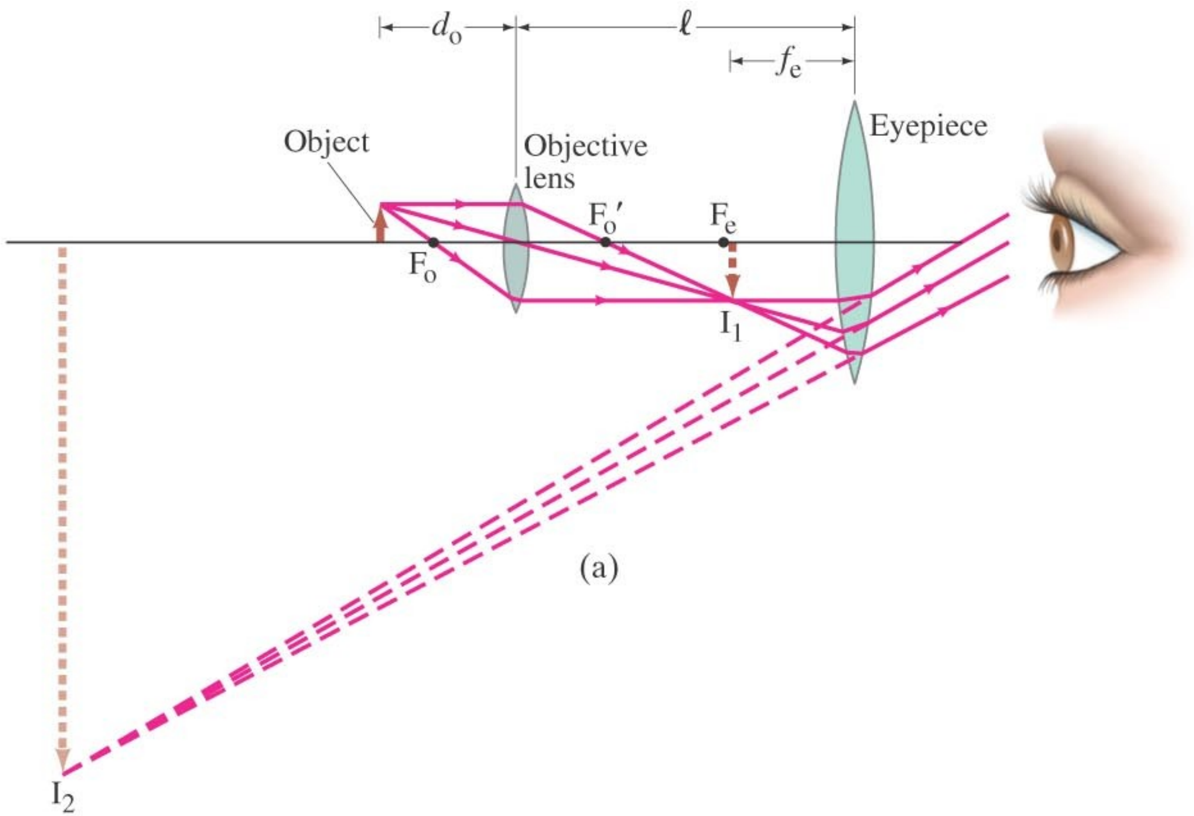
That's about the size of a car.

Resolution of Microscopes; the λ Limit

For microscopes, assuming the object is at the focal point, the resolving power is given by

$$\mathrm{RP} = s = f\theta = \frac{1.22\lambda f}{D}.$$

$$\theta = \frac{1.22\lambda}{D} \quad \text{(rad)}$$



(a)

(b)

Resolution Microscopes; the λ Limit

Typically, the focal length of a microscope lens is half its diameter, which shows that *it is not possible to resolve details smaller than the wavelength being used:*

$$\mathrm{RP} \approx \frac{\lambda}{2}.$$

Resolution of the Human Eye and Useful Magnification

The human eye can resolve objects that are about 1 cm apart at a distance of 20 m, or 0.1 mm apart at the near point.

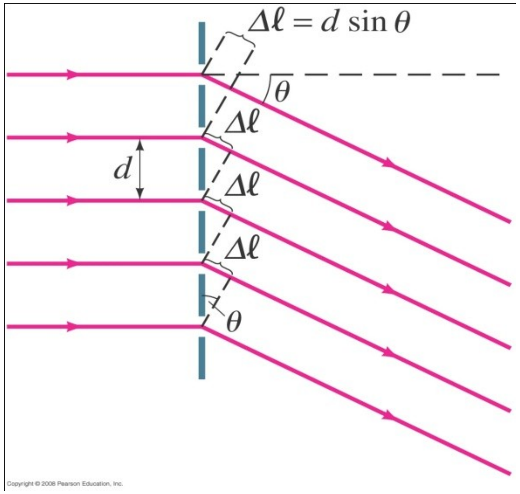
This limits the useful magnification of a light microscope to about 500x–1000x.

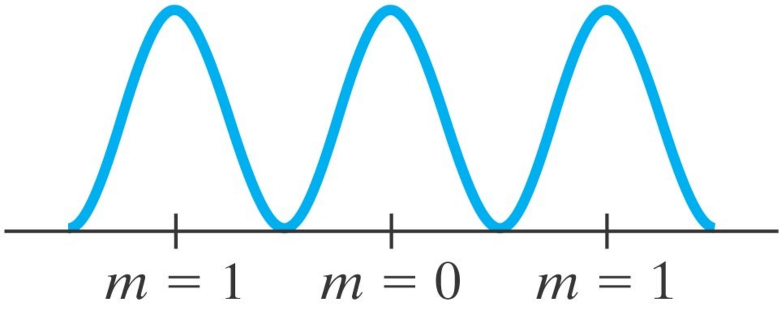
$$\frac{\lambda}{2} \approx 250\text{nm} \approx 2.5\times 10^{-7}\,m$$

$$500\times\frac{\lambda}{2} \approx 1.25\times 10^{-4}\,m \approx 0.1mm$$

Diffraction Grating

A diffraction grating consists of a large number (N) of equally spaced narrow slits or lines. A transmission grating has slits, while a reflection grating has lines that reflect light.



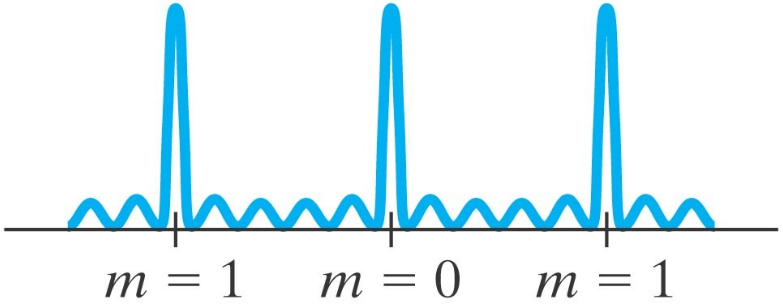


N=2

The more lines or slits there are, the narrower the peaks. $I_0 \propto N^2$.

Principal maxima (θ can be large):

$$\sin\theta = \frac{m\lambda}{d}, \qquad m = 0, 1, 2, \cdots$$



N=6

Example 35-8: Diffraction grating: lines.

Determine the angular positions of the first- and second-order maxima for light of wavelength 400 nm and 700 nm incident on a grating containing 10,000 lines/cm.

$$d = \frac{1}{10000} = 1\times10^{-4}\,cm = 1.0\times10^{-6}\,m$$

The first-order maximum:

$$\sin\theta_{400} = \frac{\lambda}{d} = \frac{4.0\times10^{-7}}{1.0\times10^{-6}} = 0.4,\quad \theta_{400} = 23.6°$$

$$\sin\theta_{700} = \frac{\lambda}{d} = \frac{7.0\times10^{-7}}{1.0\times10^{-6}} = 0.7,\quad \theta_{700} = 44.4°$$

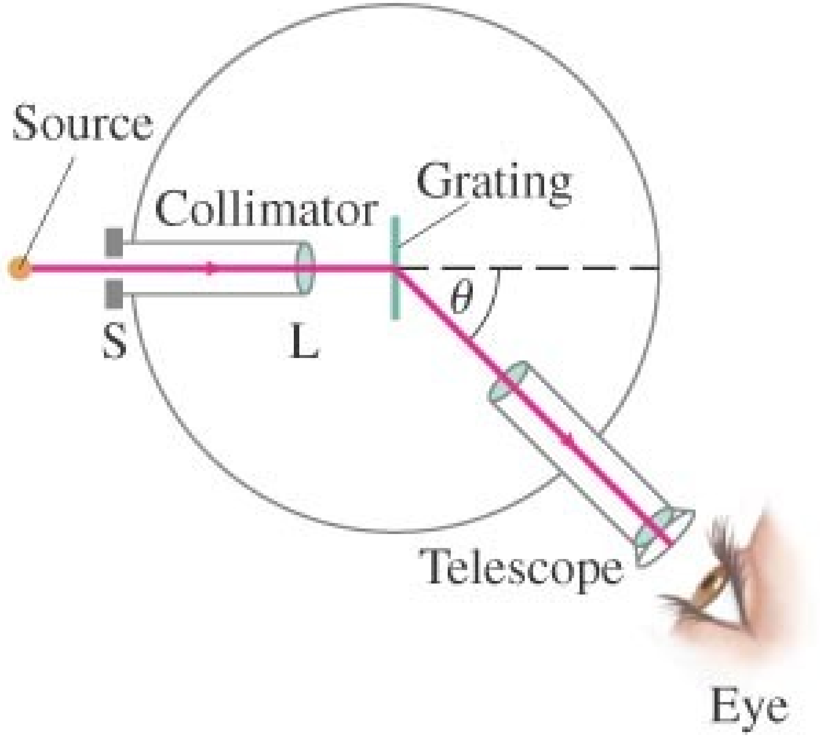
The second-order maximum:

$$\sin\theta_{400} = \frac{2\lambda}{d} = \frac{2\times4.0\times10^{-7}}{1.0\times10^{-6}} = 0.8,\quad \theta_{400} = 53.1°$$

$$\sin\theta_{700} = \frac{2\lambda}{d} = \frac{2\times7.0\times10^{-7}}{1.0\times10^{-6}} = 1.4,\quad \text{No second-order maximum.}$$

The Spectrometer and Spectroscopy

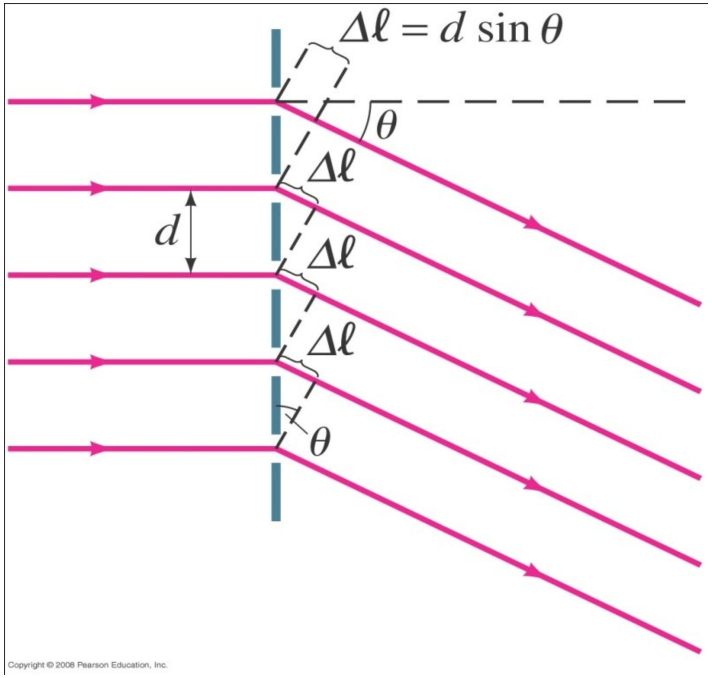
A spectrometer makes accurate measurements of wavelengths using a diffraction grating or prism.



The Spectrometer and Spectroscopy

The wavelength can be determined to high accuracy by measuring the angle at which the light is diffracted:

$$\sin\theta = \frac{m\lambda}{d}, \qquad m = 0, 1, 2, \cdots \qquad \left[\begin{array}{c}\text{diffraction grating,}\\ \text{principal maxima}\end{array}\right]$$



Spectroscopy

Atoms and molecules can be identified when they are in a thin gas through their characteristic emission lines.

Atomic hydrogen

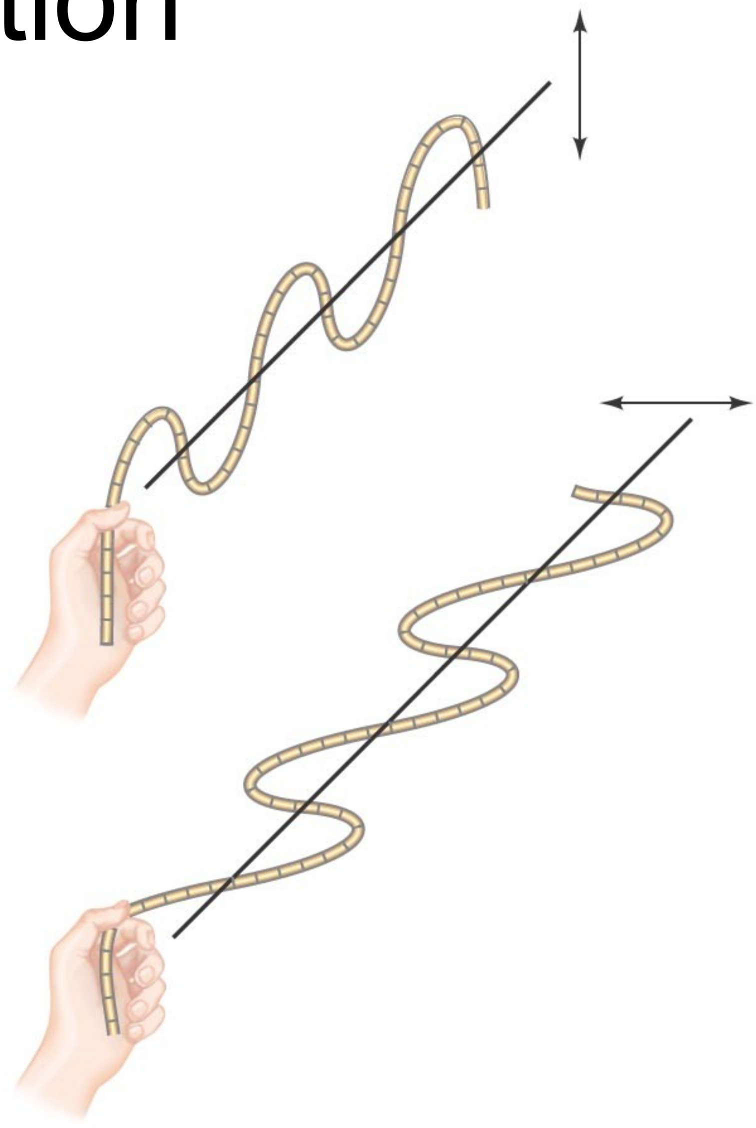
Mercury

Sodium

Solar absorption spectrum

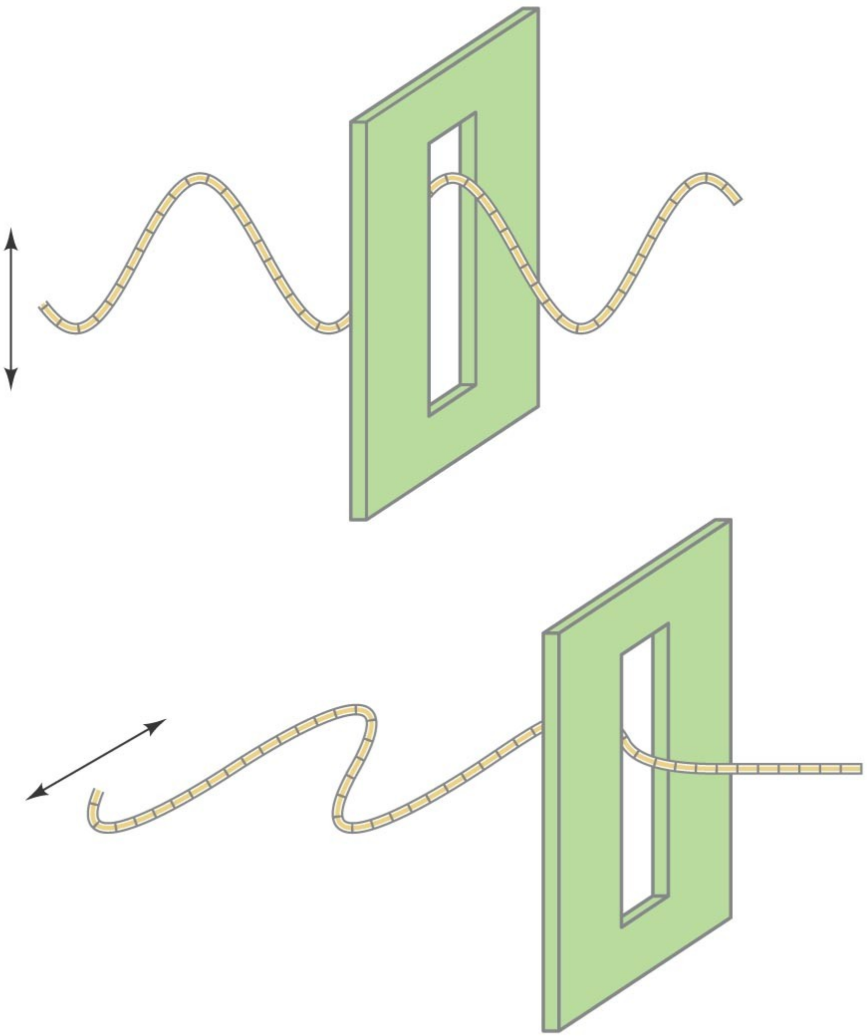
Polarization

Light is polarized when its electric fields oscillate in a single plane, rather than in any direction perpendicular to the direction of propagation.



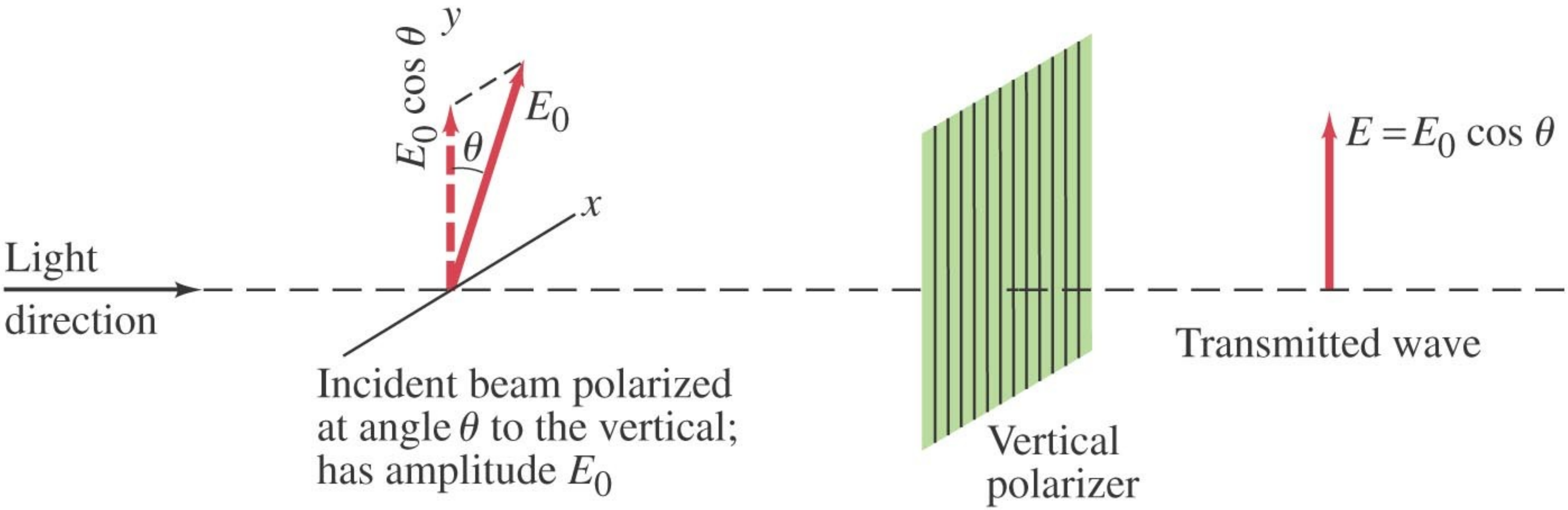
Polarization

Polarized light will not be transmitted through a polarized film whose axis is perpendicular to the polarization direction.



Polarization

When light passes through a polarizer, only the component parallel to the polarization axis is transmitted. If the incoming light is plane-polarized, the outgoing intensity is:

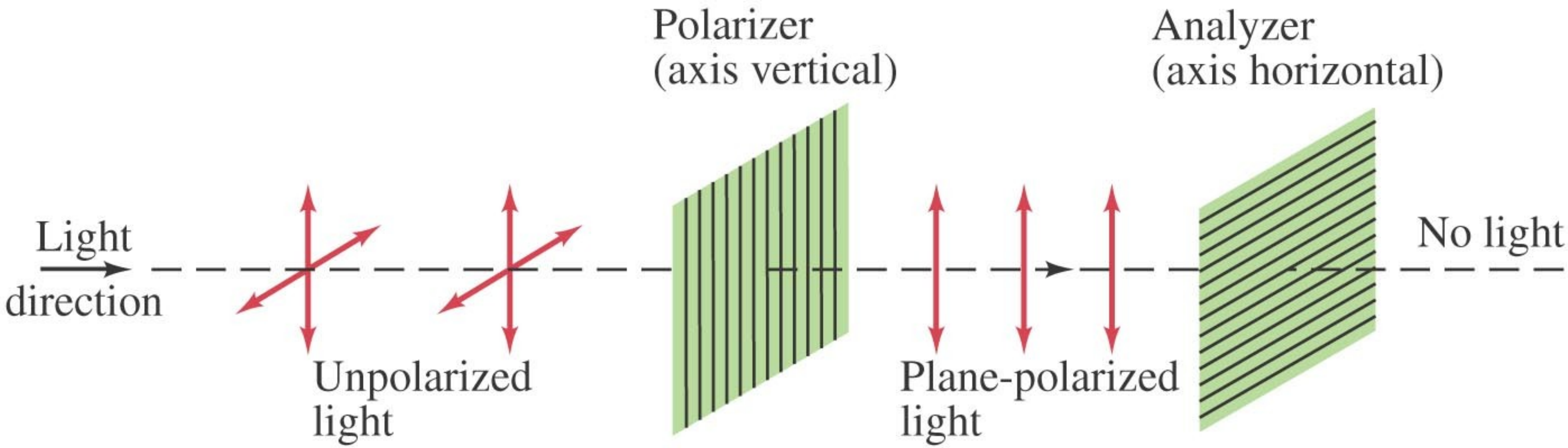


$$I = I_0 \cos^2\theta, \qquad \left[\begin{array}{c}\text{intensity of plane polarized}\\ \text{wave reduced by polarizer}\end{array}\right]$$

$$I \propto E^2$$

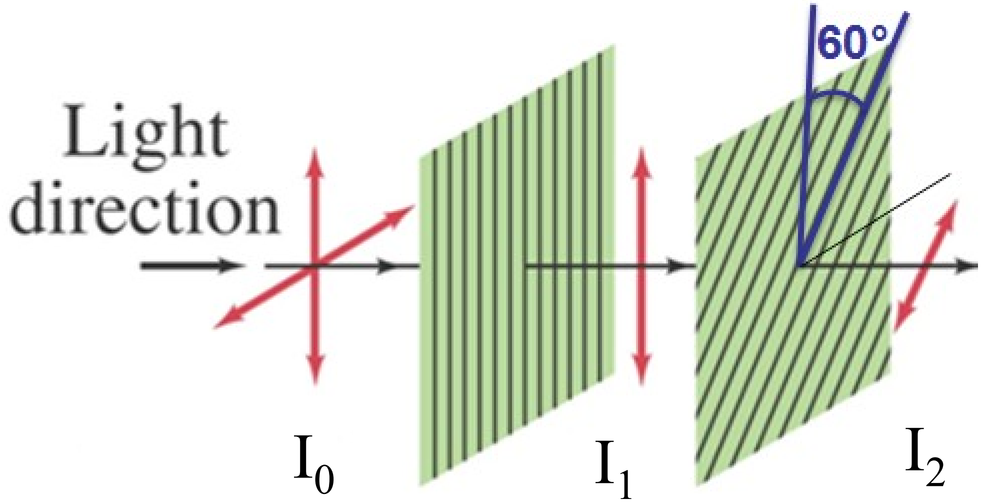
$$I = \varepsilon_0 c E^2$$

This means that if initially unpolarized light passes through crossed polarizers, no light will get through the second one.



Example 35-13: Two Polaroids at 60°.

Unpolarized light passes through two Polaroids; the axis of one is vertical and that of the other is at 60° to the vertical. Describe the orientation and intensity of the transmitted light.



$$I_1 = \frac{1}{2} I_0$$

$$I_2 = I_1 \cos^2 60° = I_1 \left(\frac{1}{2}\right)^2 = \frac{1}{4} I_1 = \frac{1}{8} I_0$$

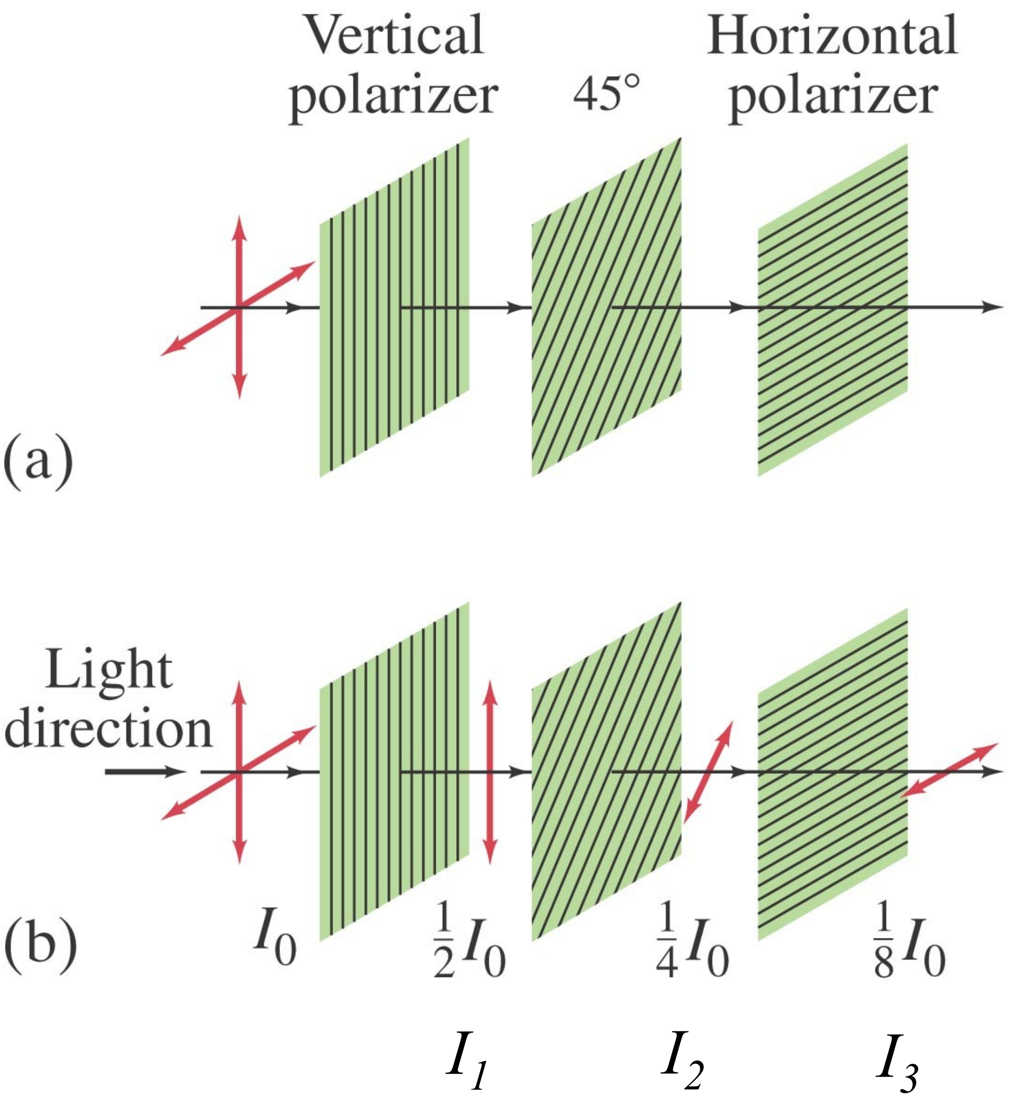
Conceptual Example 35-14: Three Polaroids.

When unpolarized light falls on two crossed Polaroids (axes at 90°), no light passes through. What happens if a third Polaroid, with axis at 45° to each of the other two, is placed between them?

$$I_1 = \frac{1}{2} I_0$$

$$I_2 = I_1 \cos^2 45° = \frac{1}{2} I_1 = \frac{1}{4} I_0$$

$$I_3 = I_2 \cos^2 45° = \frac{1}{2} I_2 = \frac{1}{8} I_0$$

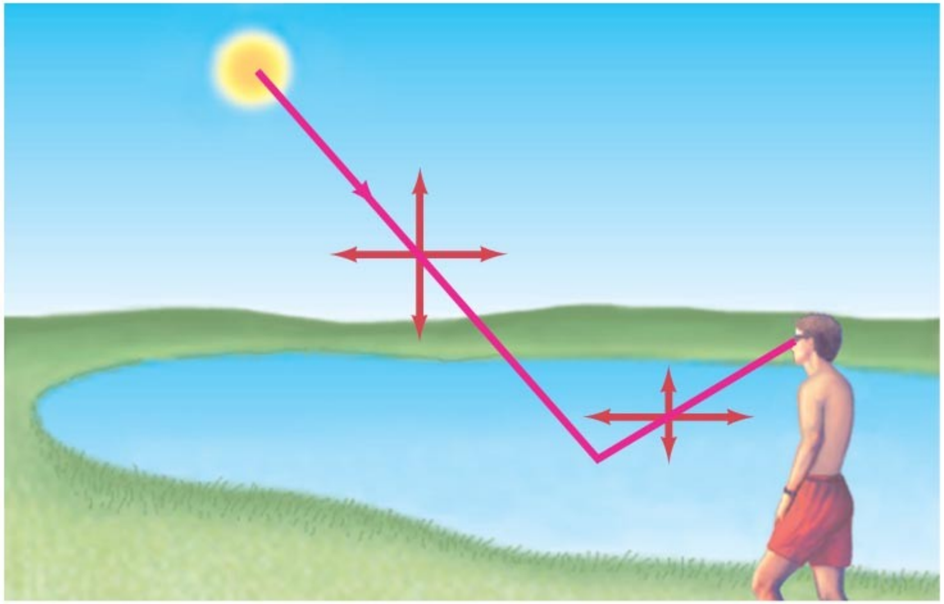


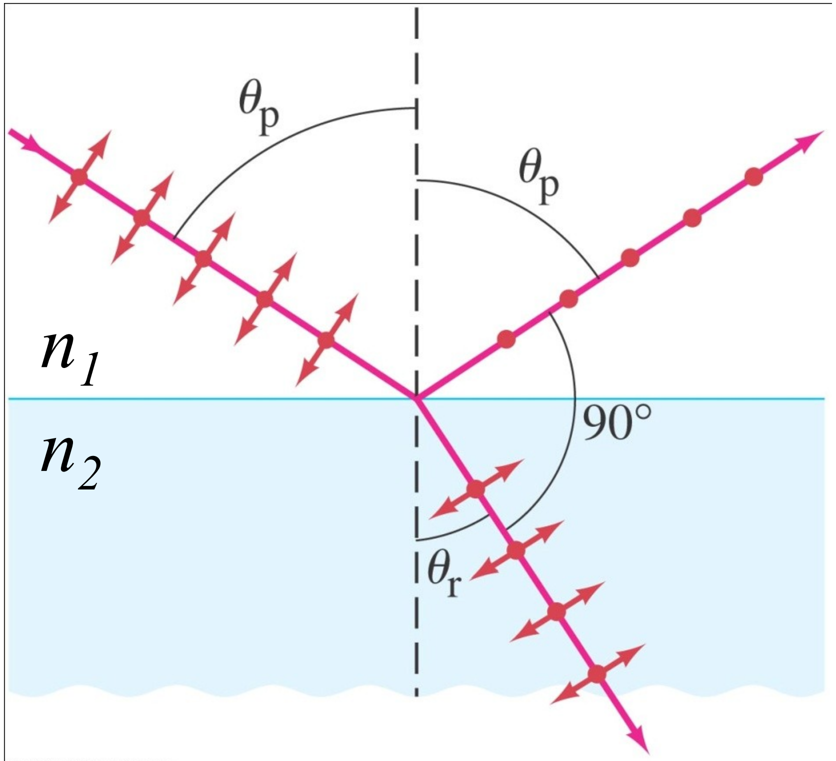
Light is also partially polarized after reflecting from a nonmetallic surface. At a special angle, called the polarizing angle or Brewster's angle, the polarization is 100%:

$$\tan\theta_p = \frac{n_2}{n_1}$$

The reflected light is polarized perpendicular to plane of incidence.

The angle between the reflected light and the refracted light is 90°.





Example 35-15: Polarizing angle.

(a) At what incident angle is sunlight reflected from a lake plane-polarized? (b) What is the refraction angle?

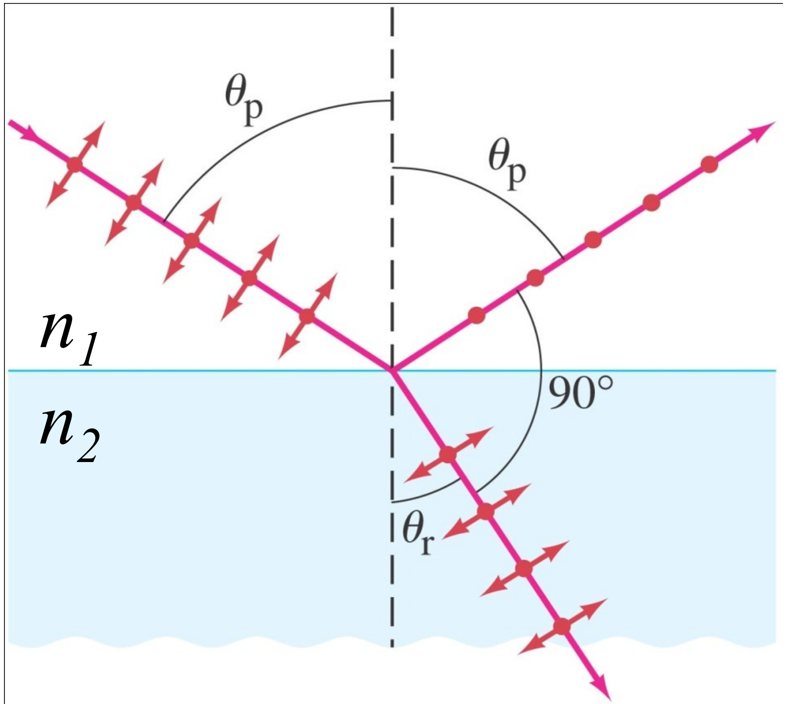
$$n_1 = 1.00, \quad n_2 = 1.33$$

$$\theta_P = tan^{-1}\left(\frac{n_2}{n_1}\right) = tan^{-1}(1.33) = 53.1°$$

$$n_1 \, sin\,\theta_P = n_2 \, sin\,\theta_r$$

$$sin\,\theta_r = \frac{n_1 \, sin\,\theta_P}{n_2} = \frac{sin\,53.1°}{1.33} = 0.601$$

$$\theta_r = sin^{-1}\,0.601 = 36.9°$$



Also, $\theta_r = 180° - 90° - \theta_P = 90° - 53.1° = 36.9°$