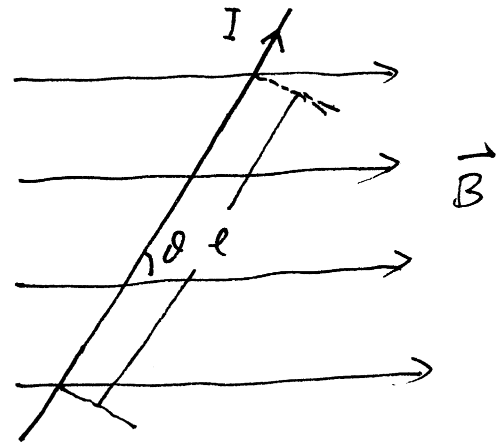
e.g. Magnetic force on a current-carrying wire.

$I = 30\ A$.

$l = 12\ cm = 0.12\ m$.

$\theta = 60°$.

$B = 0.9\ T$.



$\vec{F} = I\vec{l} \times \vec{B}$

Direction : RHR: into the page.

Magnitude : $F = IlB\sin\theta$.

$= 30 \times 0.12 \times 0.9 \times \sin 60°$

$= 2.8\ N$

Cross product of 2 vectors. (vector product)

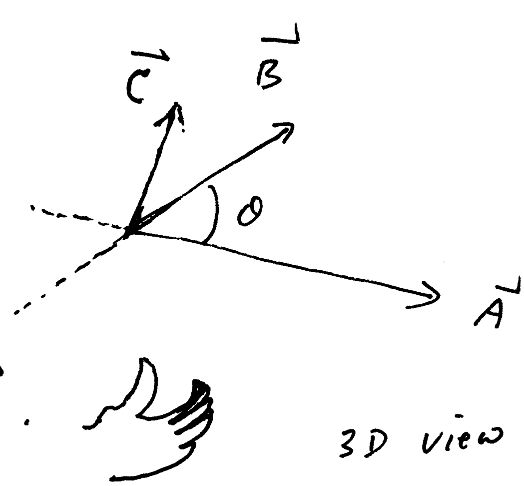
$\vec{C} = \vec{A} \times \vec{B}$.

Result: $\vec{C}$ — It's a vector.

Direction: Right-hand-Rule

Magnitude: $AB\sin\theta$

θ – angle between $\vec{A}$ and $\vec{B}$.



3D view.

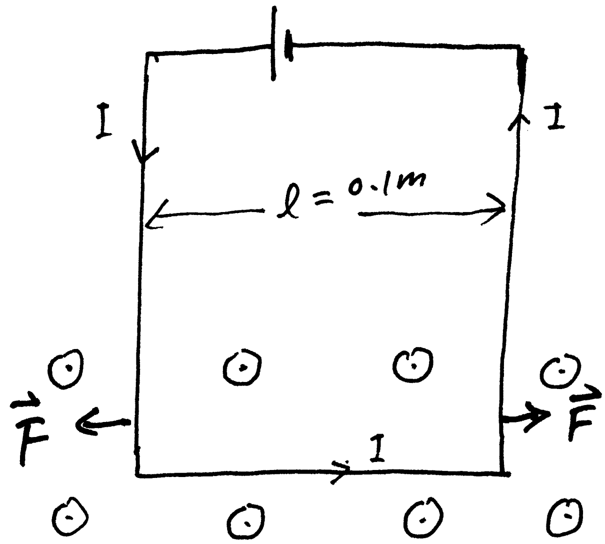
e.g. **Measuring a magnetic field.**

$l = 0.10\,m$, $\theta = 90^\circ$.

$F = 3.48 \times 10^{-2}\,N$. (downward)

$I = 0.245\,A$.

$B = ?$



$\vec{B}$ — out of page.
uniform.

$$F = I l B \sin\theta = I l B$$

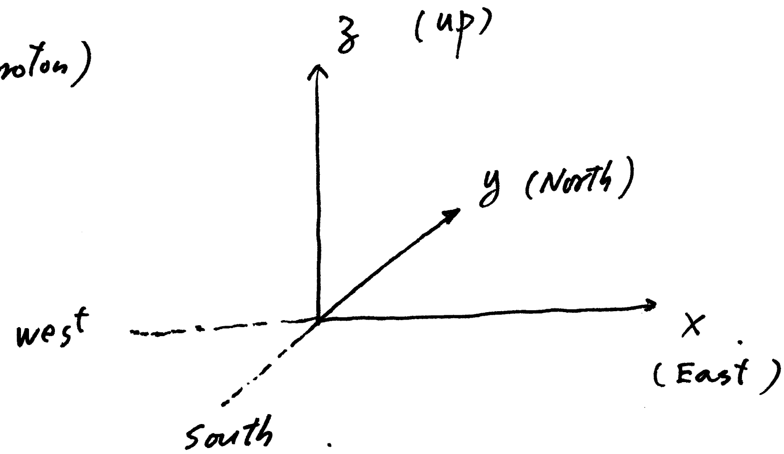
$$B = \frac{F}{I l \sin\theta} = \frac{F}{I l}.$$

$$= \frac{3.48 \times 10^{-2}}{0.245 \times 0.10 \times \sin 90^\circ}$$

$$= 1.42\ T.$$

e.g. force $F = 8 \times 10^{-14}$ N, $v = 5 \times 10^{6}$ m/s

charge: $q = 1.6 \times 10^{-19}$ C. (proton)



① when $\vec{v} \parallel \hat{z}$ (upward).

$\vec{F} \parallel (-\hat{x})$.

② when $\vec{v} \parallel \hat{y}$.

$\vec{F} = 0$

Since $|\vec{F}| = |q\vec{v} \times \vec{B}| = qvB\sin\theta$

$v \neq 0$, $B \neq 0$. If $F = 0$, $\theta = 0°$ or $180°$.

$\therefore$ $\vec{B} \parallel \hat{y}$ (or $\vec{B} \parallel (-\hat{y})$)

But from case ①, we know $\vec{B} \parallel \hat{y}$.
(By RHR) (NOT $\vec{B} \parallel (-\hat{y})$).

i.e. $\vec{B}$ is pointing to the North.

$F = qvB\sin 90°$ for case ①. $\sin 90° = 1$.

$= qvB$.

$$B = \frac{F}{qv} = \frac{8 \times 10^{-14}}{1.6 \times 10^{-19} \times 5 \times 10^{6}} = 0.10\ \text{T}.$$

e.g.

$\vec{v} \perp \vec{B}$. $v = 2 \times 10^{7}$ m/s.

$B = 0.01$ T (uniform).

$q = -e = -1.6 \times 10^{-19}$ C.

path : circular. (uniform circular motion)

on a plane $\perp \vec{B}$.

Radius:

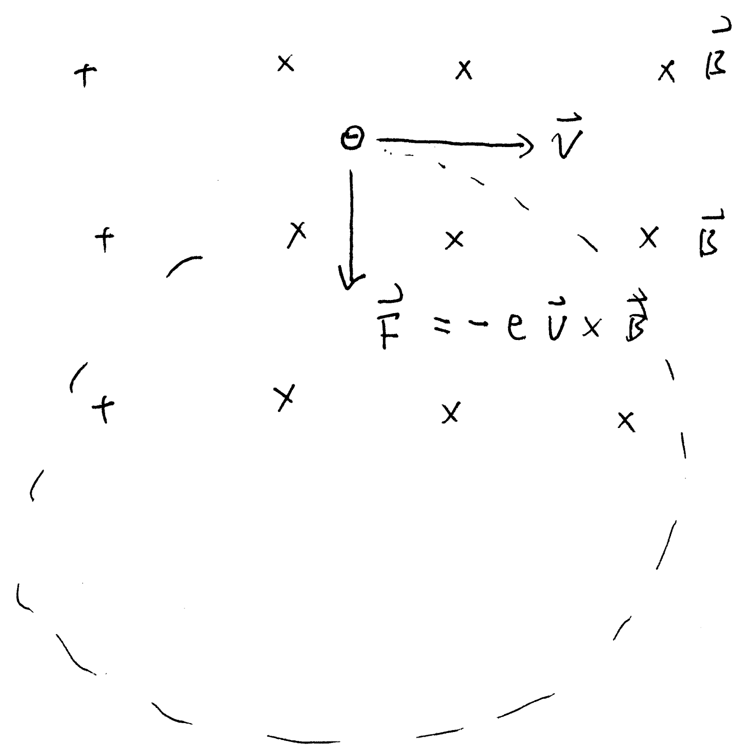
$F = ma$

$$qvB = m\frac{v^2}{R}.$$

$$R = \frac{mv}{qB}$$

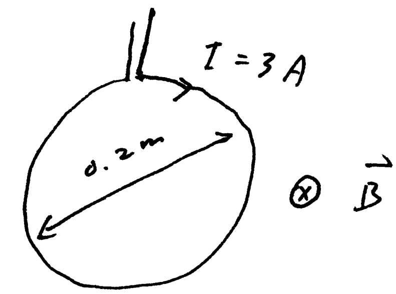
$= 1.1 \times 10^{-2}$ m

$= 1.1$ cm.



$$R \propto \frac{m}{q}.$$ (mass spectrometer).

e.g. Torque on a coil.



$\tau = NIA \cdot B \sin\theta$. (magnitude)

$A = \pi r^2 = \pi \left(\frac{0.2}{2}\right)^2 = 0.0314 \ m^2$ ⊗

$N = 10$ loops

$B = 2.0 \ T$.

$\tau = 10 \times 3 \times 0.0314 \times 2 \cdot \sin\theta \doteq 1.88 \sin\theta$

max: when $\sin\theta = 1$. $\theta = 90°$.

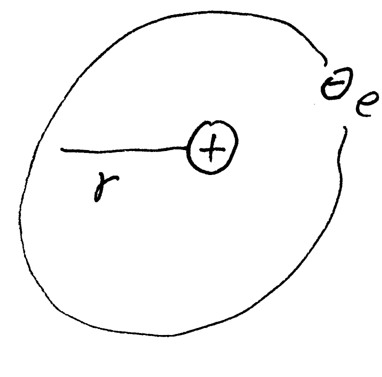
$\tau_{max} = 1.88 \ N$.

min: when $\sin\theta = 0$

$\tau_{min} = 0$.

e.g. magnetic moment of a hydrogen atom.

$r = 0.529 \times 10^{-10} \ m$.



~~F = ma~~.

$I = \frac{e}{T}$ T- period.

$T = \frac{2\pi r}{v}$.

v: F = ma: $\frac{e^2}{4\pi\varepsilon_0 r^2} = m\frac{v^2}{r}$ $\left(a = \frac{v^2}{r}\right)$

Solve for v: $v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m r}} = 2.19 \times 10^6 \ m/s$.

$$\mu = IA = \frac{e}{T} \cdot \pi r^2 = \frac{ev}{2\pi r} \pi r^2$$

$$\mu = \frac{evr}{2} = 9.27 \times 10^{-24} \ A \cdot m^2$$

$$\mu = IA = \frac{ev}{2\pi r} \cdot \pi r^2 .$$

Lorentz equation:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$evB' = m\frac{v^2}{r}$$

$$m = \frac{eB'r}{v}$$