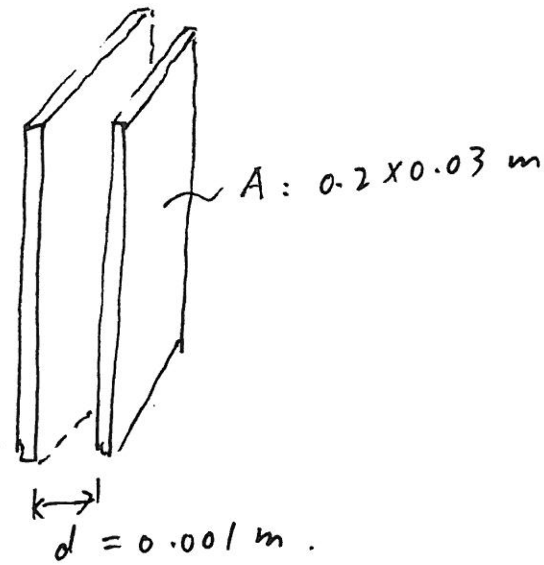
e.g. [redacted]. Capacitance of a parallel-plate capacitor



a). $C = \dfrac{A\varepsilon_0}{d}$ [redacted] $= \dfrac{0.2 \times 0.03 \times 8.85 \times 10^{-12}}{0.001}$

$= 5.3 \times 10^{-11} = 53\ pF.$

b). $V = 12.$ $Q = ?$

$Q = CV = 53 \times 10^{-12} \times 12 =$

$= 6.4 \times 10^{-10}\ C$

c). [redacted] $V = Ed$

$E = \dfrac{V}{d} = \dfrac{12}{0.001} = 1.2 \times 10^{4}\ V/m.$

d). want : $C_2 = 1\ F. = \dfrac{A_2 \varepsilon_0}{d}.$

$$\frac{C_2}{C} = \frac{A_2}{A}$$

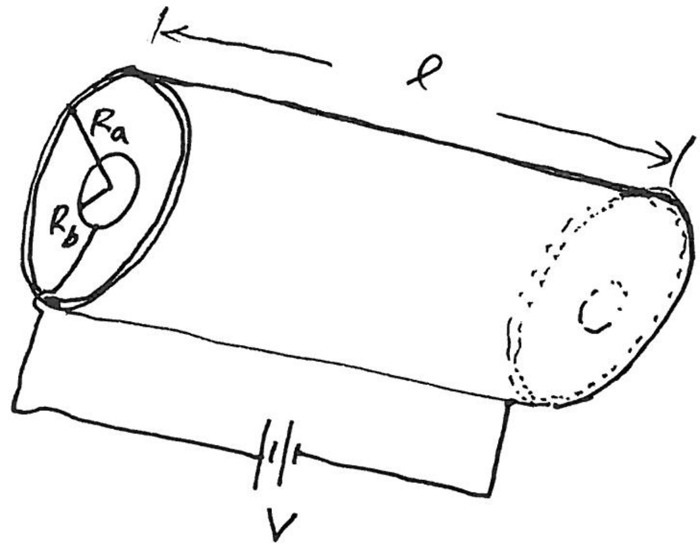
$$\frac{1\ F}{53 \times 10^{-12}\ F} = \frac{A_2}{0.2 \times 0.03}$$

$$A_2 = \frac{0.2 \times 0.03}{53 \times 10^{-12}} \approx 10^{8}\ m^2.$$

10 km x 10 km

e.g. Cylindrical Capacitor

ℓ - Very long.

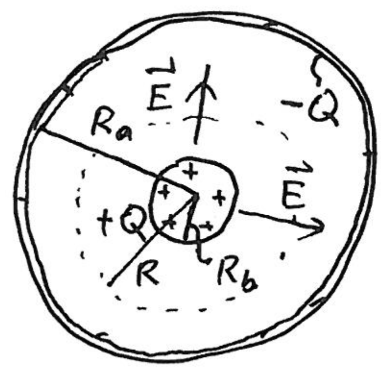


How do we find C?

- Idea:

If we apply a voltage V, we create a potential difference between the core and the shell (V).

Then, $\vec{E}$ exists from R_b to R_a.



That means, $+Q$ on R_b

$-Q$ on R_a.

Since $C = \dfrac{Q}{V}$,

we need to find relation between Q and V.

- One way to do that:

from Q find $\vec{E}$ — use Gauss's law. $E = \dfrac{Q/\ell}{2\pi\varepsilon_0 R}$

From E find V: $V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{\ell}$

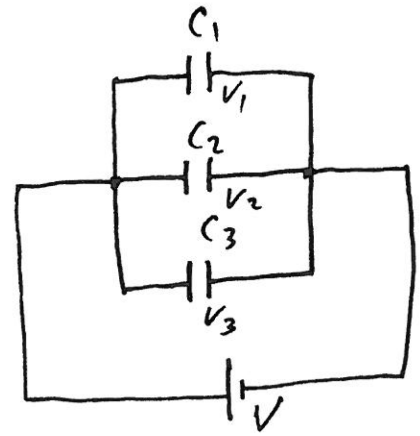
$$= -\int_{R_a}^{R_b} \frac{Q}{2\pi\varepsilon_0 \ell} \frac{dR}{R} = -\frac{Q}{2\pi\varepsilon_0\ell}\ln\frac{R_b}{R_a}$$

$$= \frac{Q}{2\pi\varepsilon_0\ell}\cdot\ln\frac{R_a}{R_b}.$$

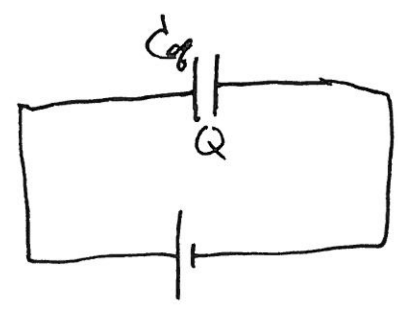
$$\therefore\ C = \frac{Q}{V} = \frac{2\pi\varepsilon_0\ell}{\ln\frac{R_a}{R_b}}$$

• Capacitors in parallel

$C_{eq.} = C_1 + C_2 + C_3$.



$\Downarrow$ equivalent to



Simple reason:

If they are parallel capacitors with the same gap distance d, then, the areas add. $A = A_1 + A_2 + A_3$.

Derivation:

parallel: $V_1 = V_2 = V_3 = V$.

$$Q = Q_1 + Q_2 + Q_3.$$

$$C = \frac{Q}{V}.$$

$$C_{eq} = \frac{Q}{V} = \frac{Q_1 + Q_2 + Q_3}{V}$$

$$= \frac{Q_1}{V_1} + \frac{Q_2}{V_2} + \frac{Q_3}{V_3}$$

$$= C_1 + C_2 + C_3.$$

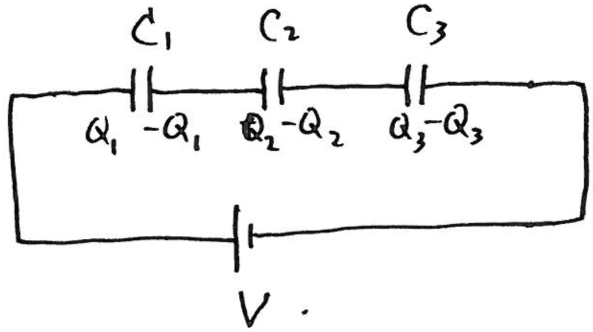
- Capacitors in Series.

Series: $Q_1 = Q_2 = Q_3 = Q$

$$V = V_1 + V_2 + V_3.$$

$$\therefore \frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$



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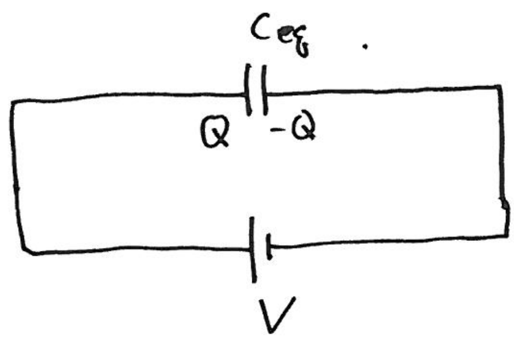


- e.g. 24-5.

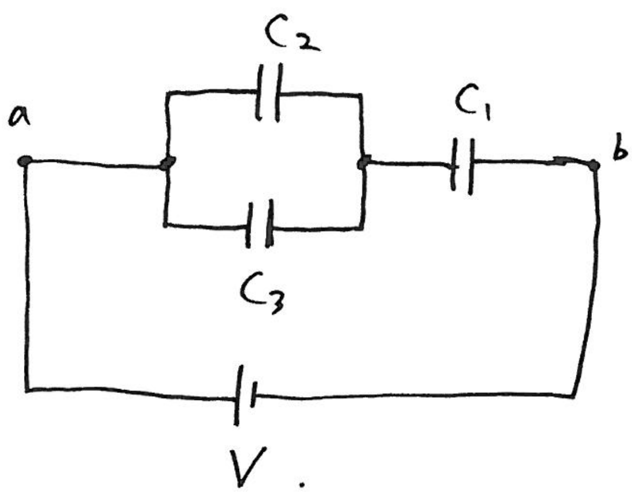
$$C_{23} = C_2 + C_3.$$

$$C_{ab}: \quad \frac{1}{C_{ab}} = \frac{1}{C_{23}} + \frac{1}{C_1}.$$

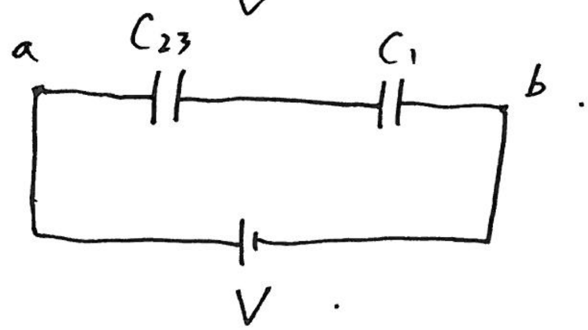
$$\frac{1}{C_{ab}} = \frac{C_1 + C_{23}}{C_{23} C_1}$$

$$C_{ab} = \frac{C_1 C_{23}}{C_1 + C_{23}}$$

$$= \frac{C_1 (C_2 + C_3)}{C_1 + C_2 + C_3}$$



$\Downarrow$

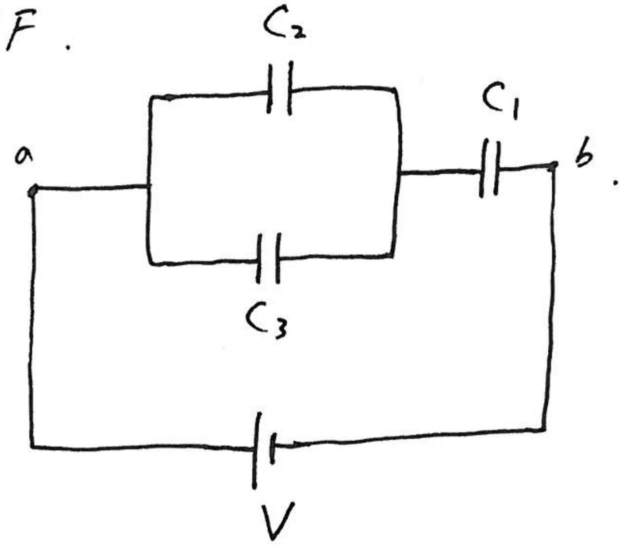


e.g. [redacted].

If $C_1 = C_2 = C_3 = C = 3.0\,\mu F$.

$V = 4.0\,V$.

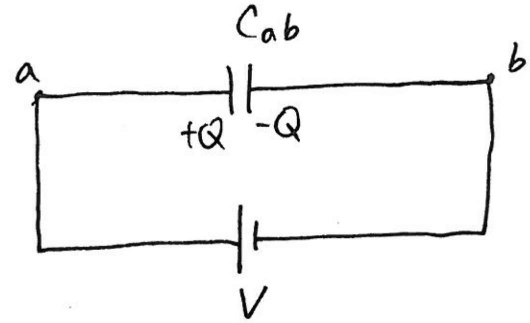
Find. $Q_1, Q_2, Q_3, V_1, V_2, V_3$.



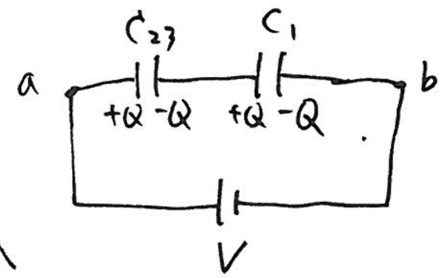
$$C_{ab} = \frac{C(C+C)}{C+C+C} = \frac{2C}{3} = 2\,\mu F.$$

$$Q = Q_{ab} \cdot V$$

$$= 2\,\mu F \cdot 4\,V = 8\,\mu C.$$



$$Q_1 = Q = 8\,\mu C.$$



$$V_1 = \frac{Q_1}{C_1} = \frac{8\,\mu C}{3\,\mu F} = 2.7\,V.$$

$$Q_{23} = Q_1 = Q = 8\,\mu C.$$

$$Q_2 = Q_3 = \frac{Q_{23}}{2} = 4\,\mu C.$$

$$V_2 = V_3 = \frac{Q_2}{C} = \frac{4\,\mu C}{3\,\mu F} = 1.3\,V.$$

$$(V_2 + V_1 = 4\,V = V)$$

e.g. [redacted].

$C_1 = 2.2\ \mu F$.

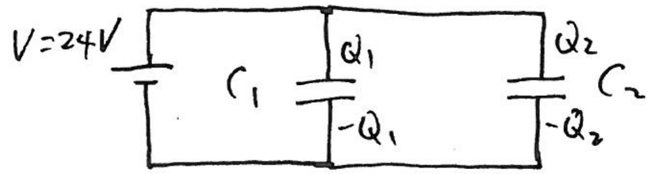
$C_2 = 1.2\ \mu F$.

$V = 24V$.

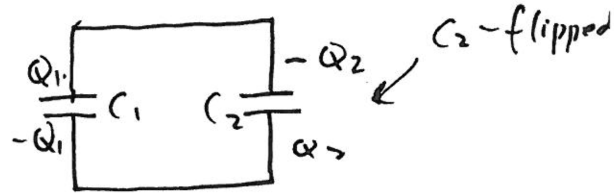
Find: q_1, q_2

V_{final}.

initial (charged)

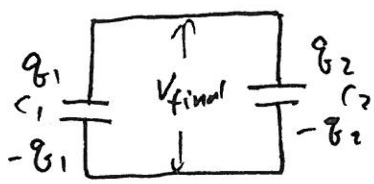


Then. (disconnected and reconnected)



↓ reach equilibrium.

Finally: (reach equilibrium)



Initial: $C_{eq} = C_1 + C_2 = 3.4\ \mu F$., $V_1 = V_2 = V = 24V$.

$$\left(Q = Q_1 + Q_2 = C_{eq} \cdot V = 3.4\ \mu F \cdot 24\ V = 81.6\ \mu C \right)$$

$$Q_1 = C_1 V_1 = 2.2\ \mu F \times 24V = 52.8\ \mu C.$$

$$Q_2 = C_2 V_2 = 1.2\ \mu F \times 24V = 28.8\ \mu C.$$

Final: $q_1 + q_2 = Q_1 - Q_2 = 24\ \mu C = q$.

$C_{eq} = C_1 + C_2 = 3.4\ \mu F$ (unchanged).

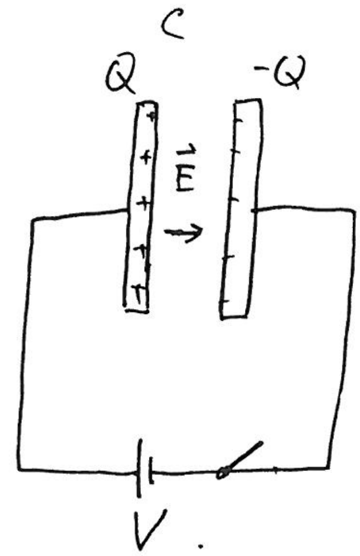
$$V_{final} = \frac{q}{C_{eq}} = \frac{24\ \mu C}{3.4\ \mu F} = 7.1V.$$

$$q_1 = C_1 V_{final} = 2.2\ \mu F \times 7.1V = 15.5\ \mu C$$

$$q_2 = C_2 V_{final} = 1.2 \times 7.1 = 8.5\ \mu C.$$

- Energy stored in a capacitor.

$U = \frac{1}{2} Q V$.



Why not $U = QV$?

where does $\frac{1}{2}$ come from ?

Remembe, if you move a charge q through a potential difference V, the potential energy difference should be $U = qV$.

Answer: Q is gradually moved across.

Initially, when $Q=0$, $\vec{E}=0$. $V=0$. easy to get across. (No energy)

Finally, when C is fully charged, $V=V$. it cost energy to get across. $U = qV$.

Average potential difference between the plates: $\frac{1}{2} V$.

$\therefore$ Total $U = Q \cdot \left(\frac{1}{2} V\right) = \frac{1}{2} Q V$.

$= \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C V^2$.

$\left(V = \frac{Q}{C}. \quad Q = CV \right)$

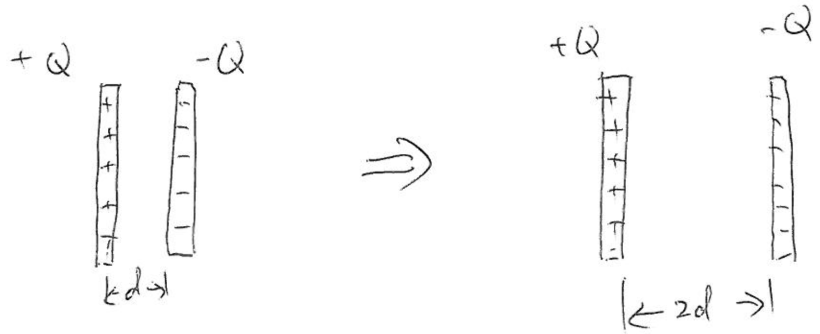
e.g. Energy stored in a capacitor.

$C = 150\ \mu F$. $V = 200\ V$.

a). $U = \frac{1}{2} C V^2 = \frac{1}{2} \times 150 \times 10^{-6} \times (200)^2 = 3.0\ J$

b). $P = \frac{U}{t} = \frac{3.0}{1 \times 10^{-3}} = 3\ KW$.

Conceptual example.



$C_1 = C_0$ $\qquad$ $C_2 = \frac{1}{2} C_0$

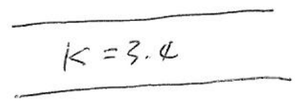
$U_1 = \frac{1}{2} \frac{Q^2}{C_0}$. $\qquad$ $U_2 = \frac{1}{2} \frac{Q^2}{\frac{1}{2} C_0} = \frac{Q^2}{C_0} = 2 U_1$.

You need to do work to the capacitor in order to increase the seperation. d.

e.g. Dielectric removal.

$A = 4.0\ m^2 \quad d = 4.0\times10^{-3}\ m.$

$K = 3.4\ ,\quad V = 100\ V.$



a). $C = \varepsilon_0 K \frac{A}{d}$

$= 8.85\times10^{-12} \times 3.4 \times \frac{4.0}{4.0\times10^{-3}}$

$= 3.0\times10^{-8}\ F \quad = 30\ nF.$

$Q = C\cdot V = 3.0\times10^{-8} \times 100 = 3.0\times10^{-6}\ C = 3.0\ \mu C.$

$E = \frac{V}{d} = \frac{100}{4.0\times10^{-3}} = 25\ kV/m.$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\times 3.0\times10^{-8}\times(100)^2 = 1.5\times10^{-4}\ J$

b). $Q = 3.0\ \mu C = 3.0\times10^{-6}\ C.$

$C = \varepsilon_0 \frac{A}{d} = 8.85\times10^{-9}\ F = 8.85\ nF$

$V = \frac{Q}{C} = \frac{3.0\times10^{-6}}{8.85\times10^{-9}} \approx 3.4\times10^2\ V = 340\ V$

$E = \frac{V}{d} = \frac{340}{4.0\times10^{-3}} = 8.5\times10^4\ V/m$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\times 8.85\times10^{-9}\times(340)^2 = 5.1\times10^{-4}\ J$

Energy stored has been increased.

The induced charges in the medium are allocated to the free charges on the plates. To move the medium out of the capacitor, we need to do work.