Phys102 Lecture 16/17
Ampere's Law

Key Points

- Ampère’s Law
- Magnetic Field Due to a Straight Wire
- Magnetic Field of a Solenoid

References

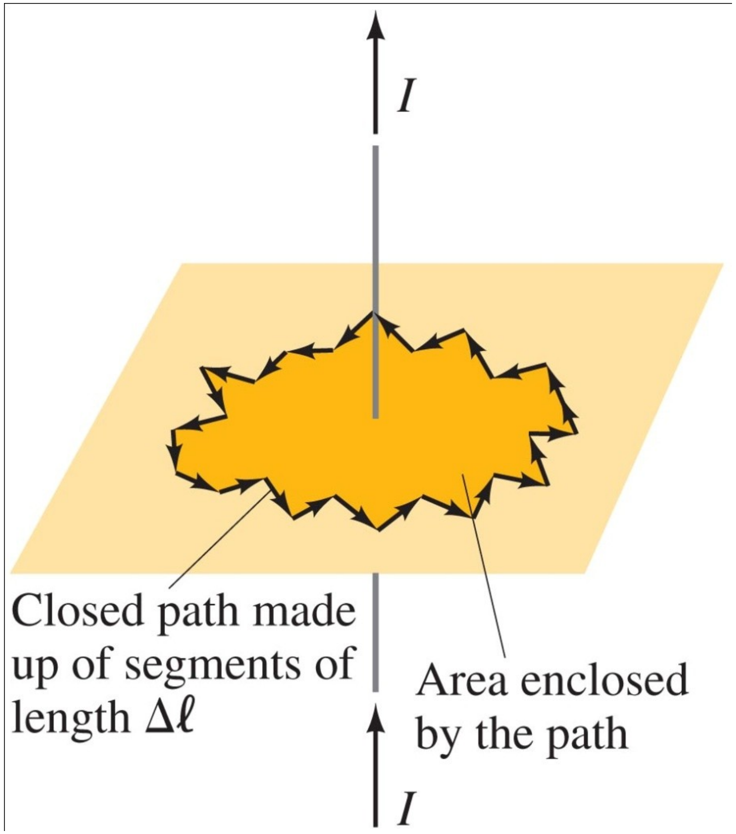
20-5,6,7.

Ampère's Law

Ampère's law relates the magnetic field around a closed loop to the total current flowing through the loop:

$$\sum \vec{B} \cdot \Delta\vec{l} = \mu_0 I_{encl.}$$

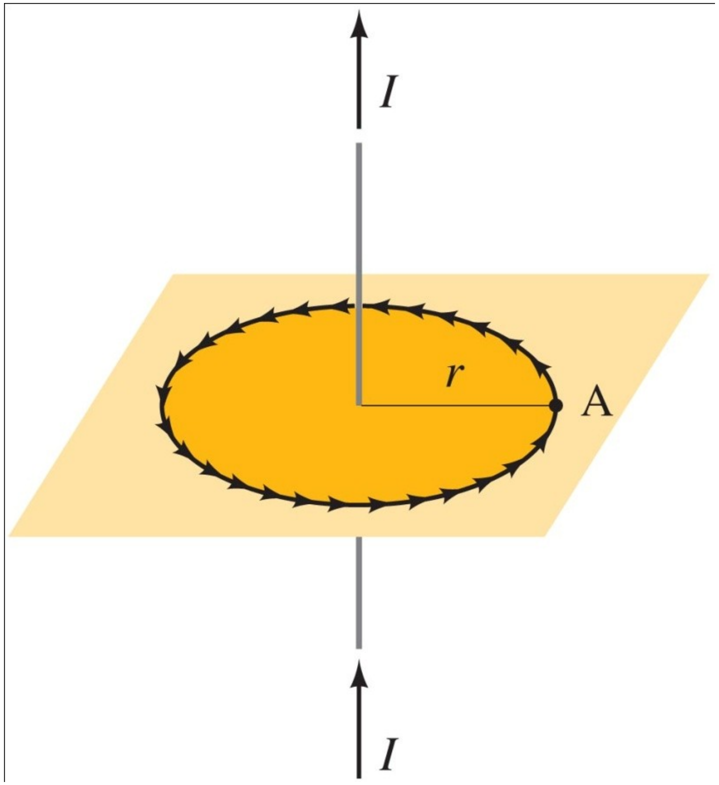
where



$$\vec{B} \cdot \Delta\vec{l} = B\Delta l \cos\theta, \qquad \theta - \text{angle between } \vec{B} \text{ and } \Delta\vec{l}$$

$\mu_0 = 4\pi \times 10^{-7}$ T·m/A is the magnetic permeability of free space.

Using Ampère's law to find the field around a long straight wire:



Use a circular path with the wire at the center; then $\vec{B}$ is tangent to $\Delta\ell$ at every point. Then

$$\sum \vec{B} \cdot \Delta\vec{l} = \mu_0 I_{encl.}$$

$$B \sum \Delta l = \mu_0 I_{encl.}$$

$$B\,2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Similar to Gauss's law, we need symmetry.

i-clicker 1. Magnetic Field

A proton beam enters a magnetic field region as shown below. What is the direction of the magnetic field *B*?

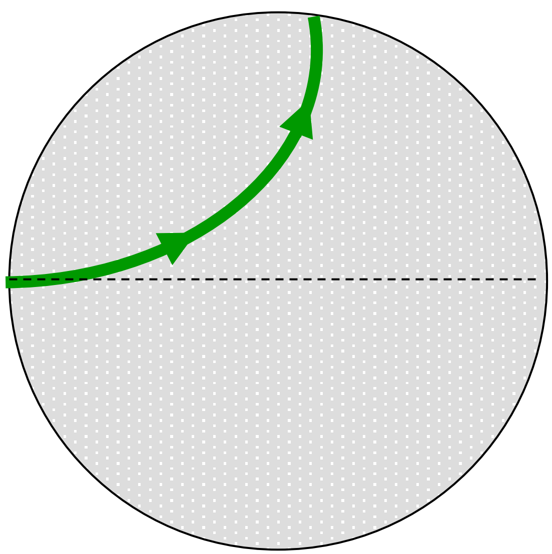
A) + *y*

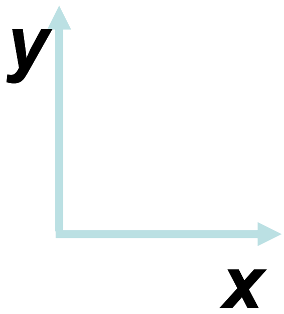
B) – *y*

C) + *x*

D) + *z* (out of page)

E) – *z* (into page)





$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}.$$

i-clicker 2 Magnetic Force on a Loop

If there is a current in the loop in the direction shown, the loop will:

A) move up

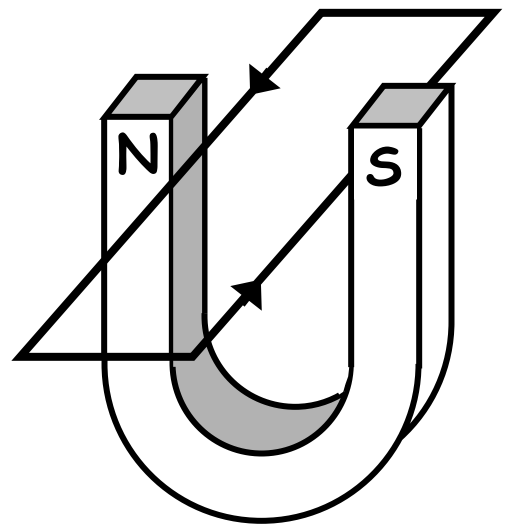
B) move down

C) rotate clockwise

D) rotate counterclockwise

E) both rotate and move

$$\vec{\mathbf{F}} = I\vec{\ell} \times \vec{\mathbf{B}}.$$



B **field out of North**
B **field into South**

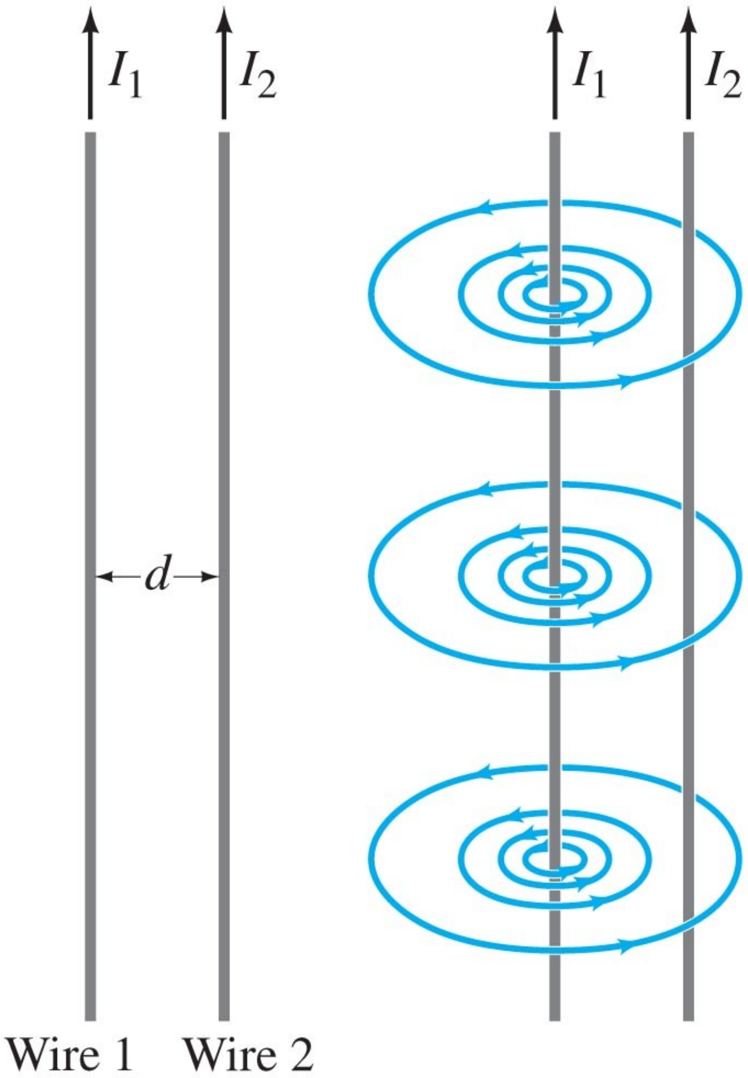
Force between Two Parallel Wires

The magnetic field produced at the position of wire 2 due to the current in wire 1 is

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{d}.$$

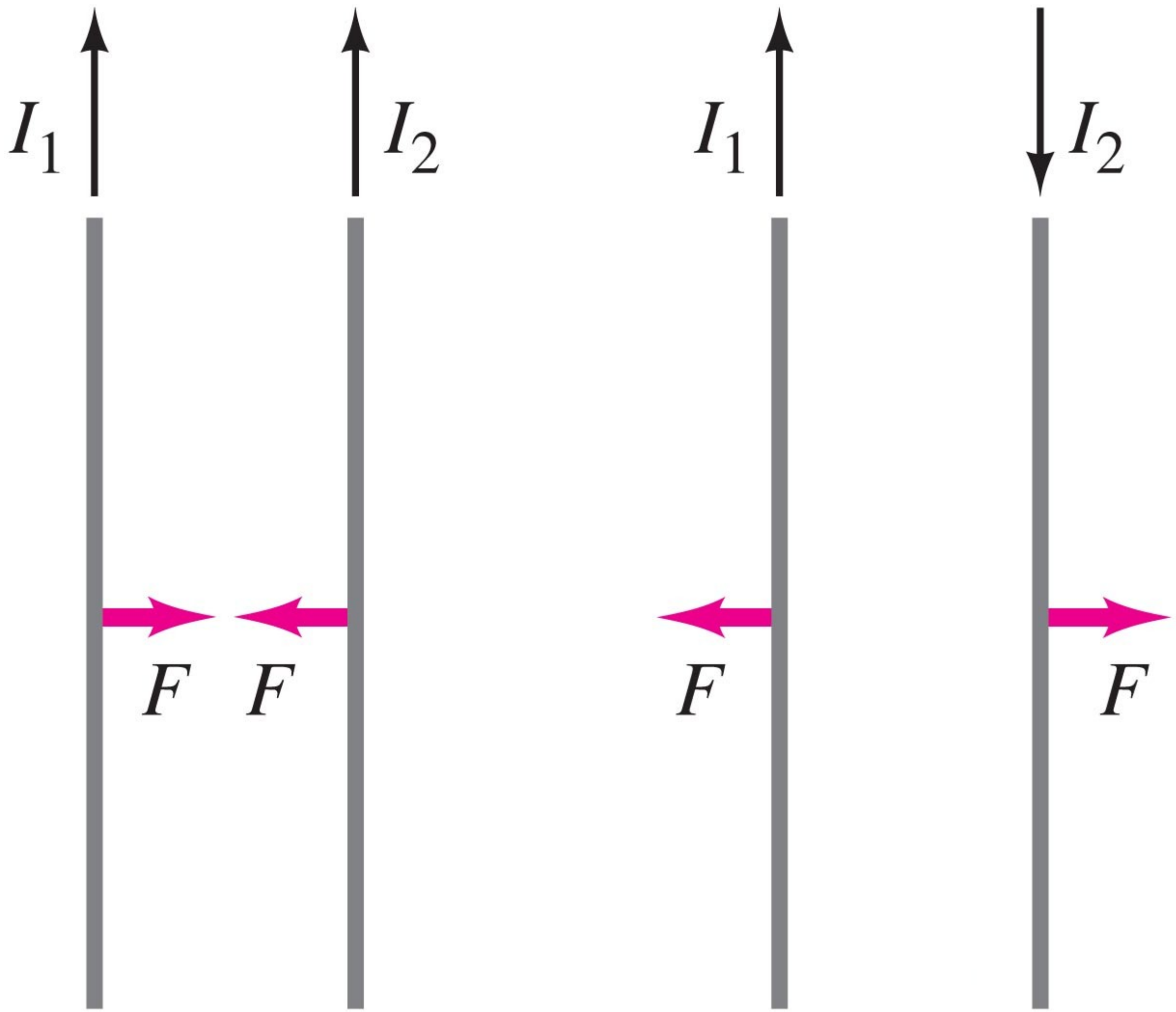
The force this field exerts on a length ℓ_2 of wire 2 is

$$F_2 = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}\ell_2$$



Force between Two Parallel Wires

Parallel currents attract; antiparallel currents repel.



Example. Force between two current-carrying wires.

The two wires of a 2.0-m-long appliance cord are 3.0 mm apart and carry a current of 8.0 A dc. Calculate the force one wire exerts on the other.

Definitions of the Ampere and the Coulomb

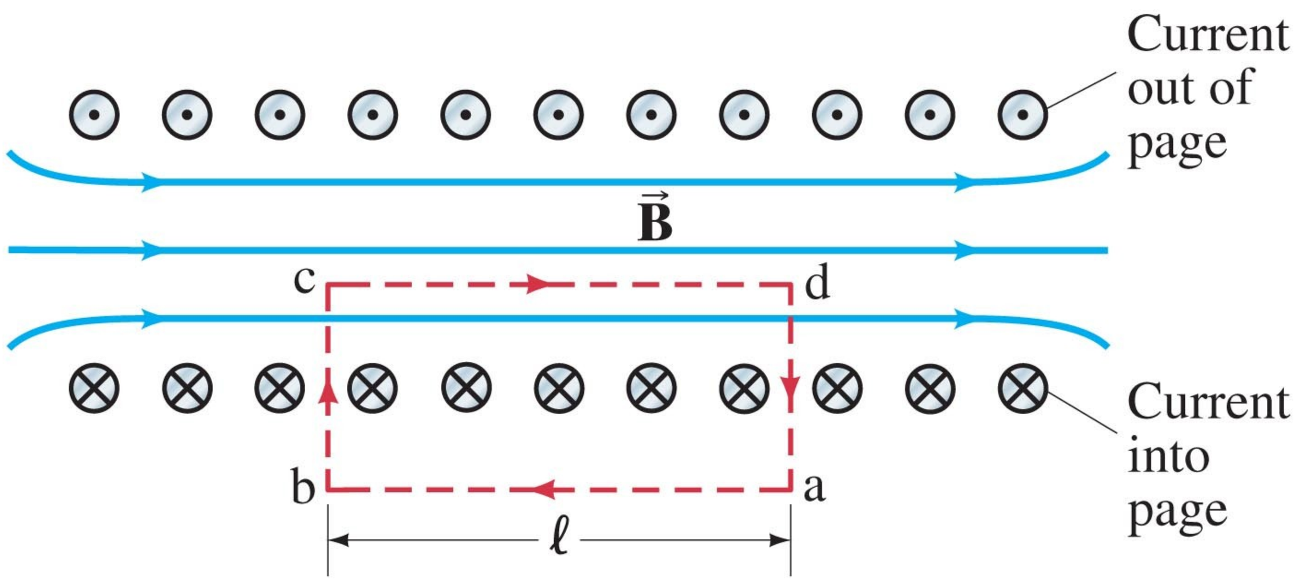
The ampere is officially defined in terms of the force between two current-carrying wires:

One ampere is defined as that current flowing in each of two long parallel wires 1 m apart, which results in a force of exactly 2 x 10^{-7} N per meter of length of each wire.

The coulomb is then defined as exactly one ampere-second.

Magnetic Field of a Solenoid

A solenoid is a coil of wire containing many loops. To find the field inside, we use Ampère's law along the path indicated in the figure.



$$\sum \vec{B} \cdot \Delta\vec{l} = \mu_0 I_{encl.}$$

$$Bl = \mu_0 nIl$$

$$B = \mu_0 nI$$

(n is the number of loops per unit length)

Example: Field inside a solenoid.

A thin 10-cm-long solenoid used for fast electromechanical switching has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the field inside near the center.

Magnetic Fields in Magnetic Materials*

If a ferromagnetic material such as iron is placed in the core of a solenoid, the magnetic field is enhanced by the field created by the ferromagnet itself. This is usually much greater than the field created by the current alone.

The magnetic field can be written as

$$B = \mu n I$$

where μ is the magnetic permeability, ferromagnets have $\mu >> \mu_0$ (e.g. $\mu \approx 2000\ \mu_0$ for electrical steel), while all other materials have $\mu \approx \mu_0$.