Force, Torque, and

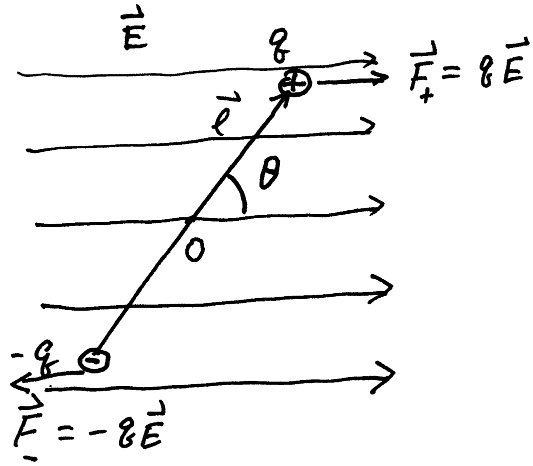
- Potential energy of a dipole in an external field.

$\vec{E}$ — uniform

$\vec{p} = q\vec{l}$ ($\vec{l}$ — from $-q$ to $+q$.)

Net force on the dipole: $\vec{F} = \vec{F}_+ + \vec{F}_- = 0$.

Torque about O:



$$\tau = -F_+ \cdot \frac{l}{2}\sin\theta - F_- \frac{l}{2}\sin\theta$$

$$\tau = -qEl\sin\theta = -pE\sin\theta$$

$F_+ = F_-$. (magnitude) $= qE$.

"−" meaning CW. (+ if it's CCW)

NOT Required

★ Text book: $\vec{\tau} = \vec{p} \times \vec{E}$. — vector. (cross product)

magnitude: $\tau = p \cdot E \cdot \sin\theta$. (always positive).

direction: Right-Hand-Rule.

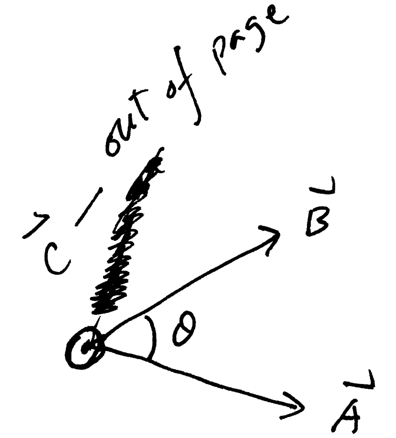
- cross product of 2 vectors: $\vec{A} \times \vec{B}$.

$$\vec{C} = \vec{A} \times \vec{B}.$$

The result is a vector $\vec{C}$.

magnitude: $C = AB \cdot \sin\theta$.

direction: RHR:



- Potential energy and work.

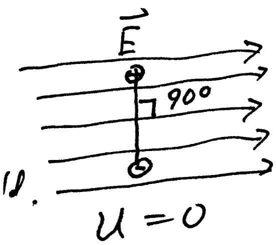
static electric force is conservative.

— work depends on positions only. (NOT path)

— we can define potential energy, U

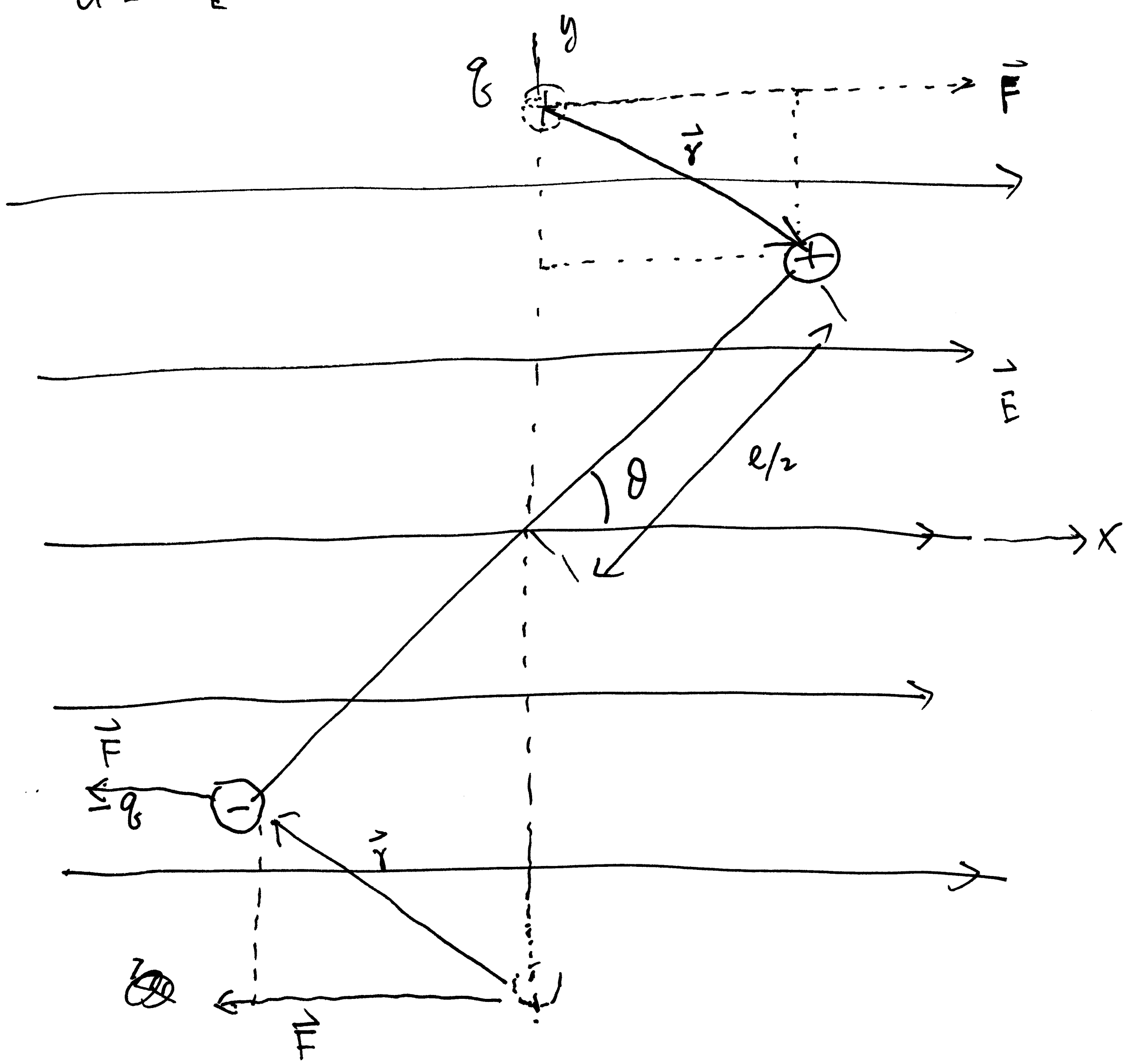
For a dipole in a uniform $\vec{E}$-field. $\left(\begin{matrix}\vec{F}=0\\ \tau\neq 0\end{matrix}\right)$.

Define $U=0$ when $\vec{p}\perp\vec{E}$, i.e. $\theta=90°$



Then $U\neq 0$ when $\theta\neq 0$.

$U=-W_E$ W_E – work done by $\vec{E}$-field.



Displacement for q : $\vec{r}$

work by $\vec{E}$ on q : $W_q = \vec{F} \cdot \vec{r} = q\vec{E} \cdot \vec{r} = qEx = qE\frac{l}{2}\cos\theta$

work by $\vec{E}$ on $-q$: $W_{-q} = qE\frac{l}{2}\cos\theta$.

work by $\vec{E}$ on dipole : $W_E = qEl\cos\theta = pE\cos\theta$

potential energy of dipole at θ :

$$U = -W_E = -qEl\cos\theta = -pE\cos\theta$$

$$= -\vec{p} \cdot \vec{E} \ .$$

θ dependence

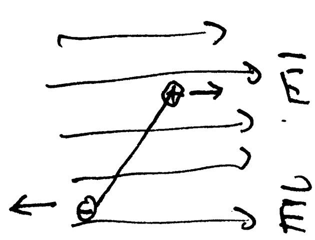
$\theta = 0$: $U(0) = -pE = -qEl$ — lowest potential energy. (stable equilibrium)
$\tau = 0$.

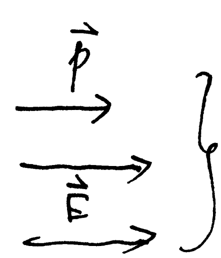
$\theta = 90°$: $U(90°) = 0$. — largest torque.
$\tau = \max$.

$\theta = 180°$. $U(180°) = pE$ — highest potential energy (unstable equilibrium)
$\tau = 0$.

Therefore, the external field $\vec{E}$ tends to line up the dipole.

i.e, make $\theta = 0$.



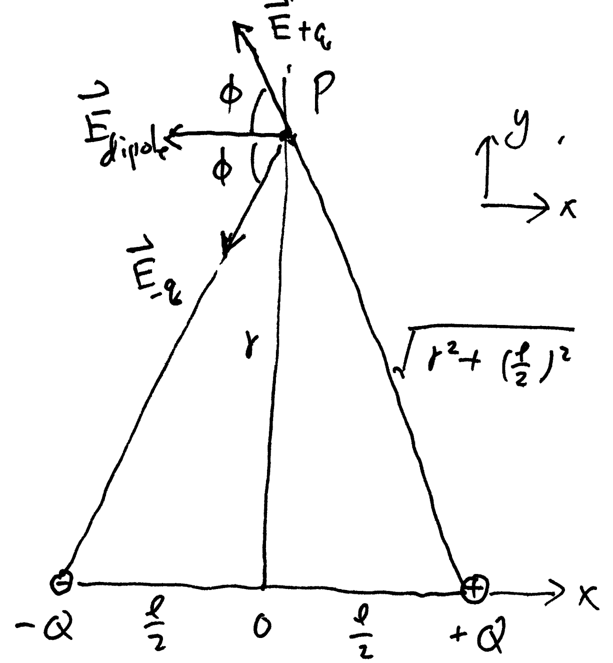


"Happiest" orientation.
(lowest potential energy .

- The $\vec{E}$-field created by a dipole.

idea: $\vec{E}_{dipole} = \vec{E}_{+q} + \vec{E}_{-q}$.

e.g. $\vec{E}_{dipole}$ at a point on the perpendicular bisector.



by symmetry:

$\vec{E}_{dipole} = -\hat{x} \cdot E$.

$$E = 2E_{+q} \cos\phi$$

$$= \frac{2Q}{4\pi\varepsilon_0 \left[r^2+(\frac{l}{2})^2\right]} \frac{l/2}{\sqrt{r^2+(\frac{l}{2})^2}}$$

$$= \frac{p}{4\pi\varepsilon_0 \left(r^2+\frac{l^2}{4}\right)^{3/2}}$$

far field: $r \gg l$.

$$E \approx \frac{p}{4\pi\varepsilon_0 r^3} \propto \frac{1}{r^3}.$$

compare to the field of a charge: $E = \frac{q}{4\pi\varepsilon_0 r^2} \propto \frac{1}{r^2}$.

The $\vec{E}$-field of a dipole decrease with r quickly.

e.g. 21-17.

Dipole of a H_2O molecule : $p = 6.1 \times 10^{-30}\ C \cdot m$.

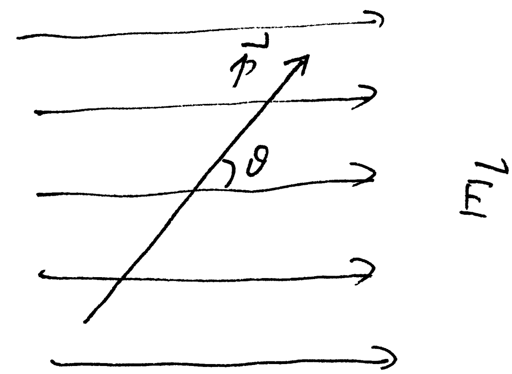
$E = 2.0 \times 10^{5}\ N/C$.

a). $\tau = pE\sin\theta$

$\tau_{max} = PE. \quad \left[\left(\sin\theta\right)_{max} = 1.\right]$

$= 6.1 \times 10^{-30} \times 2.0 \times 10^{5}$

$= 1.2 \times 10^{-24}\ (N \cdot m).$



b). at $\tau = \tau_{max}$. $\theta = 90°$.

$U = -pE\cos\theta = 0$.

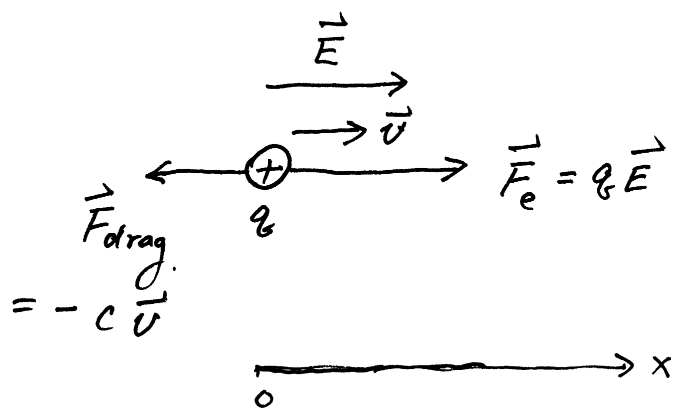
c). When $\theta = 180°$, $\cos\theta = -1$,

$U = U_{max} = PE = 6.1 \times 10^{-30} \times 2.0 \times 10^{5}$

$= 1.2 \times 10^{-24}\ (J)$.

☆. (Torque is different from potential energy !
relationship : $\tau = -\dfrac{dU}{d\theta}$)

Electrophoresis



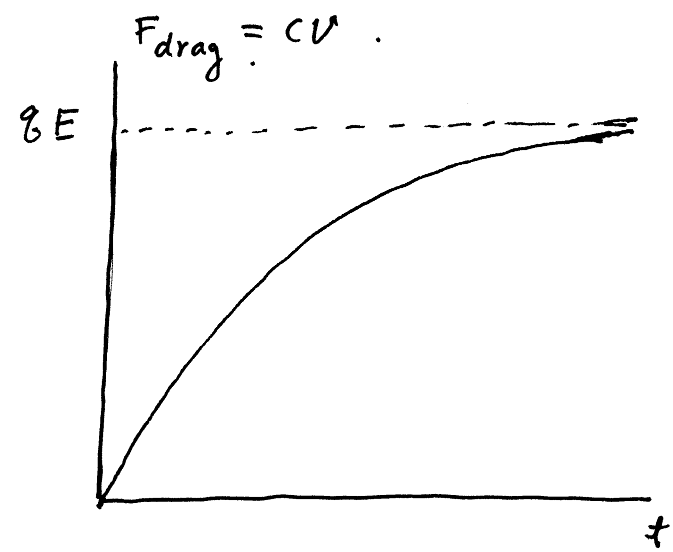
c — depends on shape, size, fluid, etc. (constant)

Net force on q:

$$\vec{F}_{net} = \vec{F}_e + \vec{F}_{drag}.$$

$$= q\vec{E} - c\vec{v}$$

x-comp: $F_{net} = qE - cv.$



as v increases.

F_{drag} increases.

until $F_{drag} = eE$.

i.e, $F_{net} = 0$.

Then v stop increasing.

∴ v = constant after a while.

quite short. because the drag in the fluid/gel is large.

finally: v: $qE - cv = 0$.

$$v = \frac{qE}{c}.$$