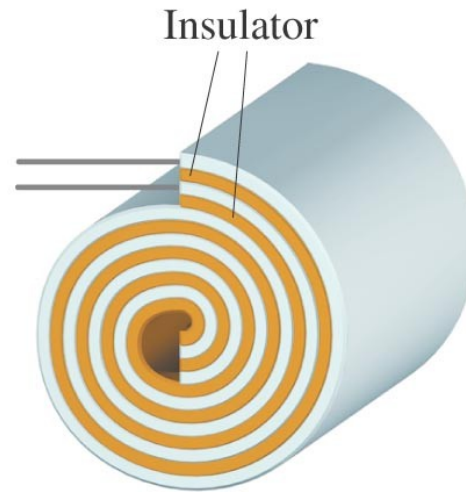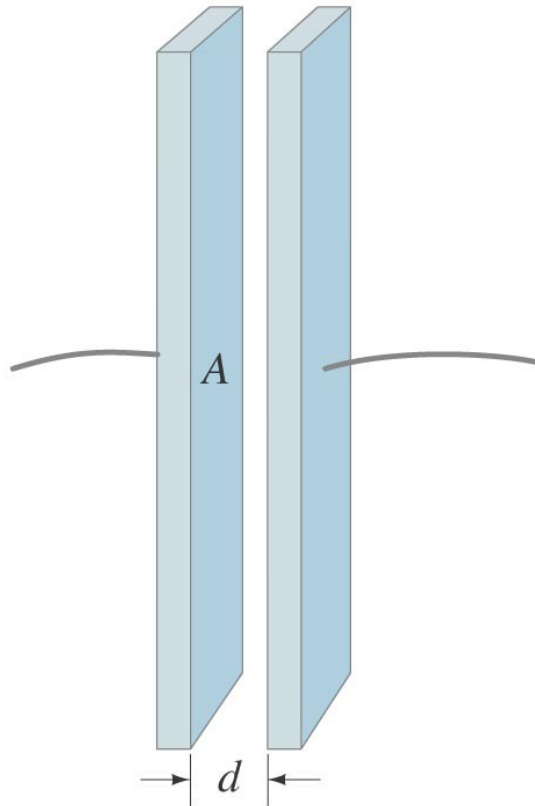**Phys102 Lecture 7/8**

Capacitors

Key Points

• Capacitors

• Determination of Capacitance

• Capacitors in Series and Parallel

• Electric Energy Storage
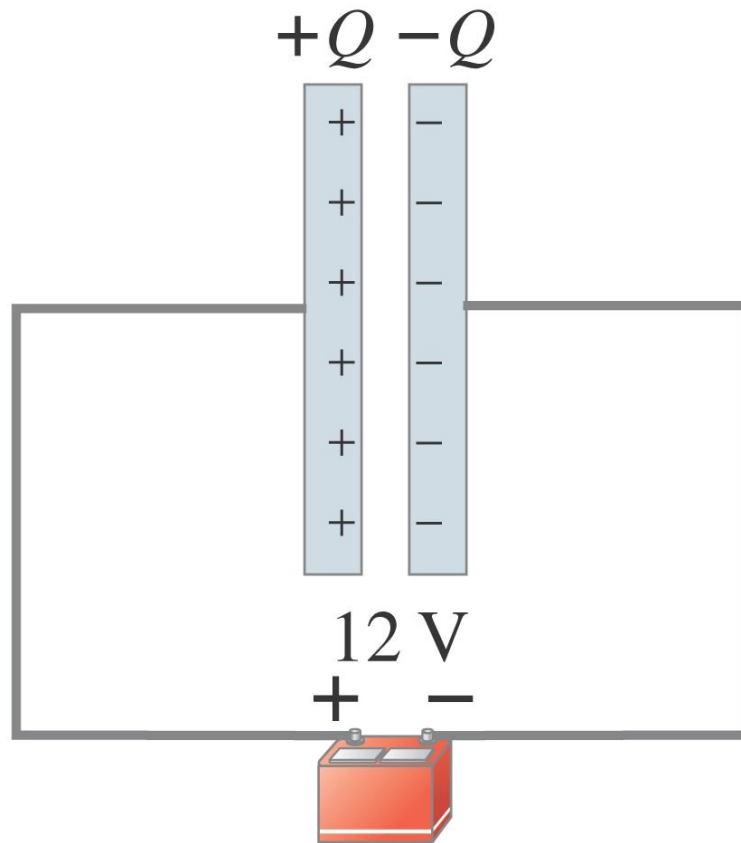
• Dielectrics
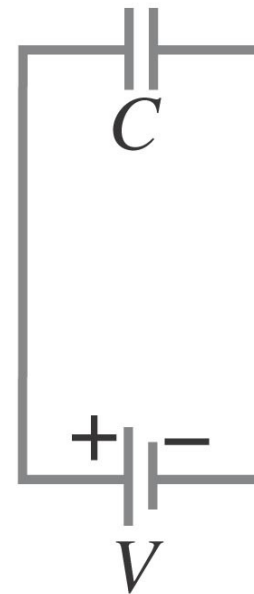
References
17-7,8,9,10+.

A capacitor consists of two conductors that are close but not touching. A capacitor has the ability to store electric charge.

Parallel-plate capacitor connected to battery. (b) is a circuit diagram.

$+Q$ $-Q$

12 V

(a)

C

V

(b)

Capacitors

When a capacitor is connected to a battery, the charge on its plates is proportional to the voltage:
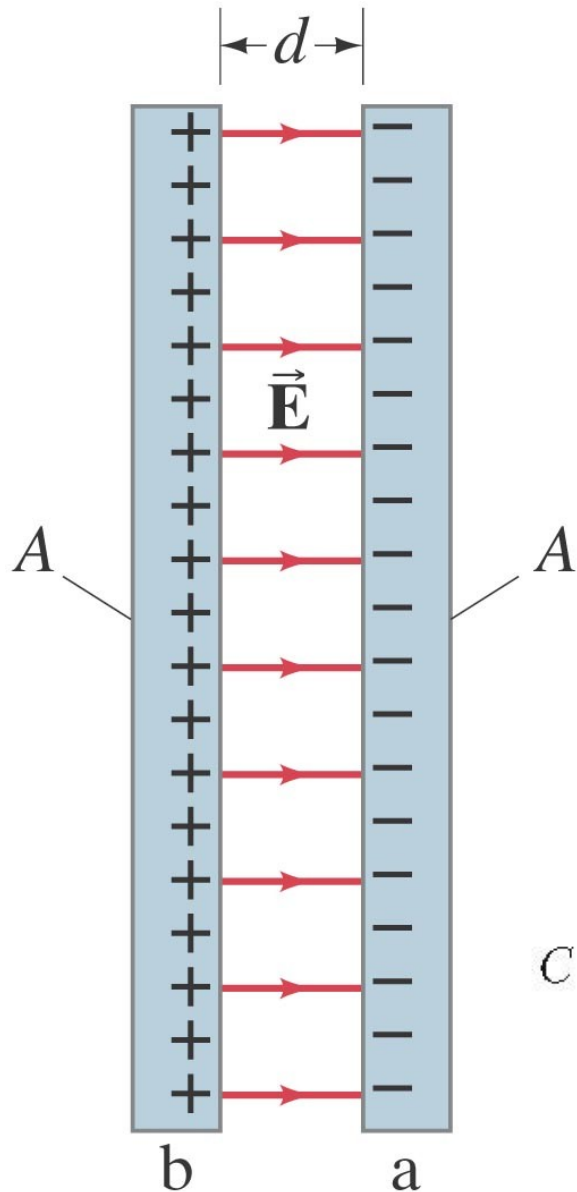
$$Q = CV.$$

The quantity C is called the capacitance.

Unit of capacitance: the farad (F):

$$1\ \text{F} = 1\ \text{C/V}.$$

Determination of Capacitance

For a parallel-plate capacitor as shown, the field between the plates is

$$E = Q/\varepsilon_0 A.$$

The potential difference:

$$V_{ba} = Ed = Qd/\varepsilon_0 A.$$

This gives the capacitance:

$$C = \frac{Q}{V} = \epsilon_0 \frac{A}{d}.$$

[parallel-plate capacitor]

Example: Capacitor calculations.

(a) Calculate the capacitance of a parallel-plate capacitor whose plates are 20 cm × 3.0 cm and are separated by a 1.0-mm air gap. (b) What is the charge on each plate if a 12-V battery is connected across the two plates? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1 F, given the same air gap d.

Capacitors are now made with capacitances of 1 farad or more, but they are not parallel-plate capacitors. Instead, they are activated carbon, which acts as a capacitor on a very small scale. The capacitance of 0.1 g of activated carbon is about 1 farad.

Some computer keyboards use capacitors; depressing the key changes the capacitance, which is detected in a circuit.

Key

Movable plate

Insulator (flexible)

Capacitor

Fixed plate
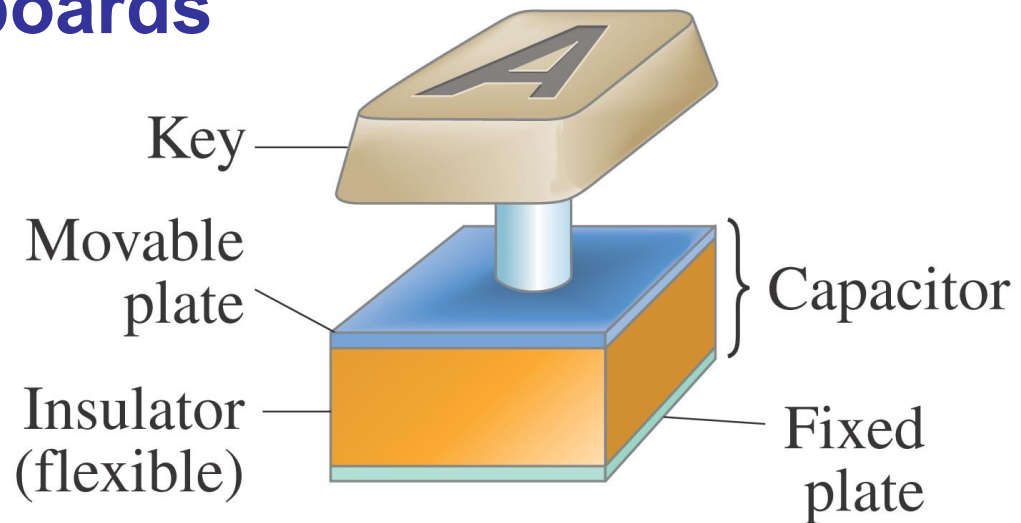
Example: Cylindrical capacitor.

A cylindrical capacitor consists of a cylinder (or wire) of radius R_b surrounded by a coaxial cylindrical shell of inner radius R_a. Both cylinders have length ℓ which we assume is much greater than the separation of the cylinders, so we can neglect end effects. The capacitor is charged (by connecting it to a battery) so that one cylinder has a charge $+Q$ (say, the inner one) and the other one a charge $-Q$. Determine a formula for the capacitance.
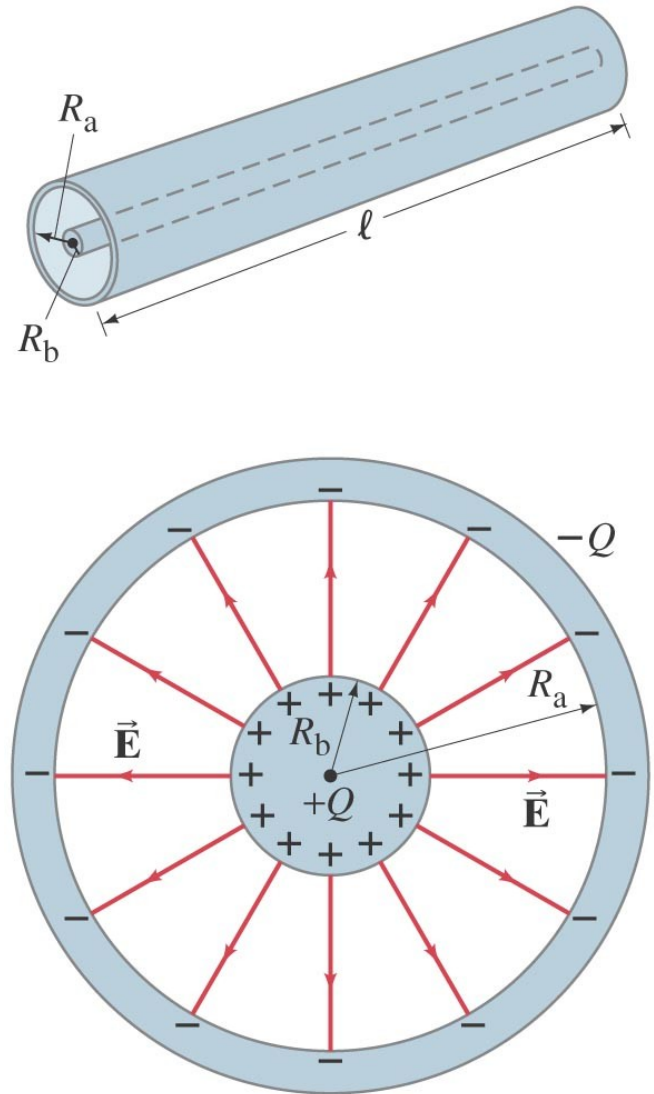
Capacitors in Series and Parallel

Capacitors in parallel have the same voltage across each one. The equivalent capacitor is one that stores the same charge when connected to the same battery:
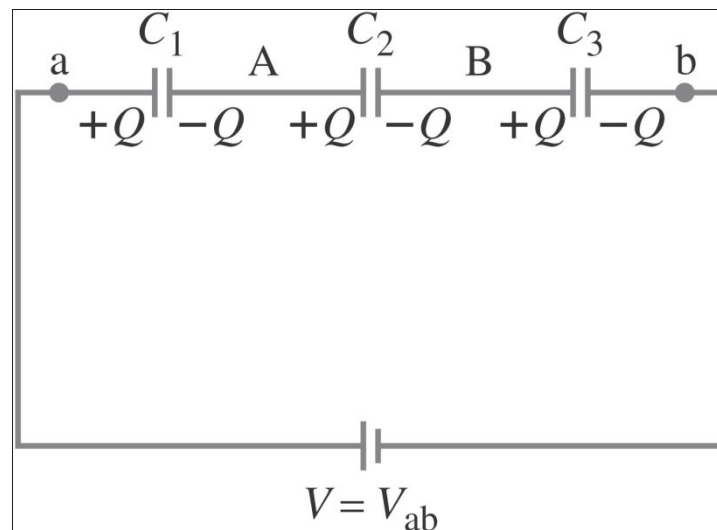

$$C_{eq} = C_1 + C_2 + C_3.$$
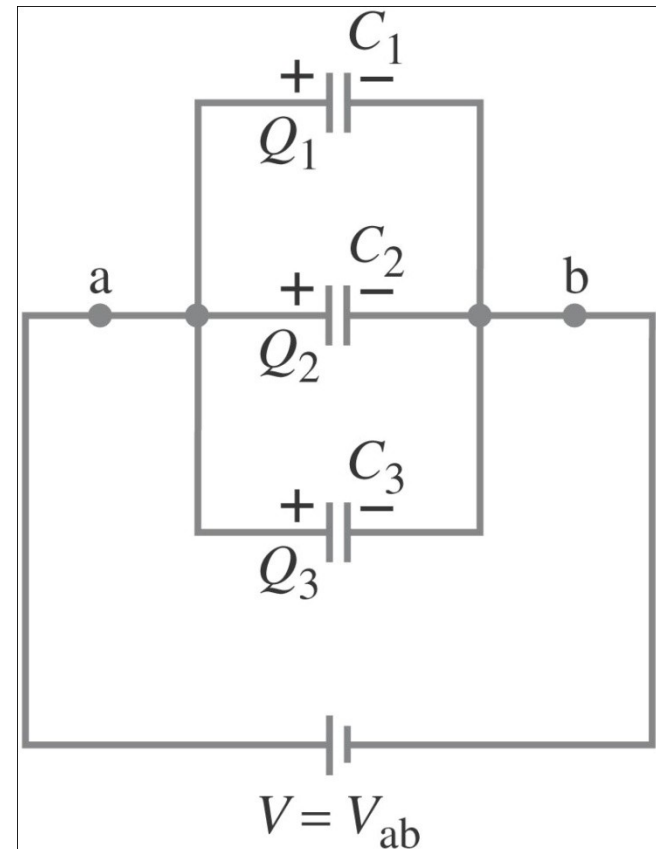
[parallel]

Capacitors in Series and Parallel

Capacitors in series have the same charge. In this case, the equivalent capacitor has the same charge across the total voltage drop. Note that the formula is for the inverse of the capacitance and not the capacitance itself!

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \qquad \text{[series]}$$

$$C_1 \quad A \quad C_2 \quad B \quad C_3 \quad b$$
$$a \quad +Q \quad -Q \quad +Q \quad -Q \quad +Q \quad -Q$$
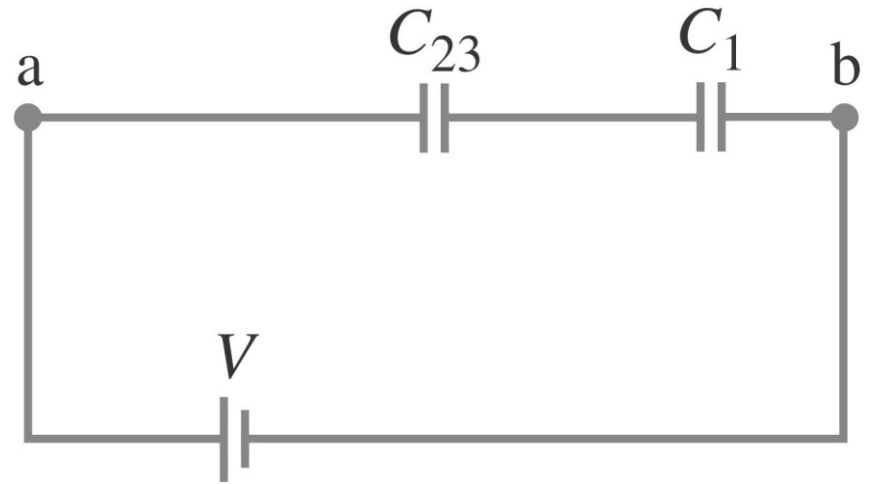$$V = V_{ab}$$

Example: Equivalent capacitance.

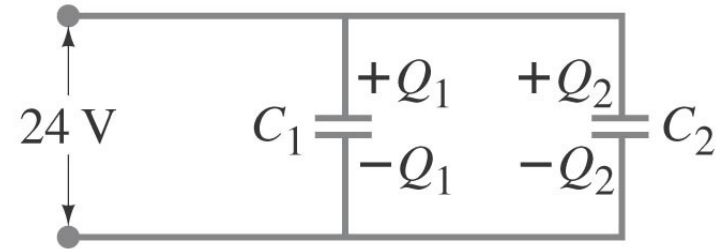**Determine the capacitance of a single capacitor that will have the same effect as the combination shown.**
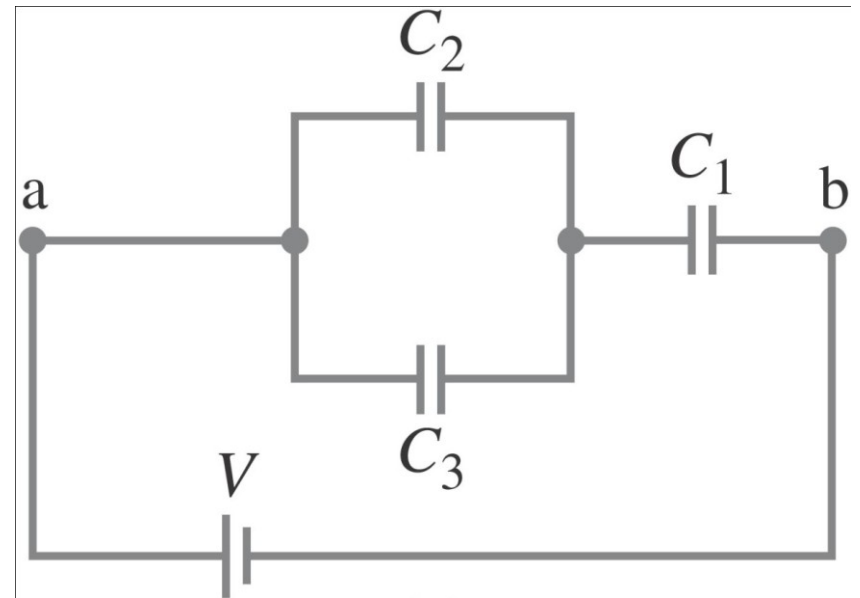
Example: Charge and voltage on capacitors.

Determine the charge on each capacitor and the voltage across each, assuming $C = 3.0$ μF and the battery voltage is $V = 4.0$ V.
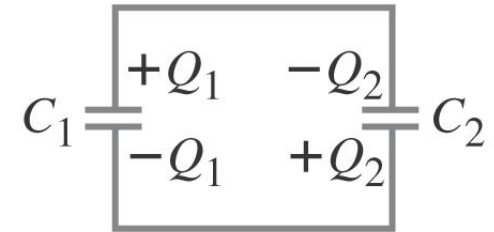
Example: Capacitors reconnected.

Two capacitors, $C_1 = 2.2$ μF and $C_2 = 1.2$ μF, are connected in parallel to a 24-V source as shown. After they are charged they are disconnected from the source and from each other and then reconnected directly to each other, with plates of opposite sign connected together. Find the charge on each capacitor and the potential across each after equilibrium is established.

24 V $C_1 =$ $\begin{array}{c}+Q_1 \\ -Q_1\end{array}$ $\begin{array}{c}+Q_2 \\ -Q_2\end{array}$ $= C_2$

(a) Initial configuration.

$C_1 =$ $\begin{array}{c}+Q_1 \\ -Q_1\end{array}$ $\begin{array}{c}-Q_2 \\ +Q_2\end{array}$ $= C_2$

(b) At the instant of reconnection only.

$\begin{array}{c}q_1 \\ -q_1\end{array}$ $\begin{array}{c}q_2 \\ -q_2\end{array}$

(c) A short time later.

Electric Energy Storage

A charged capacitor stores electric energy; the energy stored is equal to the work done to charge the capacitor:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV.$$

Electric Energy Storage

Example: Energy stored in a capacitor.

A camera flash unit stores energy in a 150-μF capacitor at 200 V. (a) How much electric energy can be stored? (b) What is the power output if nearly all this energy is released in 1.0 ms?

Conceptual Example: Capacitor plate separation increased.

A parallel-plate capacitor carries charge Q and is then disconnected from a battery. The two plates are initially separated by a distance d. Suppose the plates are pulled apart until the separation is $2d$. How has the energy stored in this capacitor changed?

i-clicker question:

A) The stored energy is decreased.

B) The stored energy is increased.

C) The stored energy does not change.

Electric Energy Storage

The energy density, defined as the energy per unit volume, is the same no matter the origin of the electric field:

$$u = \text{energy density} = \tfrac{1}{2}\epsilon_0 E^2.$$

For a parallel-plate capacitor:

$$u = \frac{U}{Ad} = \frac{\dfrac{1}{2}CV^2}{Ad} = \frac{\varepsilon_0 \dfrac{A}{d}(Ed)^2}{2Ad} = \frac{1}{2}\varepsilon_0 E^2$$

The sudden discharge of electric energy can be harmful or fatal. Capacitors can retain their charge indefinitely even when disconnected from a voltage source – be careful!

Dielectrics

A dielectric is an insulator, and is characterized by a dielectric constant K.

Capacitance of a parallel-plate capacitor filled with dielectric:

$$C = K\epsilon_0 \frac{A}{d}. \qquad \text{[parallel-plate capacitor]}$$

Using the dielectric constant, we define the permittivity:
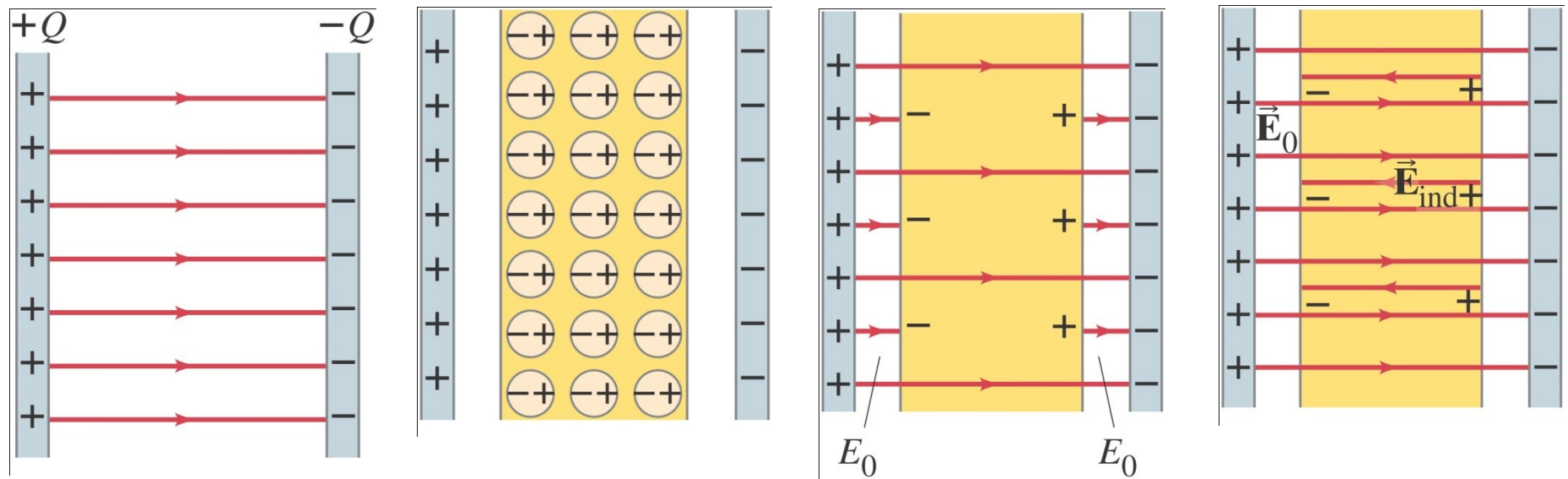
$$\epsilon = K\epsilon_0.$$

Dielectrics

Example: Dielectric removal.

A parallel-plate capacitor, filled with a dielectric with $K = 3.4$, is connected to a 100-V battery. After the capacitor is fully charged, the battery is disconnected. The plates have area A = 4.0 m² and are separated by $d = 4.0$ mm. (a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new values of capacitance, electric field strength, voltage between the plates, and the energy stored in the capacitor.
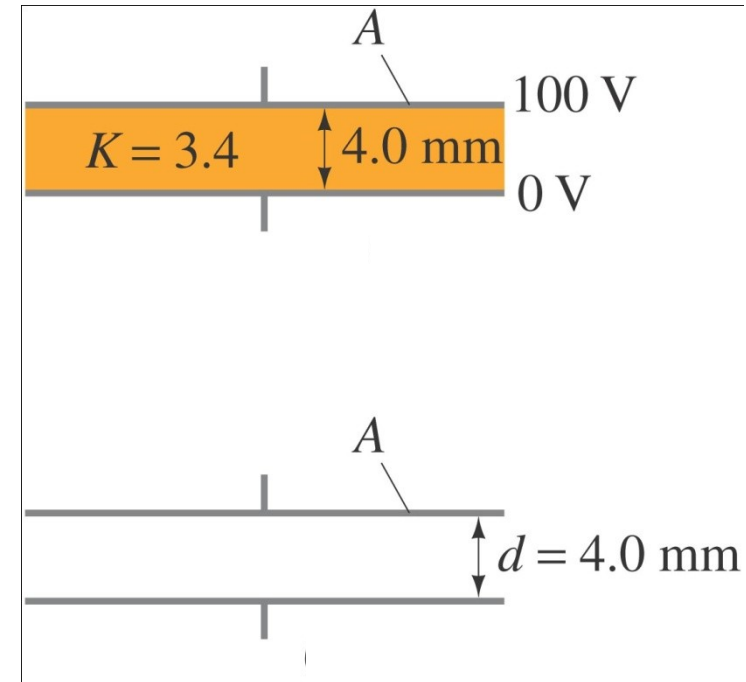
Molecular Description of Dielectrics

The molecules in a dielectric, when in an external electric field, tend to become oriented in a way that reduces the external field.

$$C = \frac{Q}{V} = \frac{Q}{E/d}; \qquad C = \varepsilon \frac{A}{d}$$