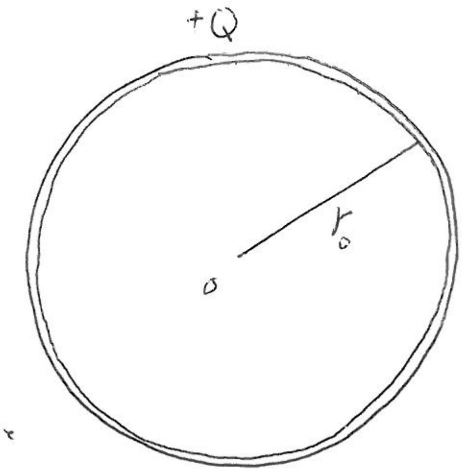
e.g. Spherical conductor

Thin shell. radius $r_0$

charge $Q$. — uniformly distributed.

$Q > 0$




Find $\vec{E}$ : (a) outside the shell.

(b). inside the shell

(c). what if it's a solid sphere (a conductor)

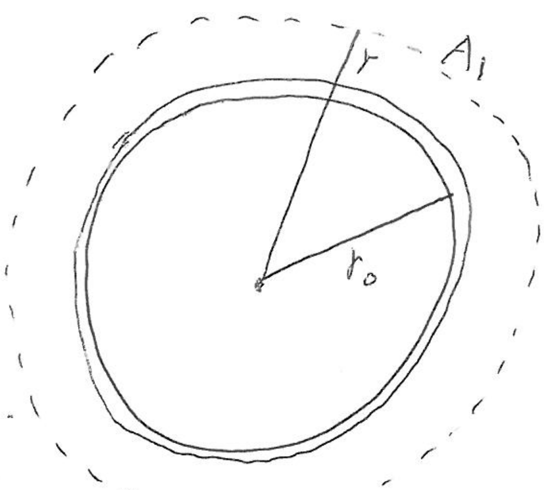
- Highly symmetrical.

— use gauss' law. $\sum \vec{E} \cdot \vec{A} = \frac{Q}{\varepsilon_0}$

choose a spherical surface as the Gauss surface. (dashed line)

a) Gaussian surface $A_1$.

radius $r$. — variable $r > r_0$.




At any point on $A_1$:

$\vec{E} \parallel \vec{r} \parallel \vec{A}_1$

$E$ = constant (magnitude)

($\because$ same r).

$$\therefore \sum_{A_1} \vec{E} \cdot \vec{A} = E \cdot A_1 = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}.$$ direction: $\vec{E} \parallel \vec{r}$.

Makes sense: just like a charge Q at the centre.

b) inside the spherical shell.

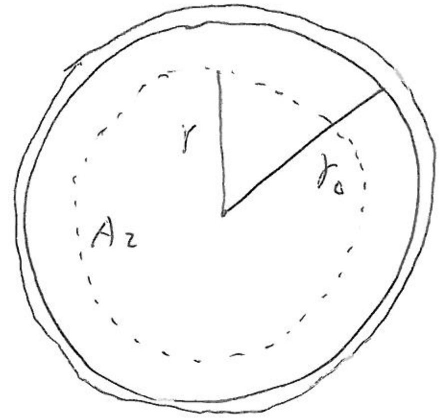
Gaussian surface $A_2$.

radius $r < r_0$

↑

variable.

(we want to find $\vec{E}$ at any point. $r$)




But now No charge is enclosed in $A_2$.

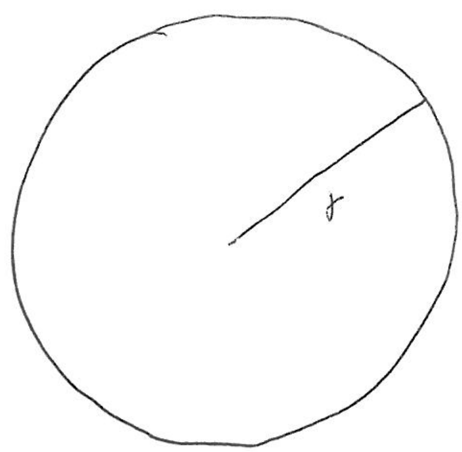
$$\therefore \quad \sum_{A_2} \vec{E} \cdot \vec{A} = E \cdot 4\pi r^2 = 0$$

$$\therefore \quad E = 0 .$$

c). What if it's a solid sphere — (still, conductor!)

The charge Q is uniformly distributed on the surface.

∴ the same as a thin shell!

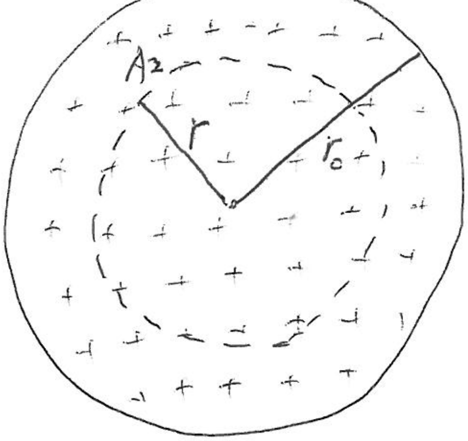


e.g. Solid sphere of charge. $Q > 0$

$Q$ is uniformly distributed throughout the sphere.

— can't be a conductor!

(has to be an insulator).




Find $\vec{E}$: a). outside $r > r_0$

b). inside $r < r_0$

a). outside. Same as before. (same as conductor)

$$\sum \vec{E} \cdot \vec{A} = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}. \quad \vec{E} \parallel \vec{r}.$$

b) inside — different!

because $Q$ is NOT distributed on the surface!

define: charge density

$$\rho = \frac{Q}{V} \quad \left\{ \begin{array}{l} \rho = \text{constant} \\ V = \text{volume} \end{array} \right\}$$

$$= Q \Big/ \tfrac{4}{3}\pi r_0^3$$

$$= \frac{3Q}{4\pi r_0^3}$$

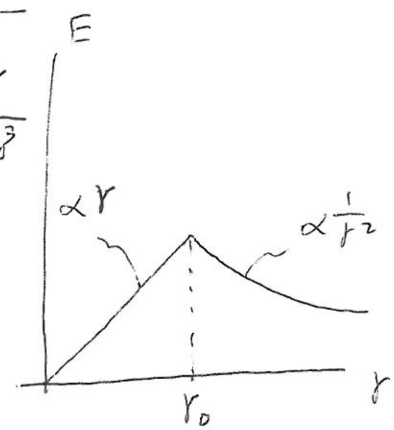
$$\sum_{A_2} \vec{E} \cdot \vec{A} = \frac{Q_{encl.}}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} \cdot \rho \frac{4}{3}\pi r^3$$

$$E = \frac{\rho r}{3\varepsilon_0} = \frac{r}{3\varepsilon_0} \cdot \frac{3Q}{4\pi r_0^3} = \frac{Q}{4\pi\varepsilon_0} \frac{r}{r_0^3}$$

$\vec{E} \parallel \vec{r}$

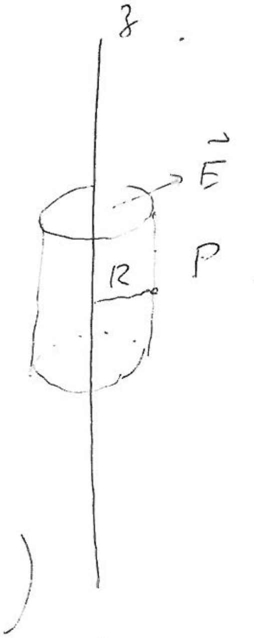
$$\therefore \vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{\vec{r}}{r_0^3} \quad (r < r_0)$$



e.g. Long uniform line of charge

Very long — infinite length $L$

linear charge density — $\lambda = \frac{Q}{L}$.




Find $\vec{E}$ near the wire. (outside) at P.

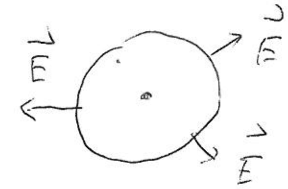
Symmetry: cylindrical.

E doesn't depend on z.

( L — very long.
z — varies in a small range )

Gaussian surface: cylinder, radius R.
height H

Top view




$$\Phi = \sum \vec{E} \cdot \vec{A}$$

$$= \Phi_{Top} + \Phi_{lateral} + \Phi_{bottom}$$

$$= 0 + 2\pi R H \cdot E + 0$$

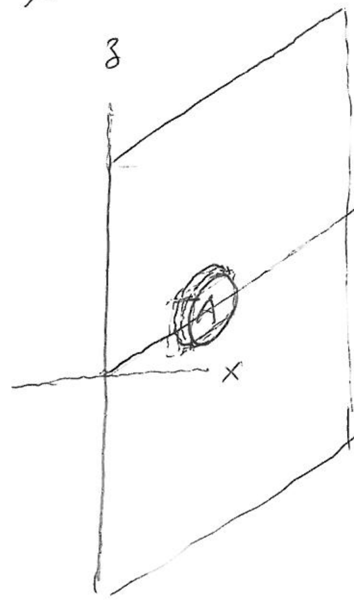
$E$ = constant.

$$\therefore \quad 2\pi R H E = \frac{Q_{encl.}}{\varepsilon_0} = \frac{\lambda H}{\varepsilon_0}.$$

$$E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{R}$$

$\vec{E}$ direction: along radius.

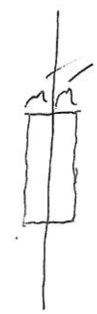
e.g. Infinite Plane of Charge. surface charge density $\sigma$.




Symmetry: No $(y, z)$ dependence.

we are interested at a point very close to the plane.

Gaussian surface: a pellet through the plane.
(drum).
(could be square shape).




$$\Phi = \Sigma \vec{E} \cdot \vec{A}$$

$$= \Phi_{left} + \Phi_{lateral} + \Phi_{right}$$

$$= A_L E + 0 + A_R E$$

$$\because \vec{E} \perp \vec{A}$$

$$= 2AE \quad (A - \text{end area})$$

$$= \frac{Q_{encl.}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}.$$

$$\therefore \quad 2AE = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

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e.g: near conducting surface. ( $E=0$ inside conductor).

$$\Phi = AE \quad \therefore \quad E = \frac{\sigma}{\varepsilon_0}$$