95. A parallel-plate capacitor with plate area $A = 2.0 \text{ m}^2$ and plate separation $d = 3.0 \text{ mm}$ is connected to a 45-V battery (Fig. 24–40a). (a) Determine the charge on the capacitor, the electric field, the capacitance, and the energy stored in the capacitor. (b) With the capacitor still connected to the battery, a slab of plastic with dielectric strength $K = 3.2$ is placed between the plates of the capacitor, so that the gap is completely filled with the dielectric. What are the new values of charge, electric field, capacitance, and the energy $U$ stored in the capacitor?

[Solution]

(a) Use Eq. 24-2 to calculate the capacitance.

$$C_o = \frac{\varepsilon_o A}{d} = \frac{\left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)(2.0 \text{ m}^2)}{\left(3.0 \times 10^{-3} \text{ m}\right)} = 5.9 \times 10^{-9} \text{ F}$$

Use Eq. 24-1 to calculate the charge.

$$Q_o = C_o V_o = \left(5.9 \times 10^{-9} \text{ F}\right)(45 \text{ V}) = 2.655 \times 10^{-7} \text{ C} \approx 2.7 \times 10^{-7} \text{ C}$$

The electric field is the potential difference divided by the plate separation.

$$E_o = \frac{V_o}{d} = \frac{45 \text{ V}}{3.0 \times 10^{-3} \text{ m}} = 15000 \text{ V/m}$$

Use Eq. 24-5 to calculate the energy stored.

$$U_o = \frac{1}{2} C_o V_o^2 = \frac{1}{2} \left(5.9 \times 10^{-9} \text{ F}\right)(45 \text{ V})^2 = 6.0 \times 10^{-6} \text{ J}$$

(b) Now include the dielectric. The capacitance is multiplied by the dielectric constant.

$$C = K C_o = 3.2 \left(5.9 \times 10^{-9} \text{ F}\right) = 1.888 \times 10^{-8} \text{ F} \approx 1.9 \times 10^{-8} \text{ F}$$

The voltage doesn’t change (it's maintained by the battery). Use Eq. 24-1 to calculate the charge.

$$Q = CV = K C_o V = 3.2 \left(5.9 \times 10^{-9} \text{ F}\right)(45 \text{ V}) = 8.496 \times 10^{-7} \text{ C} \approx 8.5 \times 10^{-7} \text{ C}$$

Since the battery is still connected, the voltage is the same as before, and so the electric field doesn’t change.

$$E = E_o = 15000 \text{ V/m}$$

Use Eq. 24-5 to calculate the energy stored.

$$U = \frac{1}{2} CV^2 = \frac{1}{2} K C_o V^2 = \frac{1}{2} \left(3.2\right) \left(5.9 \times 10^{-9} \text{ F}\right)(45 \text{ V})^2 = 1.9 \times 10^{-5} \text{ J}$$