95. A parallel-plate capacitor with plate area $A=2.0 \mathrm{~m}^{2}$ and plate separation $d=3.0 \mathrm{~mm}$ is connected to a $45-\mathrm{V}$ battery (Fig. 24-40a). (a) Determine the charge on the capacitor, the electric field, the capacitance, and the energy stored in the capacitor. (b) With the capacitor still connected to the battery, a slab of plastic with dielectric strength $K=3.2$ is placed between the plates of the capacitor, so that the gap is completely filled with the dielectric. What are the new values of charge, electric field, capacitance, and the energy $U$ stored in the capacitor?

[Solution]
(a) Use Eq. 24-2 to calculate the capacitance.

$$
C_{0}=\frac{\varepsilon_{0} A}{d}=\frac{\left(8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2}\right)\left(2.0 \mathrm{~m}^{2}\right)}{\left(3.0 \times 10^{-3} \mathrm{~m}\right)}=5.9 \times 10^{-9} \mathrm{~F}
$$

Use Eq. 24-1 to calculate the charge.

$$
Q_{0}=C_{0} V_{0}=\left(5.9 \times 10^{-9} \mathrm{~F}\right)(45 \mathrm{~V})=2.655 \times 10^{-7} \mathrm{C} \approx 2.7 \times 10^{-7} \mathrm{C}
$$

The electric field is the potential difference divided by the plate separation.

$$
E_{0}=\frac{V_{0}}{d}=\frac{45 \mathrm{~V}}{3.0 \times 10^{-3} \mathrm{~m}}=15000 \mathrm{~V} / \mathrm{m}
$$

Use Eq. 24-5 to calculate the energy stored.

$$
U_{0}=\frac{1}{2} C_{0} V_{0}^{2}=\frac{1}{2}\left(5.9 \times 10^{-9} \mathrm{~F}\right)(45 \mathrm{~V})^{2}=6.0 \times 10^{-6} \mathrm{~J}
$$

(b) Now include the dielectric. The capacitance is multiplied by the dielectric constant.

$$
C=K C_{0}=3.2\left(5.9 \times 10^{-9} \mathrm{~F}\right)=1.888 \times 10^{-8} \mathrm{~F} \approx 1.9 \times 10^{-8} \mathrm{~F}
$$

The voltage doesn't change (it's maintained by the battery). Use Eq. 24-1 to calculate the charge.

$$
Q=C V=K C_{0} V=3.2\left(5.9 \times 10^{-9} \mathrm{~F}\right)(45 \mathrm{~V})=8.496 \times 10^{-7} \mathrm{C} \approx 8.5 \times 10^{-7} \mathrm{C}
$$

Since the battery is still connected, the voltage is the same as before, and so the electric field doesn't change.

$$
E=E_{0}=15000 \mathrm{~V} / \mathrm{m}
$$

Use Eq. 24-5 to calculate the energy stored.

$$
U=\frac{1}{2} C V^{2}=\frac{1}{2} K C_{0} V^{2}=\frac{1}{2}(3.2)\left(5.9 \times 10^{-9} \mathrm{~F}\right)(45 \mathrm{~V})^{2}=1.9 \times 10^{-5} \mathrm{~J}
$$

