Phys102 Lecture 3

Gauss's Law

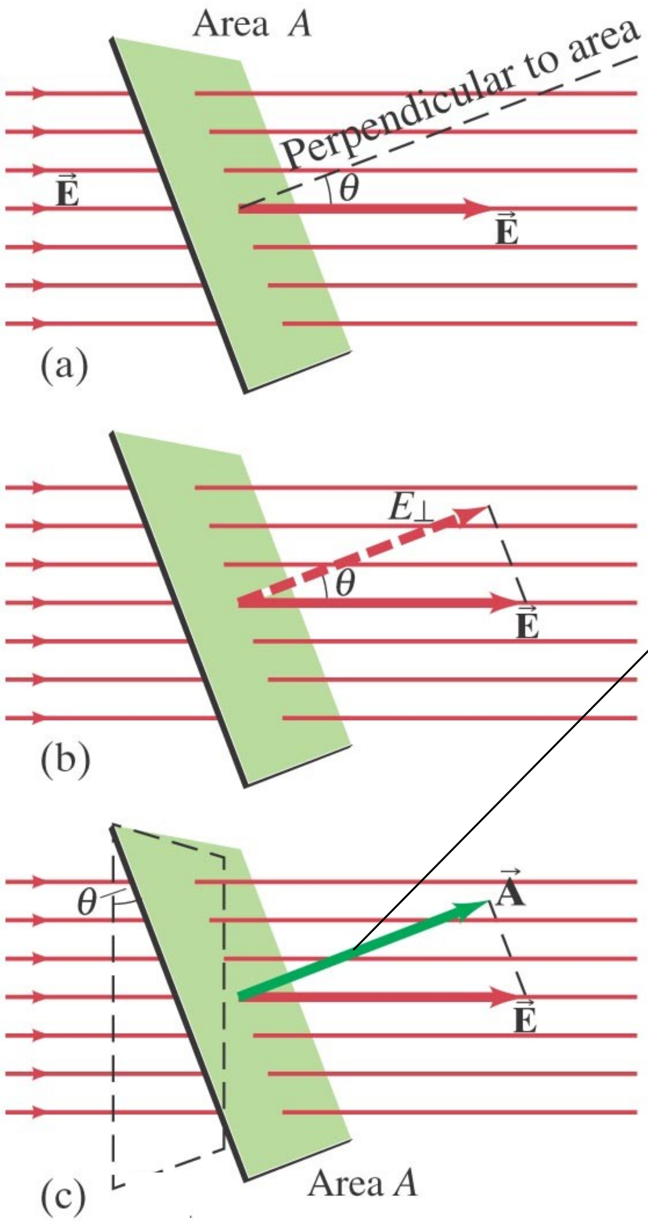
Key Points

- Electric flux
- Gauss's Law

References

6th Ed: 16-10

Electric Flux



Electric flux:

$$\Phi_E = E_\perp A = EA_\perp = EA\cos\theta, \qquad [\vec{\mathbf{E}} \text{ uniform}]$$

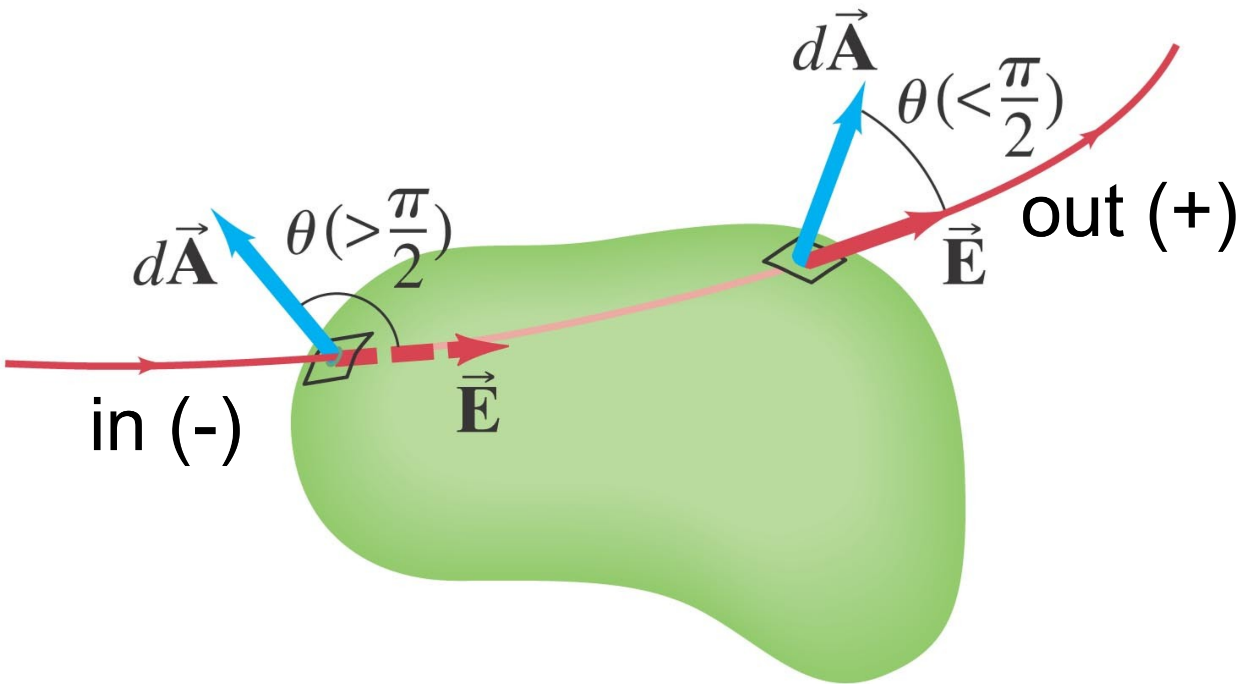
$$\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}. \qquad [\vec{\mathbf{E}} \text{ uniform}]$$

The direction of area if defined as the normal direction.

Electric flux through an area is proportional to the total number of field lines crossing the area.

Flux through a closed surface:

$$\Phi_E \approx \sum_{i=1}^{n} \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i.$$



Gauss's Law

The total flux through a closed surface is equal to the charge enclosed divided by ε_0:

$$\Phi_{Total} = \sum \vec{E} \cdot \vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

This can be used to find the electric field in situations with a high degree of symmetry.

To use Gauss's law to calculate E, we need to **choose a closed surface** such that it is easy to calculate the summation. Specifically, **E is constant on the surface**. Sometimes the surface can consists of a few parts, on each part E should be a constant or 0.

I-clicker question 3-1:

The total flux of magnetic field through a closed surface should be:

A) Always positive.

B) Always negative.

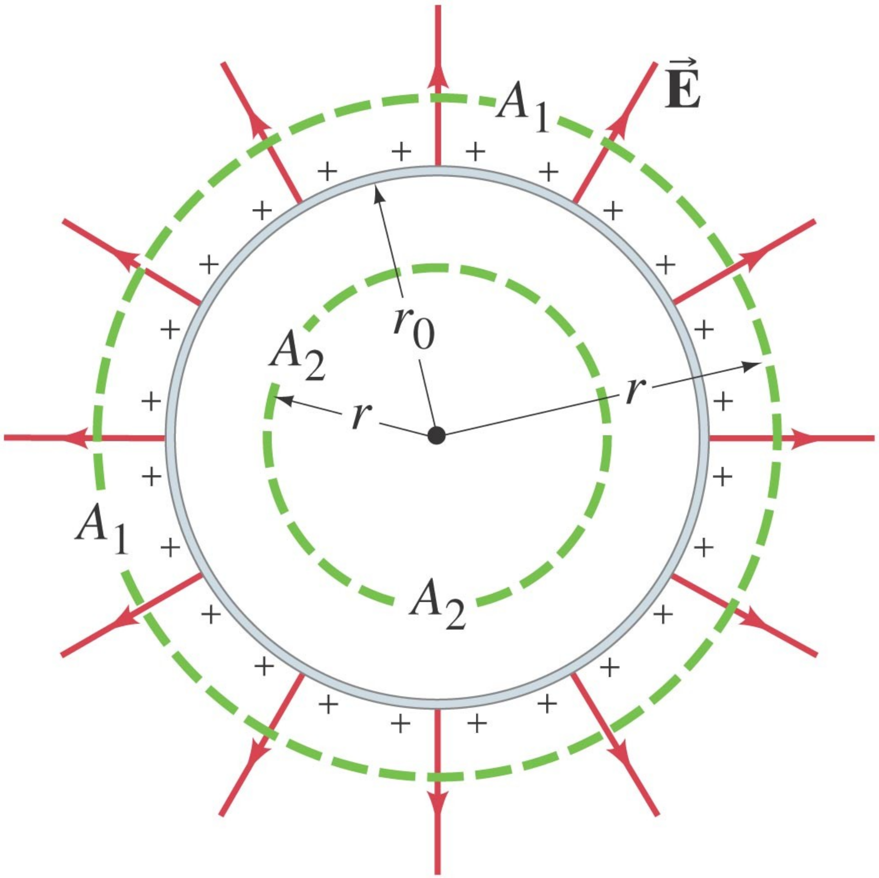
C) Always zero.

D) Can be any value.

E) No idea.

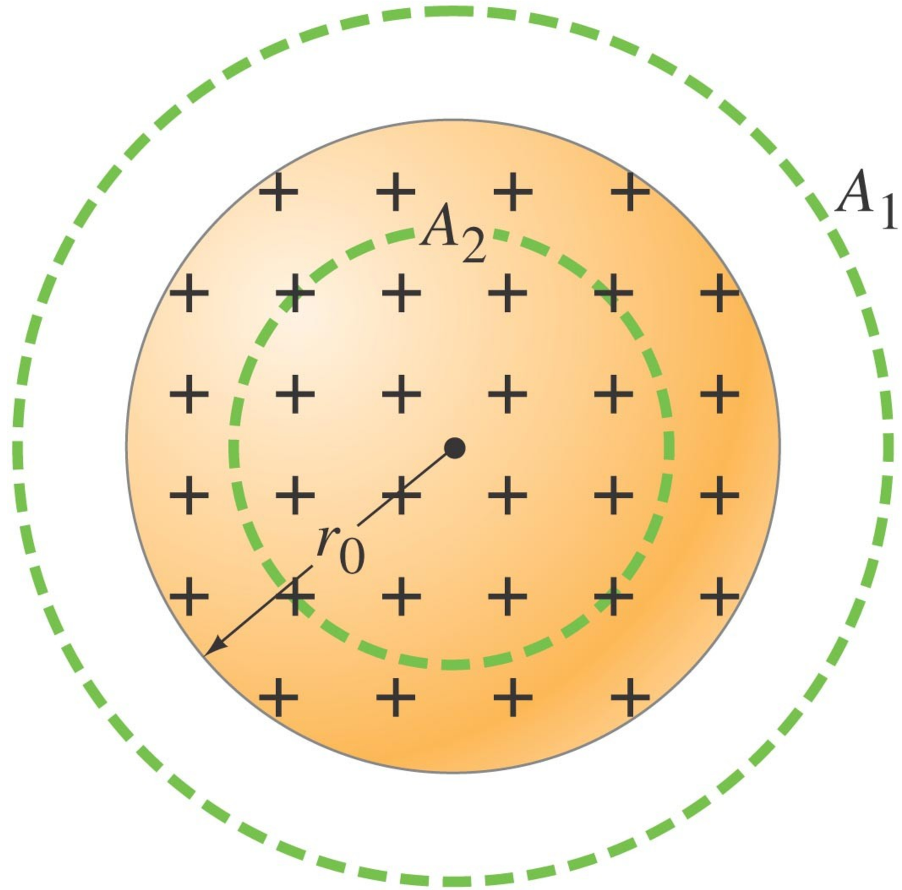
Example : Spherical conductor.

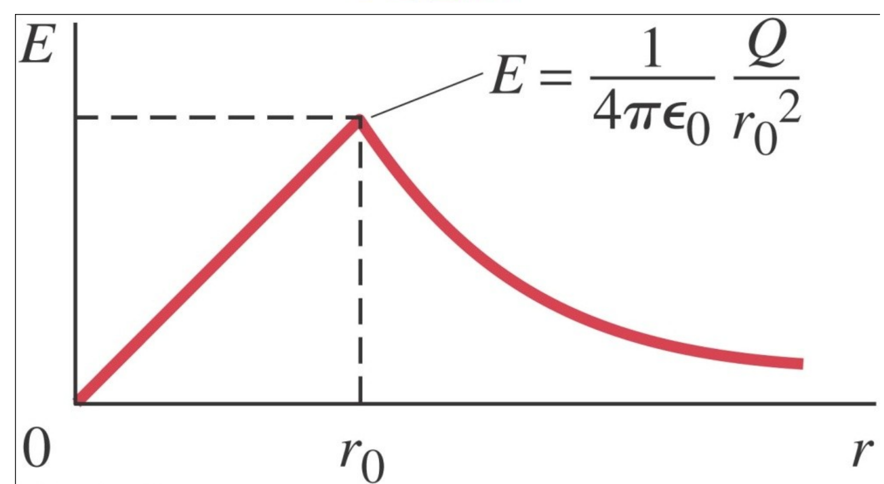
A thin spherical shell of radius r_0 possesses a total net charge Q that is uniformly distributed on it. Determine the electric field at points (a) outside the shell, and (b) within the shell. (c) What if the conductor were a solid sphere?



Example: Solid sphere of charge.

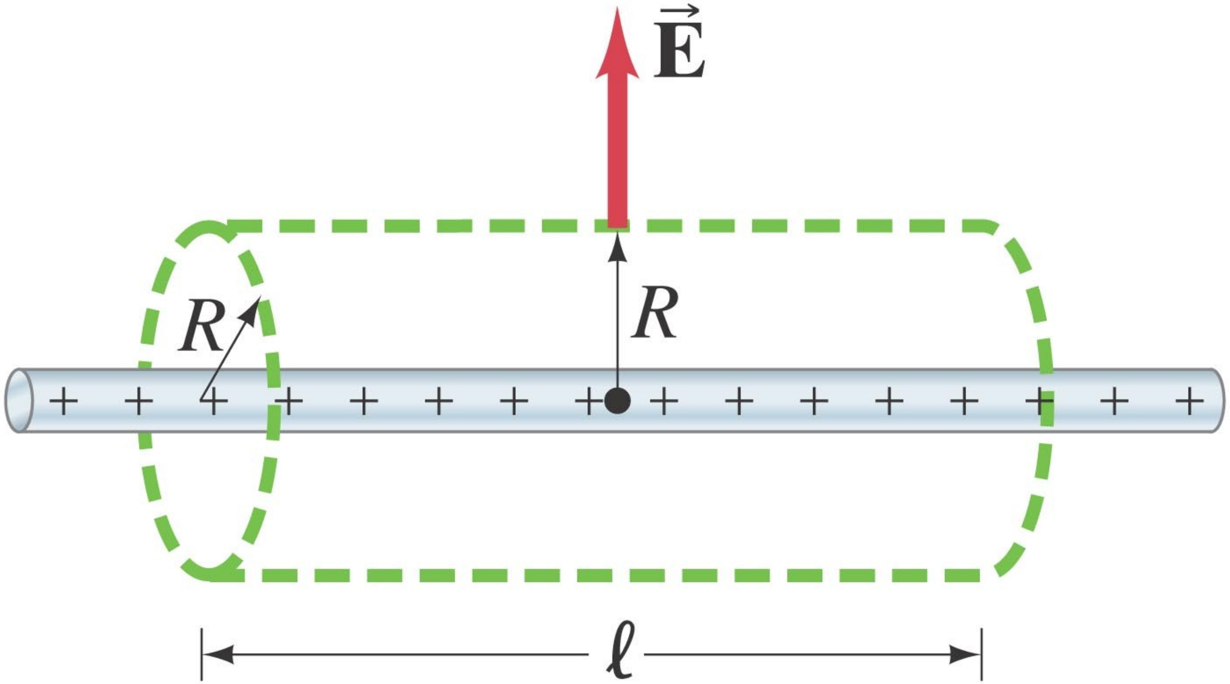
An electric charge Q is distributed uniformly throughout a nonconducting sphere of radius r_0. Determine the electric field (a) outside the sphere ($r > r_0$) and (b) inside the sphere ($r < r_0$).





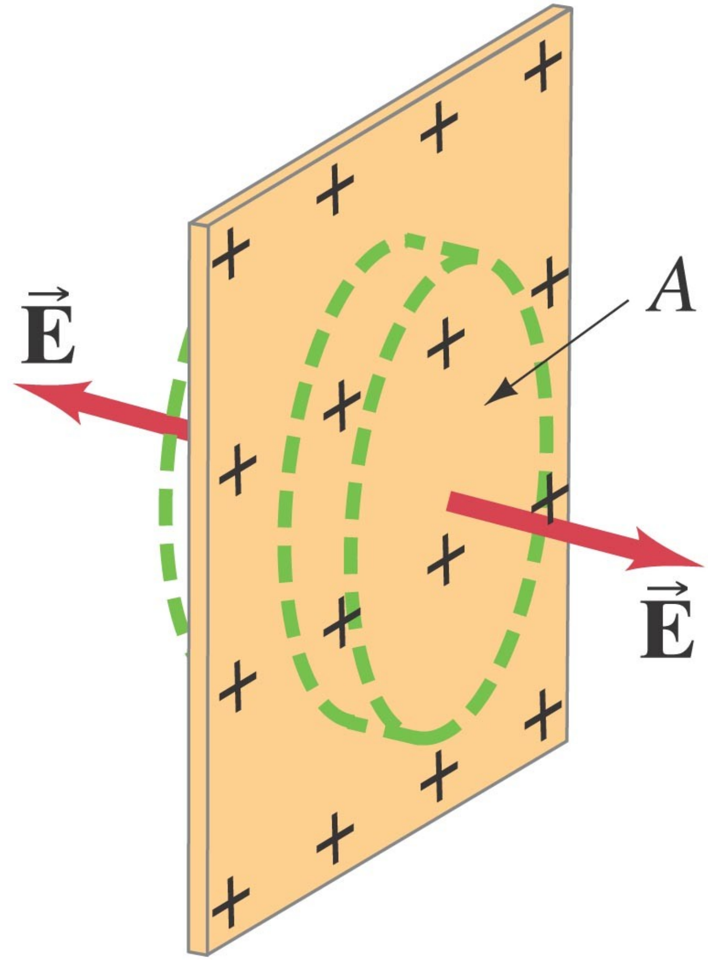
Example: Long uniform line of charge.

A very long straight wire possesses a uniform positive charge per unit length, λ. Calculate the electric field at points near (but outside) the wire, far from the ends.



Example: Large plane of charge.

Charge is distributed uniformly, with a surface charge density σ (σ = charge per unit area) over a very large but very thin nonconducting flat plane surface. Determine the electric field at points near the plane.



Example: Electric field near any conducting surface.

Show that the electric field just outside the surface of any good conductor of arbitrary shape is given by

$$E = \sigma/\varepsilon_0$$

where σ is the surface charge density on the conductor's surface at that point.