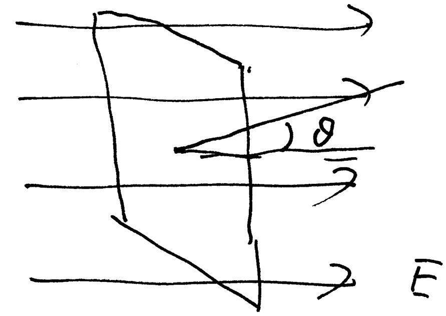
e.g. 22-1.



$\vec{E} = 200\ N/C$.

$A = 0.1 \times 0.2 = 0.02\ m^2$

$\theta = 30^\circ$.

$\vec{E}$-Flux :

$$\Phi_E = \vec{E} \cdot \vec{A}$$

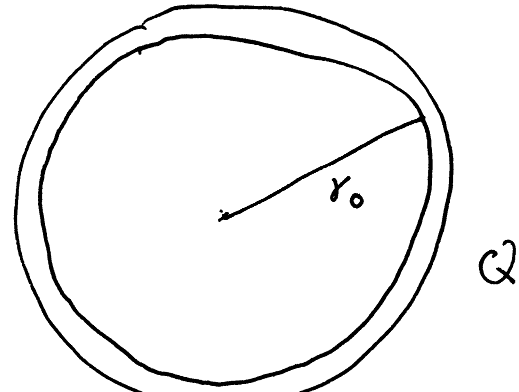
$$= E A \cos\theta$$

$$= 200 \times 0.02 \times \cos 30^\circ$$

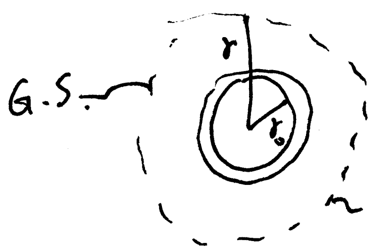
$$= 3.5\ N \cdot m^2/C$$

e.g. 22-3.

Q — uniformly distributed on outer surface.



a). outside the shell.



$\vec{E} // \hat{r}$

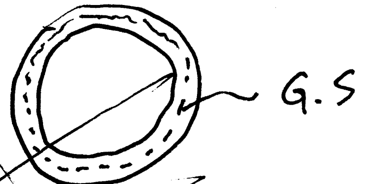
$E = \text{const}$.

$$\Phi = \Sigma \vec{E} \cdot \vec{A} = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\therefore \quad \vec{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \left(\frac{\vec{r}}{r} \right).$$

└ direction's $\frac{\vec{r}}{r} = \hat{r}$.

b). within shell.



$$\Sigma \vec{E} \cdot \vec{A} = 0$$

$$E \cdot 4\pi r^2 = 0. \qquad E = \frac{0}{4\pi r^2} = 0.$$

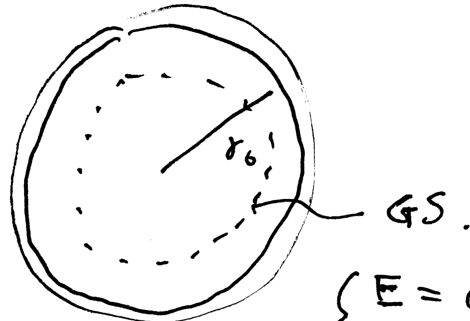
c).

b). Inside the shell.

$$\sum \vec{E}\cdot\vec{A} = 0$$

$$E\,4\pi r^2 = 0$$

$$E = \frac{0}{4\pi r^2} = 0.$$

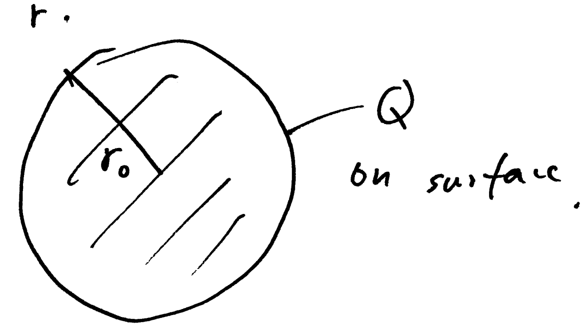


by symmetry $\begin{cases} E = \text{const}. \\ \vec{E} \parallel \vec{A} \text{ if } \vec{E} \neq 0. \end{cases}$

c). What if the conductor is a solid sphere?

outside — same. $\vec{E} = \dfrac{Q}{4\pi\varepsilon_0 r^2}\cdot\dfrac{\vec{r}}{r}$.

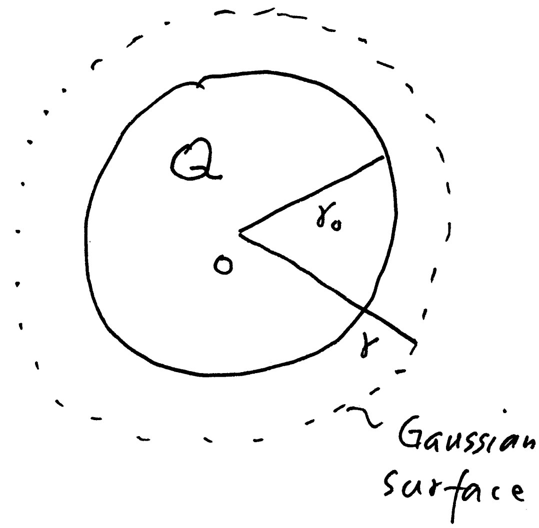
Inside — same. 0.



e.g. 22-4.

Uniformly charged solid sphere

Total charge Q.



a). Find $\vec{E}$ outside. $r > r_0$

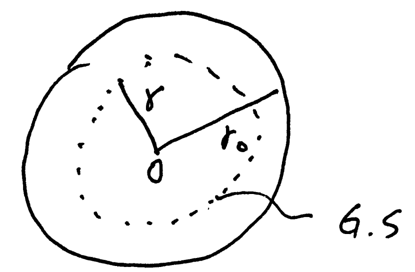
$$\sum_{G.S.} \vec{E} \cdot \vec{A} = \frac{Q}{\varepsilon_0}.$$

G.S: spherical surface.

On G.S. $E = \text{const}$. $\vec{E} \parallel \vec{r}$ — by symmetry.

$$\therefore \sum_{G.S.} \vec{E} \cdot \vec{A} = E \cdot (4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{\varepsilon_0}.$$



b). $r < r_0$. (inside).

$$\sum \vec{E} \cdot \vec{A} = E \cdot (4\pi r^2) = \frac{Q_{enclosed}}{\varepsilon_0}.$$

$Q_{enclosed} = \rho \cdot V$. (ρ — charge density, V — enclosed volume)

$$Q_{enclosed} = \frac{3Q}{4\pi r_0^3} \cdot \frac{4\pi r^3}{3}$$

$$= \frac{Q r^3}{r_0^3}.$$

$$\rho = \frac{Q}{\frac{4}{3}\pi r_0^3} = \frac{3Q}{4\pi r_0^3}.$$

$$V = \frac{4}{3}\pi r^3.$$

$$\therefore E(4\pi r^2) = \frac{Q r^3}{\varepsilon_0 r_0^3}$$

$$E = \frac{Q r}{4\pi \varepsilon_0 r_0^3}$$

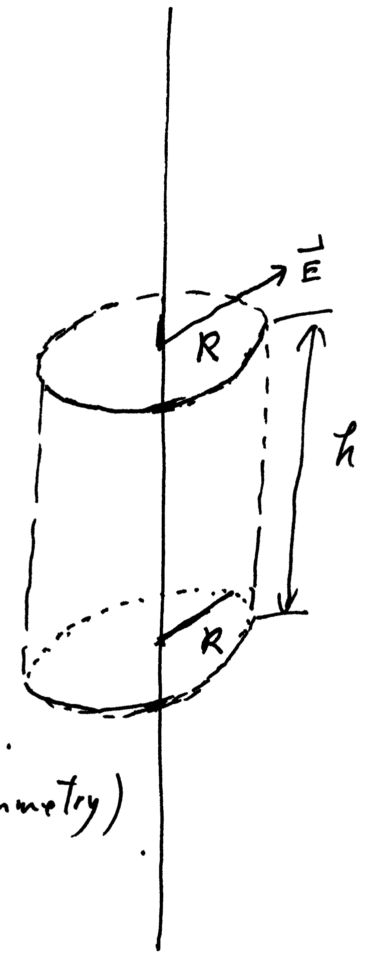
e.g. 22-6. Uniformly charged long line.
└ very long. (∞)

charge per unit length: λ.

Find $\vec{E}$ near the line.

by symmetry. $\vec{E}$ // radial direction.

Gaussian Surface: cylindrical.



Top and bottom: $\vec{E} \perp \vec{A}$.

$\vec{E} \cdot \vec{A} = 0$.

(No flux through the ends)

side surface: $\vec{E} \parallel \vec{A}$. ($\because \vec{E} \parallel \vec{R}$)

E = constant. (symmetry)

$$\therefore \sum_{G.S.} \vec{E} \cdot \vec{A} = \frac{Q}{\varepsilon_0}$$

$$E \cdot A_{lateral} = \frac{\lambda h}{\varepsilon_0}.$$

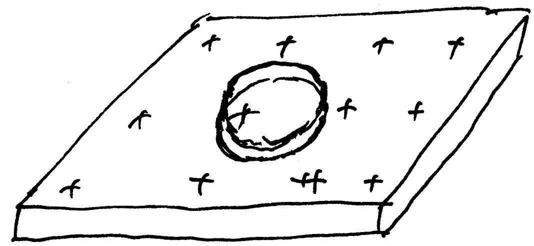
$$E \cdot 2\pi R \cdot h = \frac{\lambda h}{\varepsilon_0}$$

$$\therefore E = \frac{\lambda}{2\pi \varepsilon_0 R}$$

e.g. 22-7.

Infinite plane of charge.

charge per unit area : σ

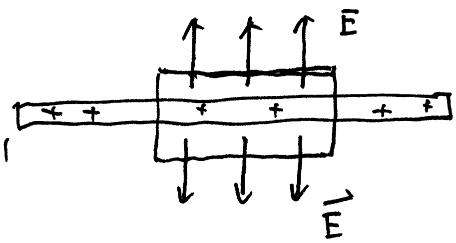


Gaussian surface : cylindrical Box.

Top and Bottom : $\vec{E} \parallel \vec{A}$.

↑ front view

Lateral : $\vec{E} \perp \vec{A}$. (No flux).



$\therefore \quad \Phi_{Total} = \Phi_{Top} + \Phi_{bottom} + \Phi_{lateral}$

$\sum \vec{E} \cdot \vec{A} = \Phi_{Top} + \Phi_{bottom}$

front view

$= EA + EA$.

$= 2EA$.

A — Top area (bottom area)

$Q_{encl.} = \sigma \cdot A$.

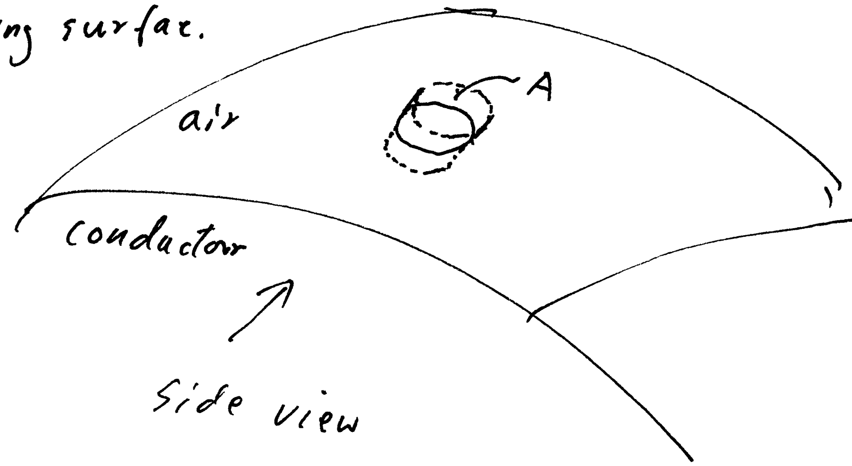
$\therefore \quad \sum \vec{E} \cdot \vec{A} = \dfrac{Q}{\varepsilon_0} \implies 2EA = \sigma A / \varepsilon_0$

$E = \dfrac{\sigma}{2\varepsilon_0}$.

Direction of $\vec{E}$: ~~p~~ perpendicular to the plane.
pointing outward.

e.g. 22-8 Near a conducting surface. — charge per unit area: σ.

Gaussian surface:
Small. Box across the conducting surface.



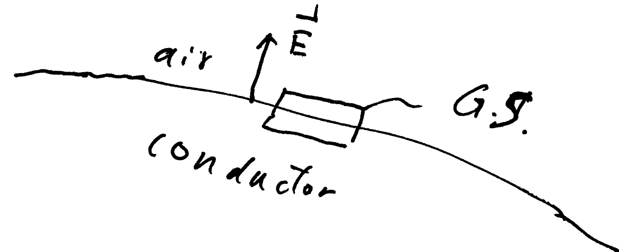
$$\Phi_{Total} = \Phi_{Top} + \Phi_{Bottom} + \Phi_{side}.$$

$$= EA + 0 + 0$$

$$Q_{encl.} = \sigma A.$$

$$\Phi_{Total} = \frac{Q_{encl.}}{\varepsilon_0}$$

$$\therefore \quad EA = \frac{\sigma A}{\varepsilon_0}$$



$$E = \frac{\sigma}{\varepsilon_0}.$$ — different from a charged insulating surface.

$$E = \frac{\sigma}{2\varepsilon_0}$$