Phys102 Lecture 29, 30, 31

Nuclear Physics and Radioactivity

Key Points
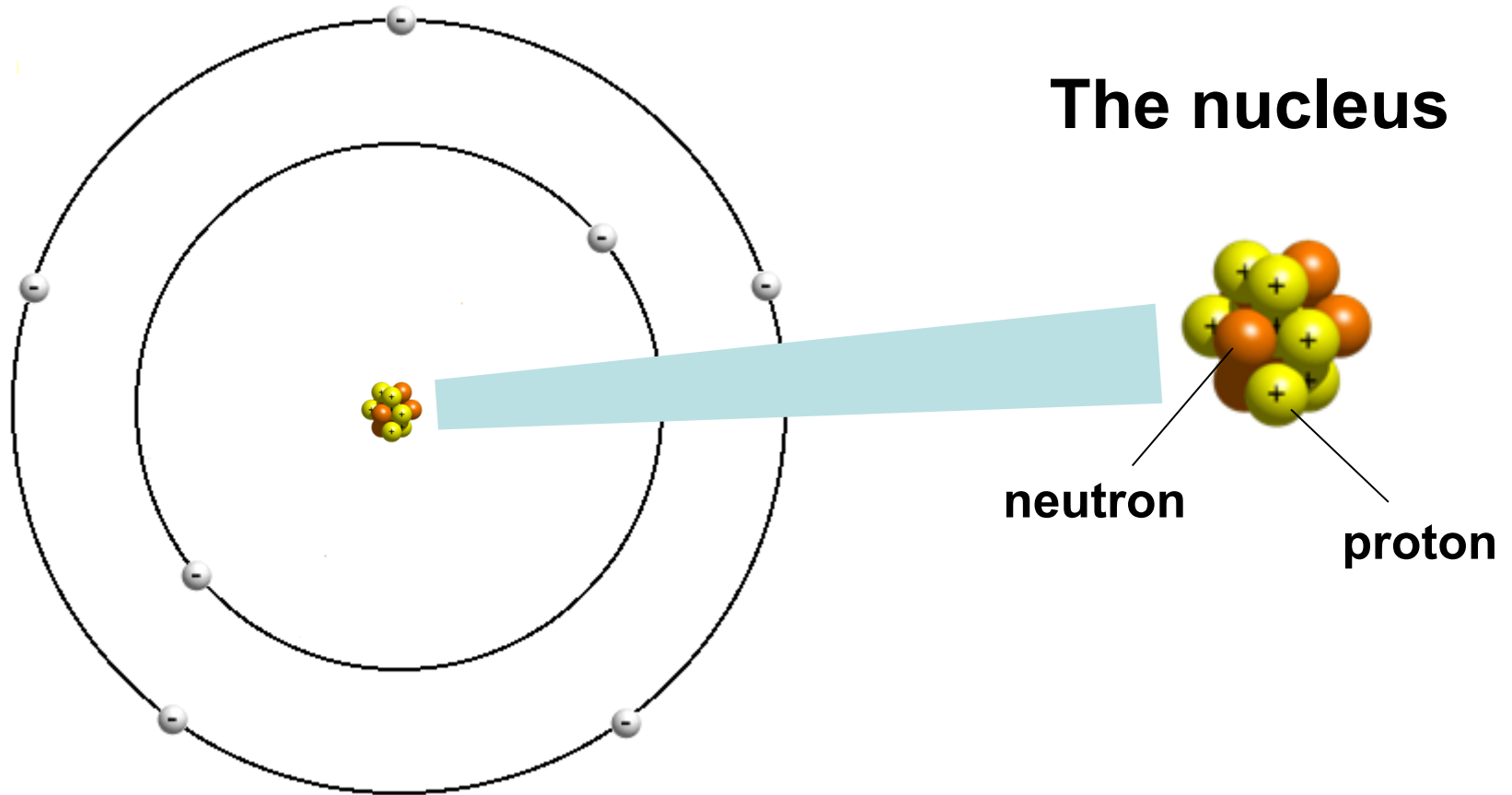
- Structure and Properties of the Nucleus

- Alpha, Beta and Gamma Decays

References

30-1,2,3,4,5,6,7.

Atomic Structure

Nitrogen (N) Atom

The nucleus

neutron

proton

Structure and Properties of the Nucleus

A nucleus is made of protons and neutrons.

A proton is positively charged. Its mass is:

$$m_\mathrm{p} = 1.67262 \times 10^{-27}\,\mathrm{kg}$$

A neutron is electrically neutral:

$$m_\mathrm{n} = 1.67493 \times 10^{-27}\,\mathrm{kg}$$

Structure and Properties of the Nucleus

Neutrons and protons are collectively called nucleons.

The different nuclei are referred to as nuclides.

Number of protons: atomic number, Z

Number of nucleons: atomic mass number, A

Neutron number: $N = A - Z$. Therefore, A and Z are sufficient to specify a nuclide.

Nuclides are symbolized as : $^A_Z X$

X is the chemical symbol for the element; it contains the same information as Z but in a more easily recognizable form.

For example, $^{15}_{7}N$ refers to a specific type of nucleus containing 7 protons and 8 neutrons (Nitrogen -15).

Structure and Properties of the Nucleus

Nuclei with the same Z – so they are the same element – but different N are called isotopes.

For many elements, several different isotopes exist in nature.

Natural abundance is the percentage of a particular element that consists of a particular isotope in nature.

For example, the most stable nitrogen isotopes are:

nuclide symbol	Z(p)	N(n)	mass (u)	half-life	Natural abundance
^{13}N	7	6	13.005739	9.965 min	
^{14}N	7	7	14.003074	Stable	99.632%
^{15}N	7	8	15.000109	Stable	0.368%

Structure and Properties of the Nucleus

Because of wave-particle duality, the size of the nucleus is somewhat fuzzy. Measurements of high-energy electron scattering yield:

$$r \approx \left(1.2 \times 10^{-15}\, \text{m}\right)\left(A^{\frac{1}{3}}\right) \qquad \textbf{(30-1)}$$

Example 30-1:   Nuclear sizes. Estimate the diameter of   (a) $^{1}_{1}\text{H}$, (b) $^{40}_{20}\text{Ca}$, (c) $^{208}_{82}\text{Pb}$, (d) $^{235}_{92}\text{U}$.

[Solution] $$r \approx (1.2 \times 10^{-15}\,\text{m})\left(A^{\frac{1}{3}}\right)$$

(a) For hydrogen, $A = 1$, $A^{\frac{1}{3}} = 1$, $d = 2r = 2.4 \times 10^{-15}\,m = 2.4\ fm$

(b) For calcium, $d = 2r = 2.4 \times 10^{-15} \times (40)^{\frac{1}{3}} = 8.2 \times 10^{-15}\,m$

(c) For lead, $d = 2r = 2.4 \times 10^{-15} \times (208)^{\frac{1}{3}} = 14 \times 10^{-15}\,m$

(d) For uranium, $d = 2r = 2.4 \times 10^{-15} \times (235)^{\frac{1}{3}} = 15 \times 10^{-15}\,m = 15\ fm$

Size of an atom ~ 10^{-10} m

Structure and Properties of the Nucleus

Masses of atoms are measured with reference to the carbon-12 atom, which is assigned a mass of exactly 12u. A u is a *unified atomic mass unit*.

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2$$

Mass-energy relation: $E = mc^2$

Structure and Properties of the Nucleus

From the following table, you can see that the electron is considerably less massive than a nucleon.

TABLE 30–1
Rest Masses in Kilograms, Unified Atomic Mass Units, and MeV/c^2

Object	Mass		
	kg	u	MeV/c^2
Electron	9.1094×10^{-31}	0.00054858	0.51100
Proton	1.67262×10^{-27}	1.007276	938.27
$^1_1$H atom	1.67353×10^{-27}	1.007825	938.78
Neutron	1.67493×10^{-27}	1.008665	939.57

Binding Energy and Nuclear Forces

The total mass of a stable nucleus is always less than the sum of the masses of its separate protons and neutrons.

Where has the mass gone?

Binding Energy and Nuclear Forces

It has become energy, such as radiation or kinetic energy, released during the formation of the nucleus. Again, E = mc^2 .

This difference between the total mass of the constituents and the mass of the nucleus is called the total binding energy of the nucleus. (The binding energy is the energy needed to break a nucleus into individual nucleons.)

$$E_{Binding} = \left[\left(total \ mass \ of \ nucleons \right) - \left(mass \ of \ nucleus \right) \right] c^2$$

i-clicker   Binding Energy I

What weighs more, an
electron and a proton,
or a hydrogen atom?

1) electron and proton

2) hydrogen atom

3) both the same

i-clicker   Binding Energy I

What weighs more, an electron and a proton, or a hydrogen atom?

1) electron and proton

2) hydrogen atom

3) both the same

The total energy (or mass) of a hydrogen atom must be *less* than the energies (or masses) of the electron plus the proton individually in order for the electron to be bound.

Example 30-3: $^{4}_{2}He$ mass compared to its constituents.

Compare the mass of a $^{4}_{2}He$ atom to the total mass of its constituent particles.

Appendix B gives the mass of neutral atoms, including the electrons outside the nucleus:

$\text{Mass of a neutral } ^{4}_{2}He \text{ atom} = 4.002603\, u$

$$\text{Total mass of constituent particles} = 2m_n + 2m_p + 2m_e$$
$$= 2m_{n1} + 2\left(mass\ of\ ^{1}_{1}H\right)$$
$$= 2(1.008665) + 2(1.007825)$$
$$= 4.032980\, u$$

$\text{Mass difference} = 4.032980 - 4.002603 = 0.030377\, u$

$\text{Binding energy} = (\Delta m)c^2 = 0.030377u \cdot 931.5 \dfrac{MeV}{u\,c^2} c^2 = 28.30 MeV$

$\text{Binding energy per nucleon} = \dfrac{28.30}{4} = 7.075 MeV$

Binding Energy and Nuclear Forces

To compare how tightly bound different nuclei are, we divide the binding energy by A to get the binding energy per nucleon.

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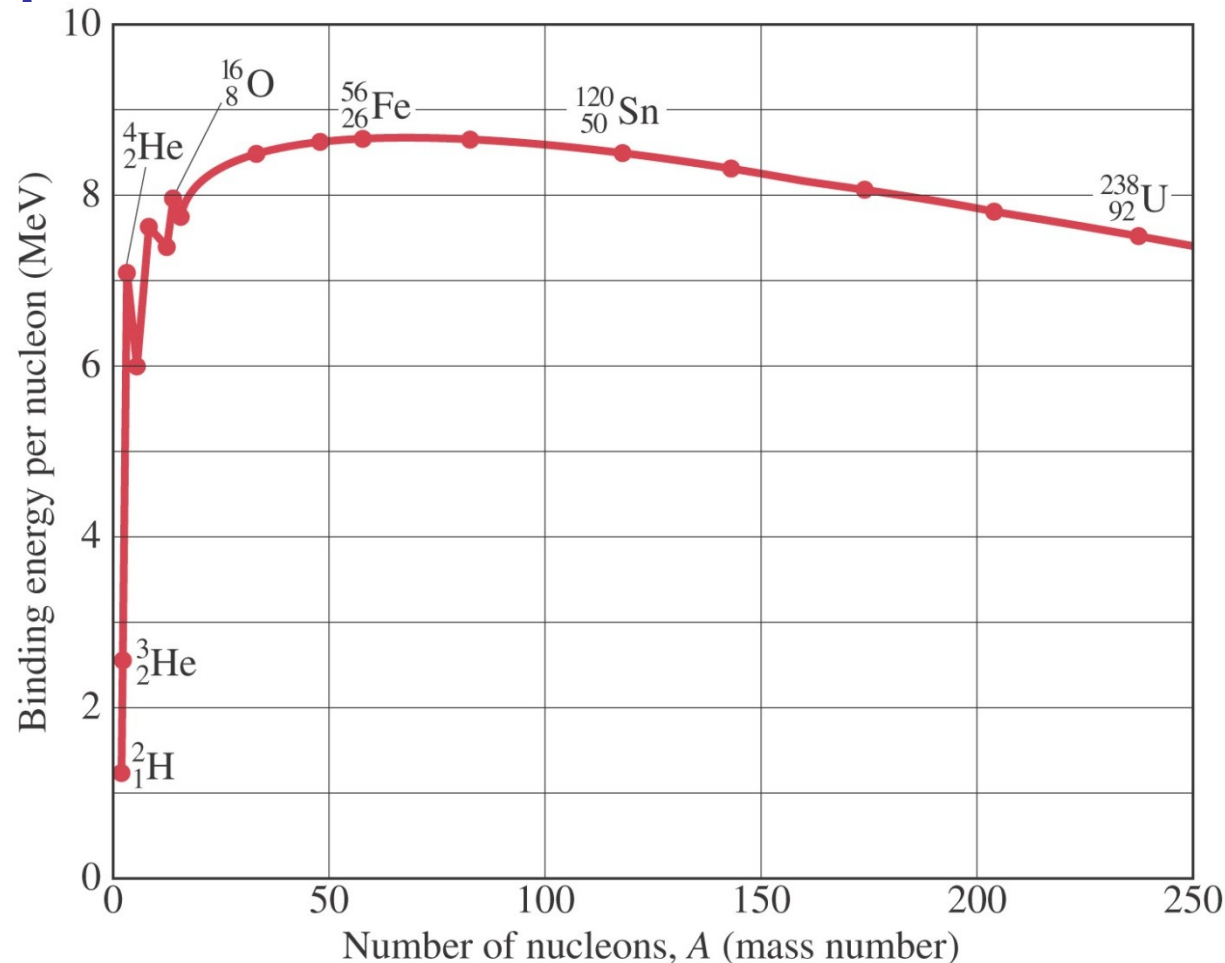
Binding Energy and Nuclear Forces

Example 30-4: Binding energy for iron.

Calculate the total binding energy and the binding energy per nucleon for $^{56}_{26}\text{Fe}$, the most common stable isotope of iron.

$$\text{Mass of a } ^{56}_{26}\text{F}_\text{e} \text{ atom} = 55.93494 \, u$$

$$\text{Mass of 30 neutrons} = 30.25995 \, u$$

$$\text{Mass of 26 } ^{1}_{1}H \text{ atom} = 26.20345 \, u$$

$$\Delta m = 26.20345 + 30.25995 - 55.93494 = 0.52846 \, u$$

$$\text{Total binding energy} : 0.52846 \, u \times 931.5 MeV / u = 492.26 MeV$$

$$\text{Binding energy/nucleon} : 492.26 MeV / 56 = 8.79 MeV$$

Example 30-5: Binding energy of last neutron.

What is the binding energy of the last neutron in $^{13}_{6}C$?

Mass of a $^{12}_{6}C$ atom $= 12.000000\, u$

Mass of a free neutron $= 1.008665\, u$

Mass of a $^{13}_{6}C$ atom $= 13.003355\, u$

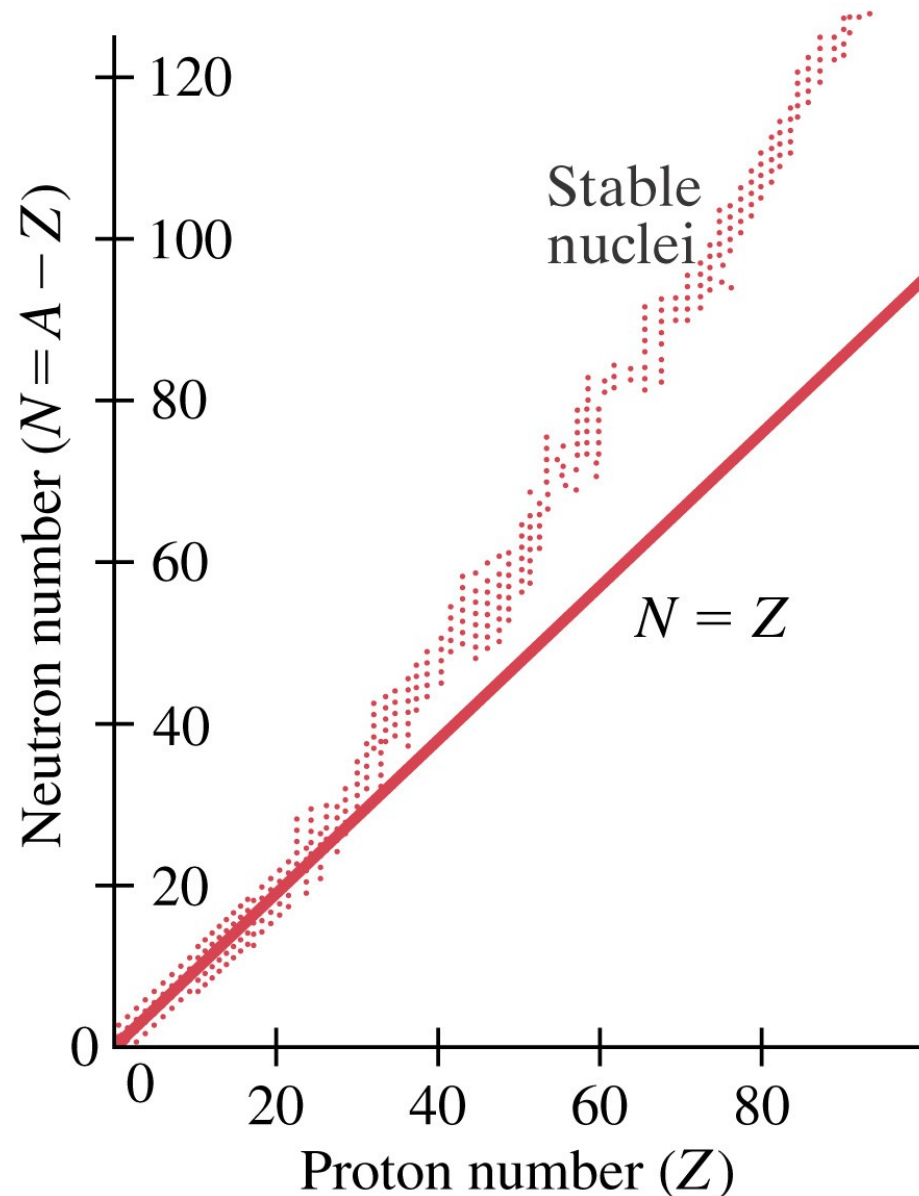$\Delta m = 12.000000 + 1.008665 - 13.003355 = 0.005310\, u$

The binding energy of the last neutron (the energy needed to remove a neutron from $^{13}_{6}C$):

$$E = \left(\Delta m\right)c^{2} = 0.005310u \cdot \frac{931.5 MeV}{uc^{2}}\, c^{2} = 4.95 MeV$$

Binding Energy and Nuclear Forces

The higher the binding energy per nucleon, the more stable the nucleus.

More massive nuclei require extra neutrons to overcome the Coulomb repulsion of the protons in order to be stable.

Stable nuclei

$N = Z$

Neutron number ($N = A - Z$)

Proton number (Z)

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Binding Energy and Nuclear Forces

The force that binds the nucleons together is called the <u>strong nuclear force</u>. It is a very strong, but short-range, force. It is essentially zero if the nucleons are more than about 10^{-15} m apart. The Coulomb force is long-range; this is why extra neutrons are needed for stability in high-Z nuclei.

Binding Energy and Nuclear Forces

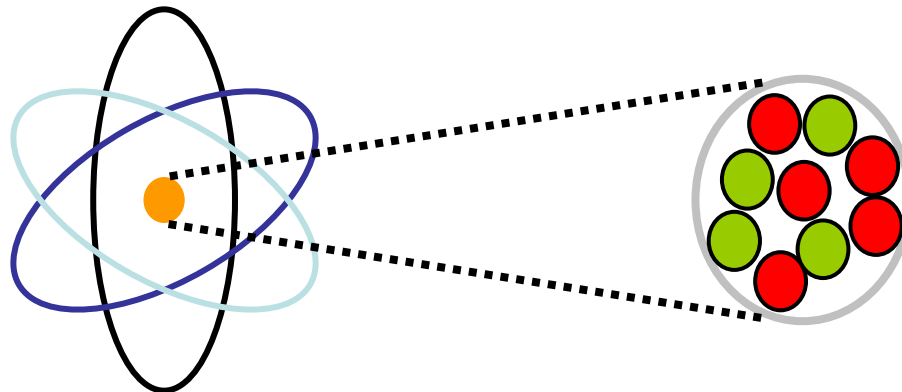Nuclei that are unstable decay; many such decays are governed by another force called the weak nuclear force.

i-clicker   The Nucleus

There are 82 protons in a lead nucleus.

Why doesn't the lead nucleus burst apart?

1) **Coulomb repulsive force doesn't act inside the nucleus**
2) **gravity overpowers the Coulomb repulsive force inside the nucleus**
3) **the negatively charged neutrons balance the positively charged protons**
4) **protons lose their positive charge inside the nucleus**
5) **none of the above**

The Coulomb repulsive force is overcome by the even stronger *nuclear force*!
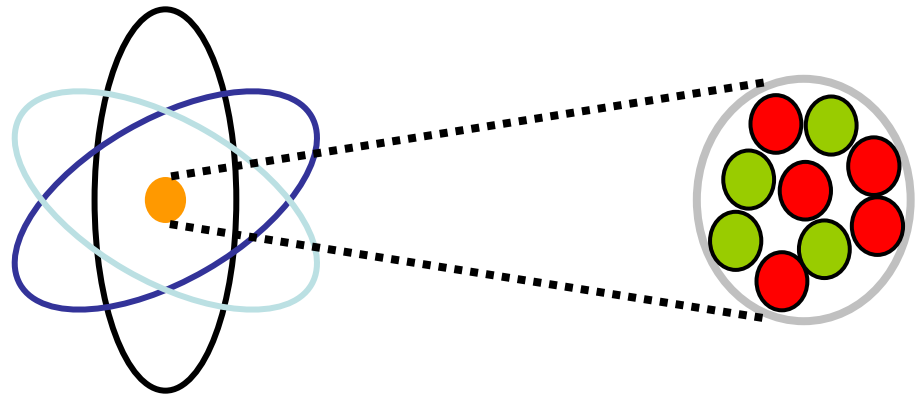
Radioactivity

Towards the end of the 19th century, minerals were found that would darken a photographic plate even in the absence of light.

This phenomenon is now called radioactivity.

Marie and Pierre Curie isolated two new elements that were highly radioactive; they are now called polonium and radium.

Radioactivity

Radioactive rays were observed to be of three types:

1. Alpha rays, which could barely penetrate a piece of paper

2. Beta rays, which could penetrate 3 mm of aluminum

3. Gamma rays, which could penetrate several centimeters of lead

We now know that alpha rays are helium nuclei, beta rays are electrons, and gamma rays are electromagnetic radiation.
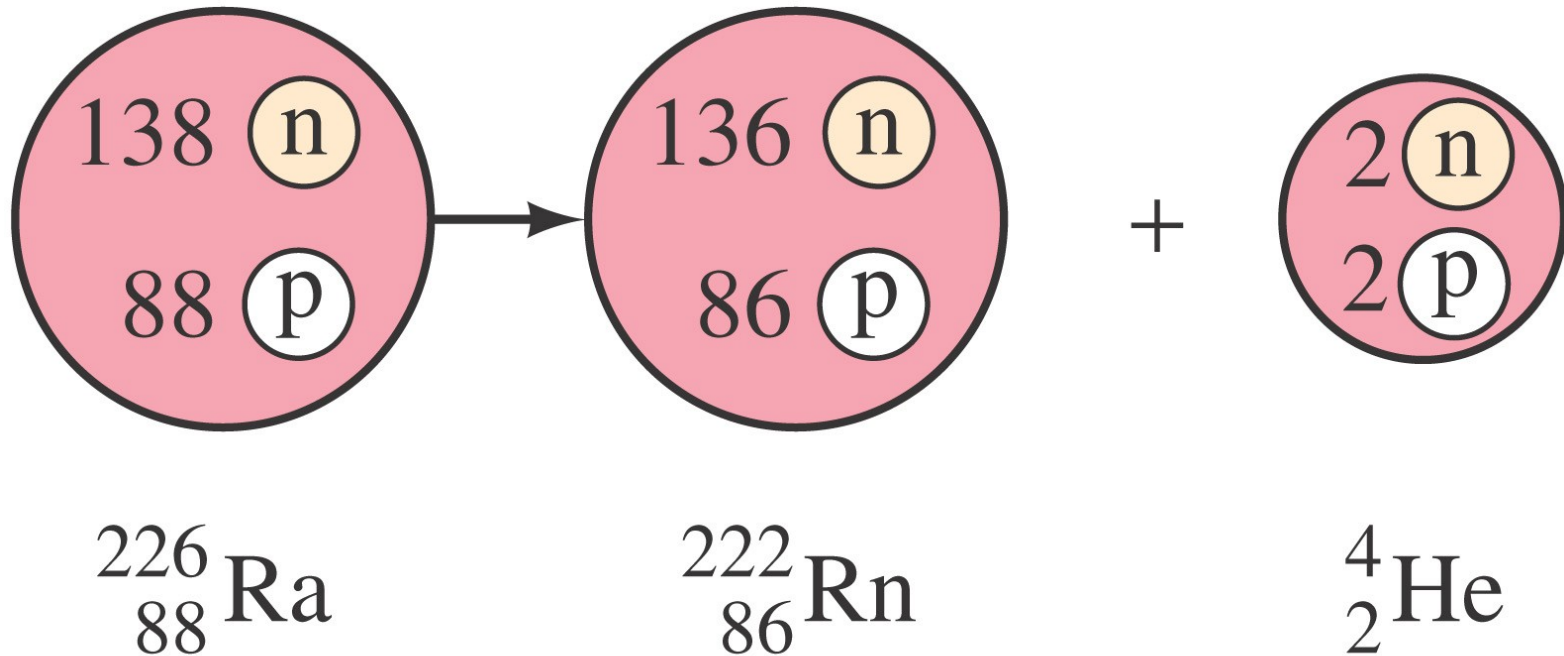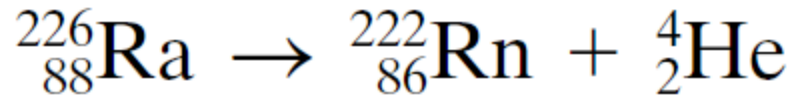
Radioactivity

Alpha and beta rays are bent in opposite directions in a magnetic field, while gamma rays are not bent at all.
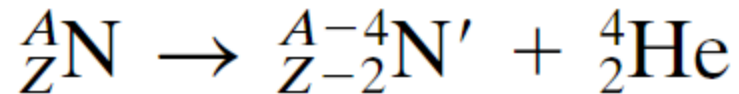

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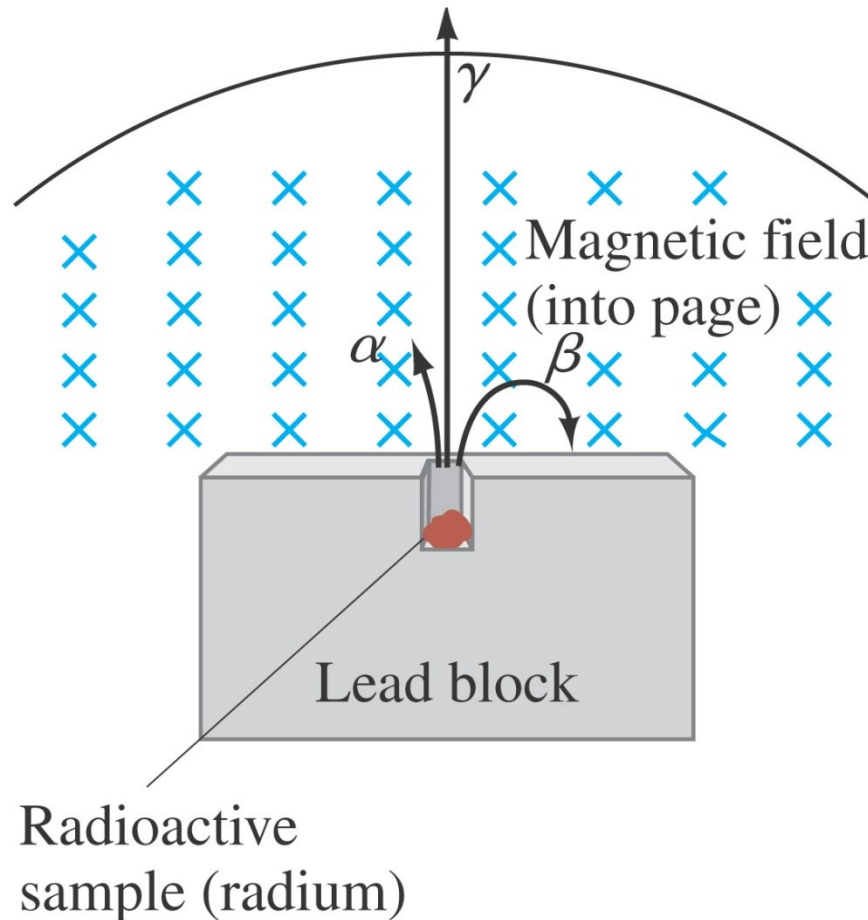
Alpha Decay

Example of alpha decay:

Radium-226 will alpha-decay to radon-222

$$^{226}_{88}\text{Ra} \rightarrow \ ^{222}_{86}\text{Rn} + \ ^{4}_{2}\text{He}$$

$^{226}_{88}\text{Ra}$ $^{222}_{86}\text{Rn}$ $^{4}_{2}\text{He}$

Alpha Decay

In general, alpha decay can be written:

$$^A_Z N \rightarrow ^{A-4}_{Z-2} N' + ^4_2 He$$

Alpha decay occurs when the strong nuclear force cannot hold a large nucleus together. The mass of the parent nucleus is greater than the sum of the masses of the daughter nucleus and the alpha particle; this difference is called the disintegration energy.

Why Alpha Decay?

Alpha decay is so much more likely than other forms of nuclear disintegration because the alpha particle itself is quite stable.

Alpha Decay

Example : Uranium decay energy release.

Calculate the disintegration energy when $^{232}_{92}$U (mass = 232.037156 u) decays to $^{228}_{90}$Th (228.028741 u) with the emission of an α particle. (As always, masses given are for neutral atoms.)

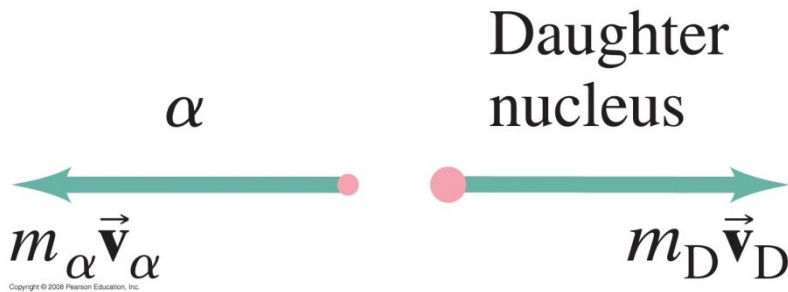$$\text{Mass of a } ^4_2\text{H}_e \text{ atom} = 4.002603\,u$$

$$\Delta m = 232.037156 - (228.028741 + 4.002603) = 0.005812\,u$$

$$Q = 0.005812\,u \times 931.5 MeV\,/\,u \approx 5.414 MeV \approx 5.4 MeV$$

Alpha Decay

Example: Kinetic energy of the α in $^{232}_{92}U$ decay.

For the above decay, how much of the 5.4-MeV disintegration energy will be carried off by the α particle?

α Daughter nucleus

$m_\alpha \vec{\mathbf{v}}_\alpha$ $m_D \vec{\mathbf{v}}_D$

$m_\alpha v_\alpha = m_D v_D$

$$\frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_D v_D^2 = Q$$
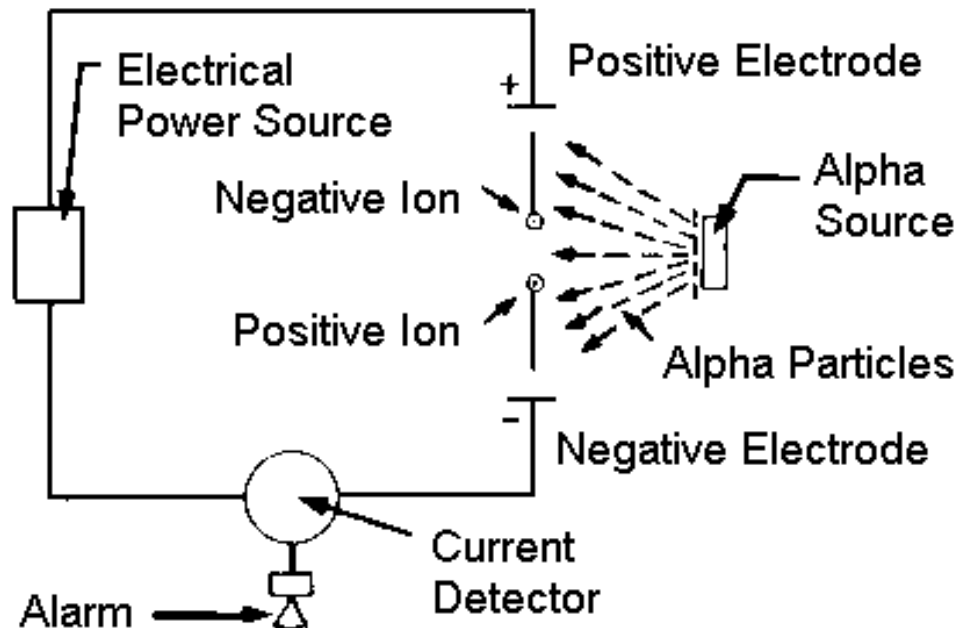
$$v_D = \frac{m_\alpha}{m_D} v_\alpha$$

$$\frac{1}{2} m_\alpha v_\alpha^2 + \frac{m_\alpha^2}{2 m_D} v_\alpha^2 = Q$$

$$v_\alpha^2 = \frac{2 Q m_D}{m_\alpha (m_D + m_\alpha)}$$

$$KE_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 = \frac{Q m_D}{(m_D + m_\alpha)}$$

$$= \frac{5.4 \times 228.028731}{228.028731 + 4.002603}$$

$$= 5.3 MeV$$

Alpha Decay

One type of smoke detector uses alpha radiation from an isotope of americium – the presence of smoke is enough to absorb the alpha rays and keep them from striking the collector plate. The interruption in the flow of alpha rays triggers the alarm.

Beta Decay

Beta decay occurs when a nucleus emits an electron. An example is the decay of carbon-14:

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + e^{-} + \text{a neutrino}$$
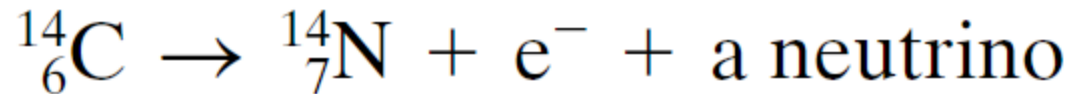
The nucleus still has 14 nucleons, but it has one more proton and one fewer neutron.

This decay is an example of an interaction that proceeds via the weak nuclear force.

Beta Decay

The electron in beta decay is not an orbital electron; it is created in the decay.

The fundamental process is a neutron decaying to a proton, electron, and neutrino:

$$n \rightarrow p + e^- + a \text{ neutrino}.$$
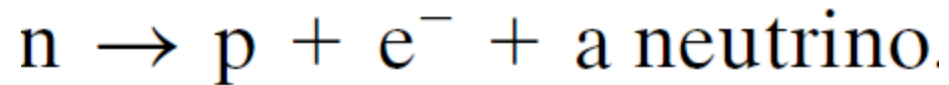
The need for a particle such as the neutrino was discovered through analysis of energy and momentum conservation in beta decay – it could not be a two-particle decay.

Beta Decay

Neutrinos are notoriously difficult to detect, as they interact only weakly, and direct evidence for their existence was not available until more than 20 years had passed.

The symbol for the neutrino is the Greek letter nu (ν); using this, we write the beta decay of carbon-14 as:

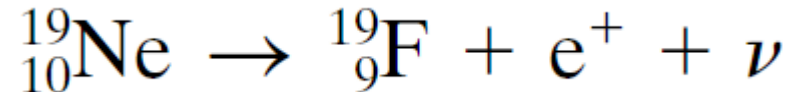$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^{-} + \bar{\nu}$$

Beta Decay

Example : Energy release in $^{14}_{6}\text{C}$ decay.

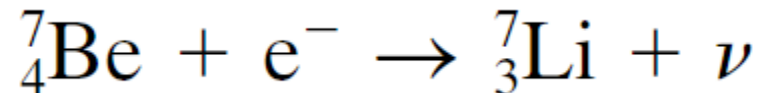**How much energy is released when $^{14}_{6}\text{C}$ decays to $^{14}_{7}\text{N}$ by β emission?**

$$^{14}_{6}\text{C} \rightarrow ^{14}_{7}\text{N} + e^{-} + \bar{\nu}$$

Beta Decay

Beta decay can also occur where the nucleus emits a positron rather than an electron (β⁺ decay):

$$_{10}^{19}\text{Ne} \rightarrow {}_{9}^{19}\text{F} + e^+ + \nu$$

And a nucleus can capture one of its inner electrons (electron capture):

$$_{4}^{7}\text{Be} + e^- \rightarrow {}_{3}^{7}\text{Li} + \nu$$

Beta Decay

In general, we can write beta decay in the following form:

$$^A_Z N \rightarrow\ ^A_{Z+1} N' + e^- + \bar{\nu} \quad [\beta^- \text{ decay}],$$

$$^A_Z N \rightarrow\ ^A_{Z-1} N' + e^+ + \nu \quad [\beta^+ \text{ decay}].$$
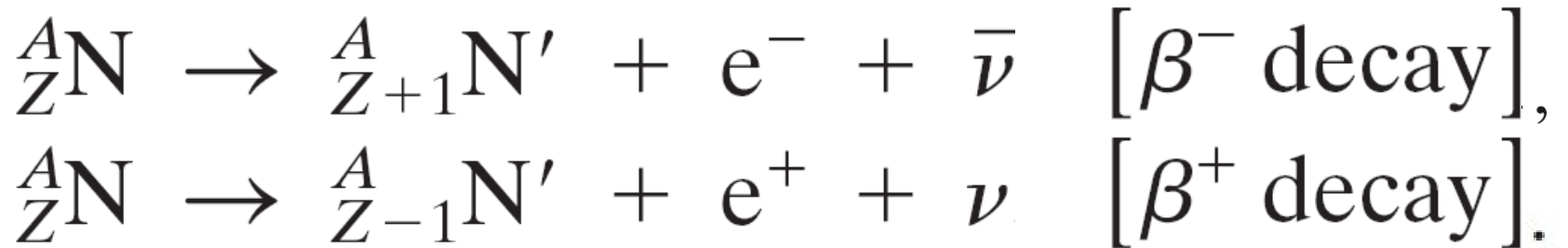
And similarly for electron capture:

$$^A_Z N + e^- \rightarrow\ ^A_{Z-1} N' + \nu.$$
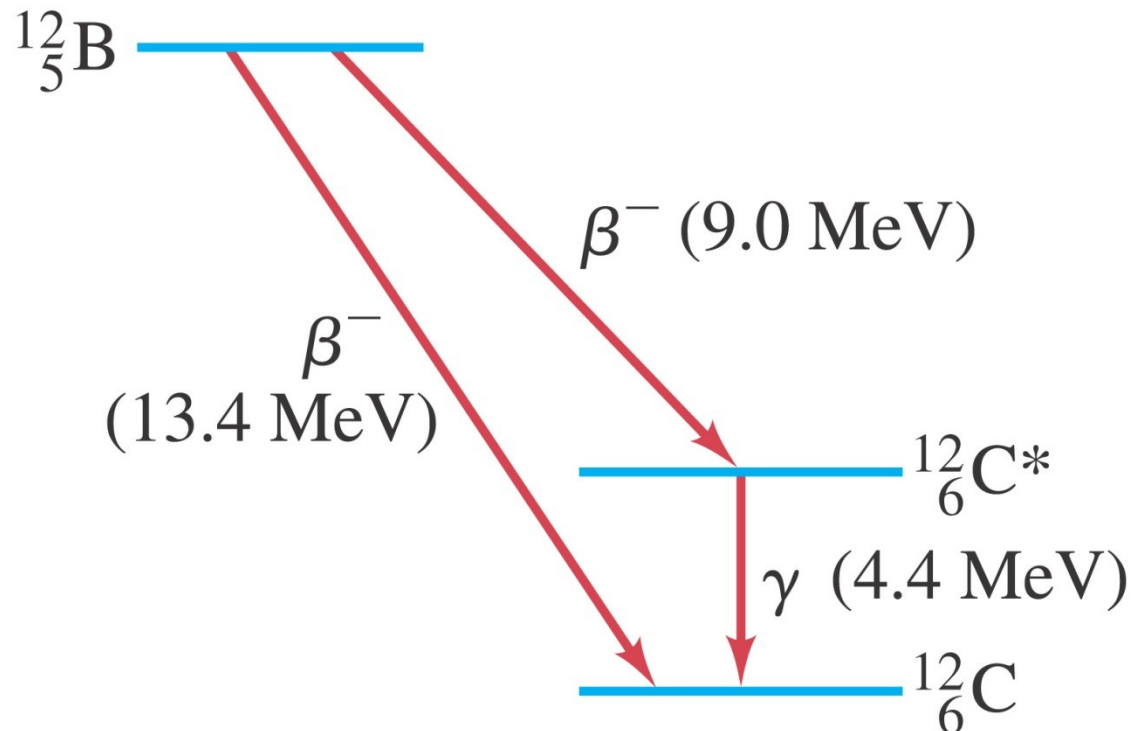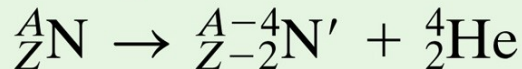
Gamma Decay

Gamma rays are very high-energy photons. They are emitted when a nucleus decays from an excited state to a lower state, just as photons are emitted by electrons returning to a lower state.
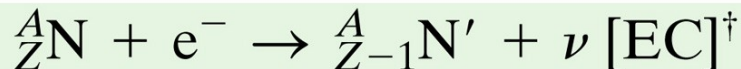

$^{12}_{5}\text{B}$

β^- (9.0 MeV)

β^- (13.4 MeV)

$^{12}_{6}\text{C*}$

γ (4.4 MeV)

$^{12}_{6}\text{C}$

Conservation of Nucleon Number and Other Conservation Laws

TABLE 30–2 The Three Types of Radioactive Decay

α *decay:*

$$_Z^A\text{N} \rightarrow \,_{Z-2}^{A-4}\text{N}' + \,_2^4\text{He}$$

β *decay:*

$$_Z^A\text{N} \rightarrow \,_{Z+1}^{A}\text{N}' + e^- + \bar{\nu}$$

$$_Z^A\text{N} \rightarrow \,_{Z-1}^{A}\text{N}' + e^+ + \nu$$

$$_Z^A\text{N} + e^- \rightarrow \,_{Z-1}^{A}\text{N}' + \nu \,[\text{EC}]^\dagger$$
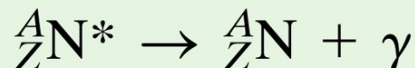
γ *decay:*

$$_Z^A\text{N}* \rightarrow \,_Z^A\text{N} + \gamma$$

† Electron capture.

$*$ Indicates the excited state of a nucleus.

A new law that is evident by studying radioactive decay is that the total number of nucleons cannot change.