Phys102 Lecture 12
Kirchhoff's Rules

Key Points

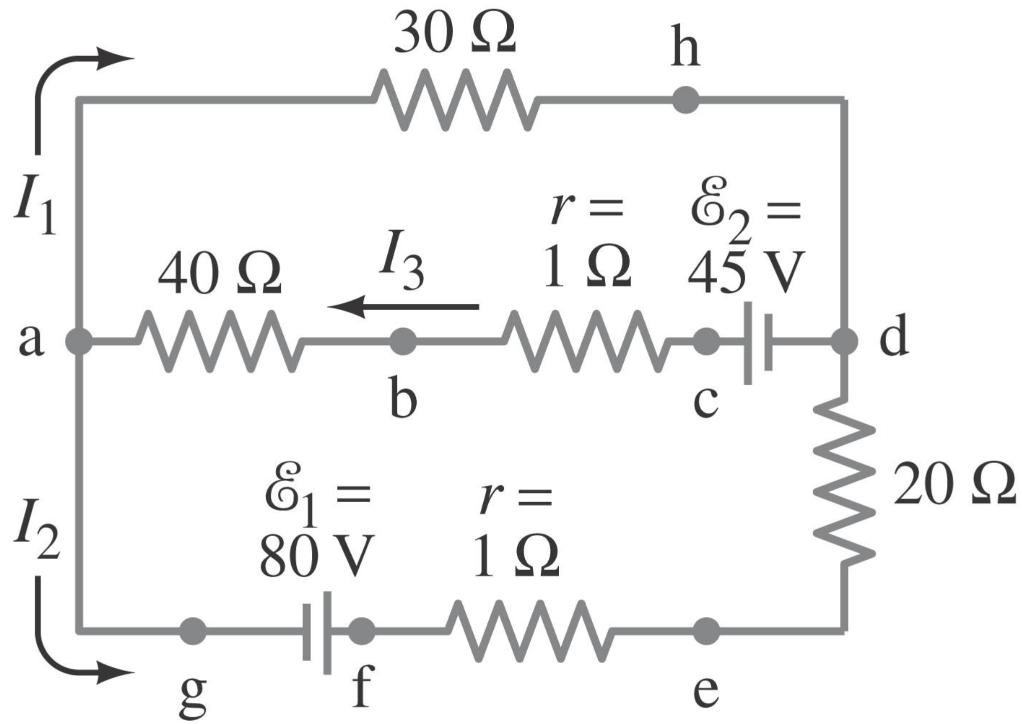
- Kirchhoff's Junction Rule
- Kirchhoff's Loop Rule
- Solving Linear Algebraic Equations

References

19-3.

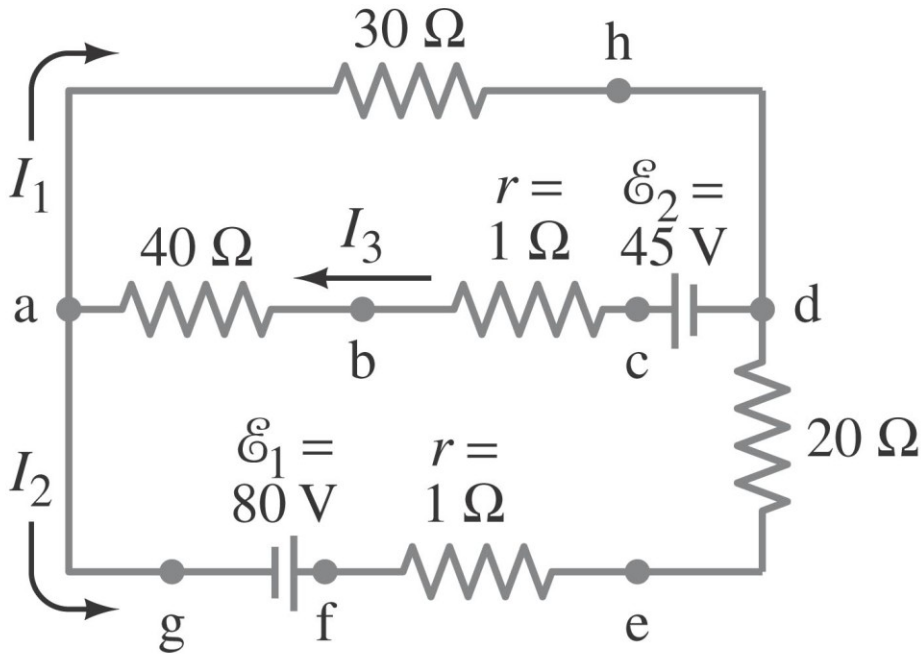
Kirchhoff's Rules

Some circuits cannot be broken down into series and parallel connections. For these circuits we use Kirchhoff's rules.



19-3 Kirchhoff's Rules

Junction rule: The sum of currents entering a junction equals the sum of the currents leaving it.

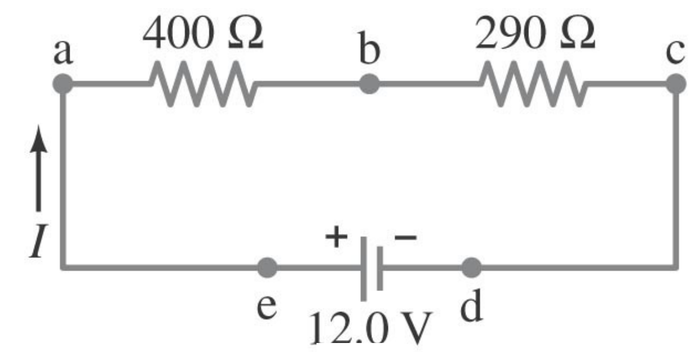


$$I_3 = I_1 + I_2$$

It's a consequence of charge conservation.

19-3 Kirchhoff's Rules

Loop rule: The sum of the changes in potential around a closed loop is zero.



$$V_{ba} + V_{cb} + V_{ed} = 0$$

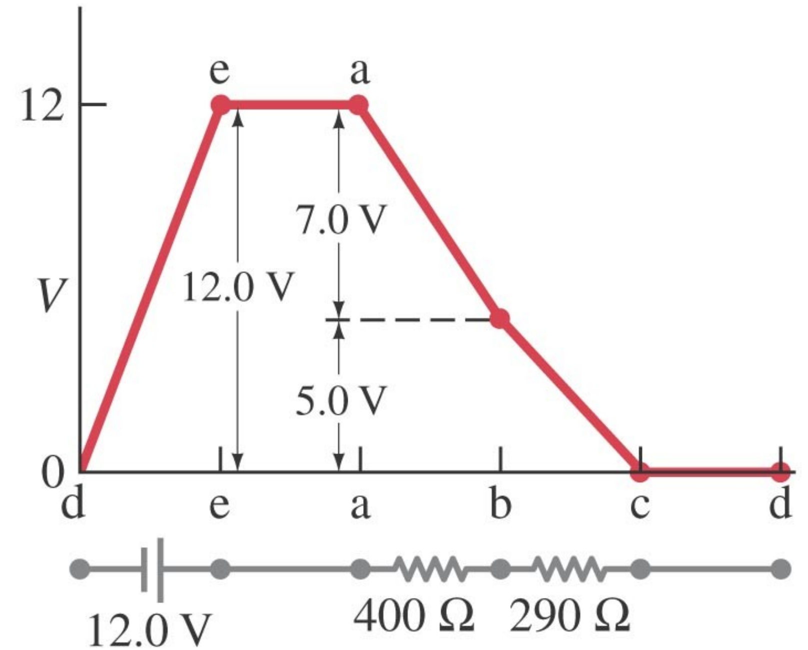
It's a consequence of energy conservation.

$$I = \frac{12V}{400\Omega + 290\Omega} = 0.017A$$

$$V_{ba} = V_b - V_a = -400I = -7.0V$$

$$V_{cb} = V_c - V_b = -290I = -5.0V$$

$$V_{ed} = V_e - V_d = 12V$$



+: potential increase

-: potential decrease

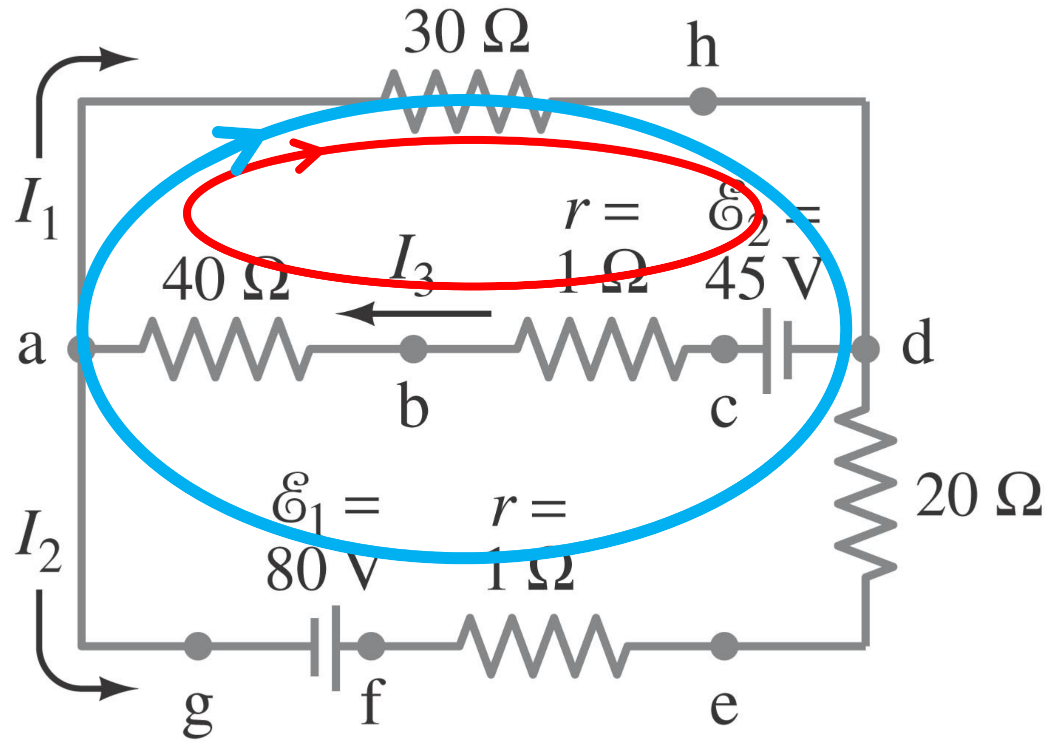
19-3 Kirchhoff’s Rules

Problem Solving: Kirchhoff’s Rules

1. **Label** each current, including its direction.
2. Identify unknowns.
3. Apply junction and loop rules; you will need as many independent equations as there are unknowns.
4. Solve the equations, being careful with signs. If the solution for a current is negative, that current is in the opposite direction from the one you have chosen.

Example: Using Kirchhoff's rules.

Calculate the currents I_1, I_2, and I_3 in the three branches of the circuit in the figure.



Junction:

$$I_{in} = I_{out}$$

$$\text{a}: \quad I_3 = I_1 + I_2$$

$$\text{d}: \quad I_1 + I_2 = I_3$$

They're not independent!

We'll use one of them.

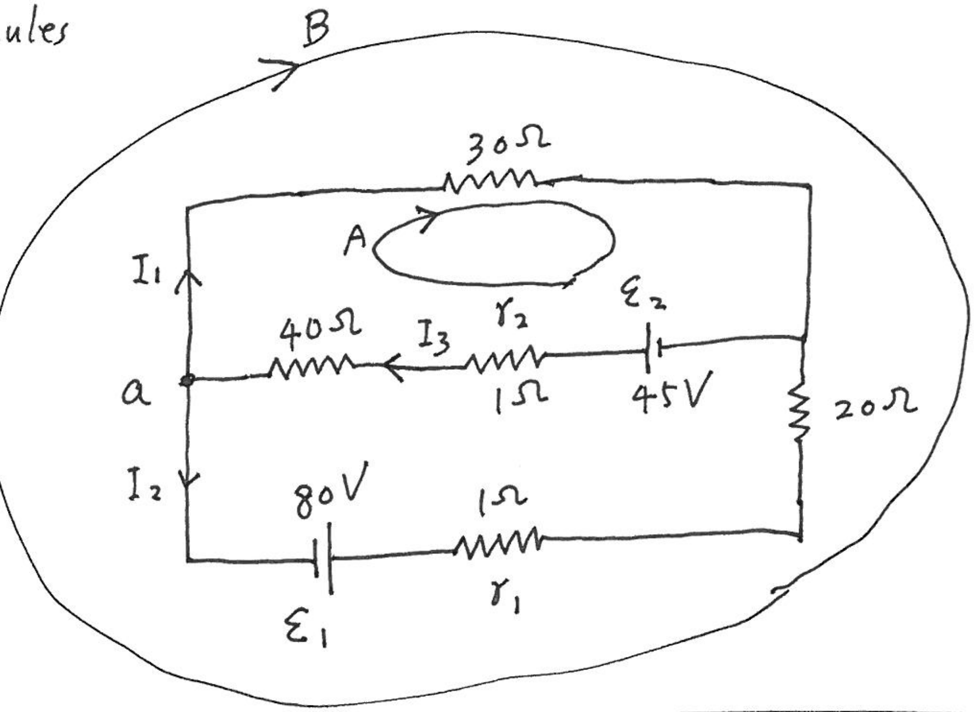
Loop:

$$\sum V = 0$$

$$\text{Upper:} \quad -30I_1 + 45 - 1I_3 - 40I_3 = 0$$

$$\text{Large:} \quad -30I_1 + 20I_2 + 1I_2 - 80 = 0$$

e.g. Kirchhoff's Rules



$$\begin{cases} I_3 = I_1 + I_2 \\ 45 - 1 \cdot I_3 - 40 I_3 - 30 I_1 = 0 \\ -30 I_1 + 20 I_2 + 1 \cdot I_2 - 80 = 0 \end{cases}$$

$$\begin{cases} I_1 + I_2 - I_3 = 0 & (1) \\ 30 I_1 + 41 I_3 = 45 & (2) \\ -30 I_1 + 21 I_2 = 80 & (3) \end{cases}$$

Eliminate I_3 to get a 2x2 system (with I_1 and I_2)

(1) × 41 + (2):

$$41 I_1 + 30 I_1 + 41 I_2 = 45$$

$$\begin{cases} 71 I_1 + 41 I_2 = 45 & (4) \\ -30 I_1 + 21 I_2 = 80 & (3) \end{cases}$$

(4) $\times \frac{30}{71}$: $30 I_1 + \frac{1230}{71} I_2 = \frac{1350}{71}$

+(3). $-30 I_1 + 21 I_2 = 80$.

$$I_2 \left(\frac{1230}{71} + 21 \right) = \frac{1350}{71} + 80$$

$$38.3\ I_2 = 99.0$$

$$I_2 = \frac{99}{38.3} = 2.59\ A.$$

(3): $I_1 = \frac{21 I_2 - 80}{30}$

$= \frac{21 \times 2.59 - 80}{30}$

$= -0.85 A$.

(1). $I_3 = I_1 + I_2 = -0.85 + 2.59 = 1.74 A$.