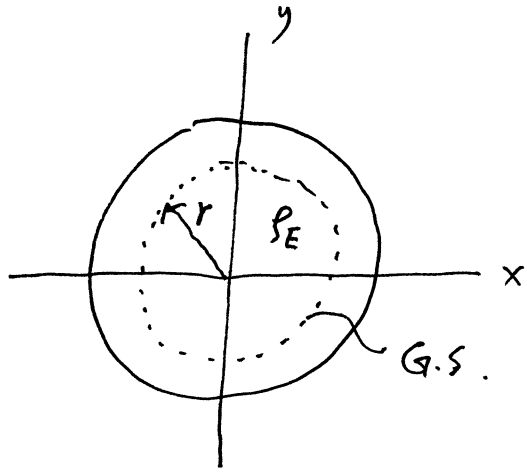
Physics 02 Written Assignment #3.

[Solution].

First, consider the electric field produced by a uniformly charged sphere. $\vec{E} \parallel \vec{r}$



$$\Phi = 4\pi r^2 \cdot E = \frac{Q_{encl.}}{\varepsilon_0}.$$

$$Q_{encl.} = \rho_E \cdot \frac{4}{3}\pi r^3$$

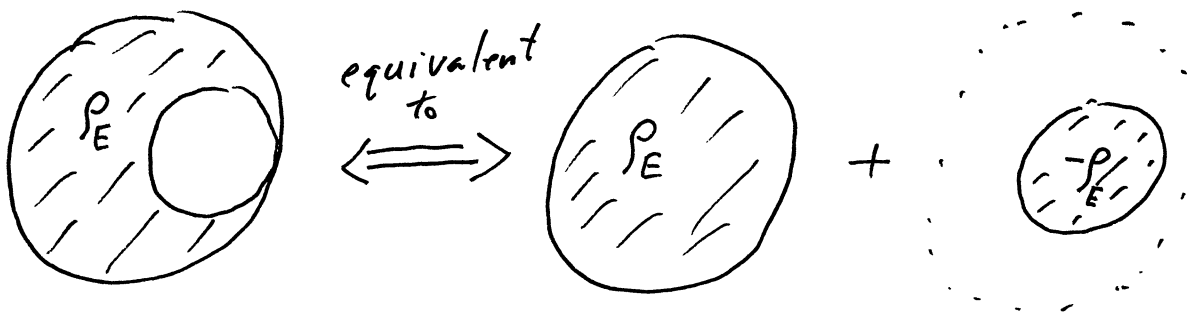
$$4\pi r^2 E = \frac{\rho_E 4\pi r^3}{3\varepsilon_0}.$$

$$E = \frac{\rho_E r}{3\varepsilon_0}$$

$$\vec{E} = \frac{\rho_E \vec{r}}{3\varepsilon_0}$$

Second, creating a cavity is equivalent to adding a negatively charged object.

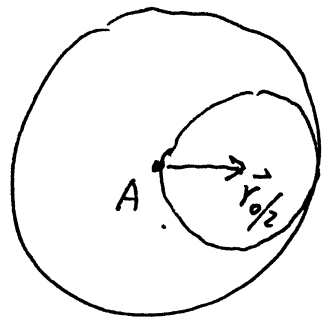
i.e.



$$\therefore \quad \vec{E} = \vec{E}_{\rho_E} + \vec{E}_{-\rho_E}.$$

a)

At point A :



$$\vec{E}_{\rho_E} = 0 \quad (\text{since } \vec{r} = 0)$$

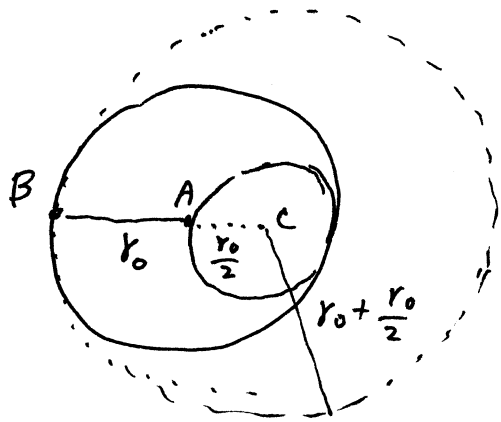
$$\vec{E}_{-\rho_E} = \frac{\rho_E \vec{r}_0/2}{3\varepsilon_0} = \frac{\rho_E \vec{r}_0}{6\varepsilon_0}$$

$$\therefore \quad \vec{E}_A = \frac{\rho_E \vec{r}_0}{6\varepsilon_0} \qquad (\text{direction : to the right})$$

b)

At point B.

$$\left|\vec{E}_{\rho_E}\right| = \frac{\rho_E r_0}{3\varepsilon_0} \qquad \text{to the left.} \quad \vec{E}_{\rho_E} \parallel \hat{x}$$



~~$\left|\vec{E}_{\rho_E}\right| = \frac{\rho_E (r_0 + \frac{r_0}{2})}{3\varepsilon_0}$ to the right.~~

$$\vec{E}_{-\rho_E}: \quad -E_{-\rho_E} \cdot 4\pi \left(r_0 + \frac{r_0}{2}\right)^2 = -\frac{\rho_E}{\varepsilon_0} \frac{4}{3} \cdot \pi \left(\frac{r_0}{2}\right)^3$$

$$E_{-\rho_E} = \frac{\rho_E \; r_0}{3\varepsilon_0 \left(\frac{3}{2}\right)^2} = \frac{4\rho_E r_0}{27\varepsilon_0 8} = \frac{\rho_E r_0}{54\varepsilon_0}$$

$\vec{E}_{-\rho_E}$ at B : to the right. $\vec{E}_{-\rho_E} \parallel -\hat{x}$.

$$\therefore \vec{E}: \quad E = \frac{\rho_E r_0}{3\varepsilon_0} - \frac{\rho_E r_0}{54\varepsilon_0} = \frac{\rho_E r_0}{\varepsilon_0}\left(\frac{18-1}{54}\right) = \frac{17\rho_E r_0}{54\varepsilon_0}$$

direction of $\vec{E}$ at B: to the left.