**PHYS 221 Midterm Examination #2**

July 4, 2008

Time: 50 minutes

Name Key

Student No. 27 + 1

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1 (1/20 marks). Write down Maxwell's equations in the differential and integral forms:

$$\nabla \cdot \vec{D} = \rho_v \qquad \oint_S \vec{D} \cdot d\vec{s} = Q$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint \vec{B} \cdot d\vec{s} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \oint_C \vec{H} \cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \cdot d\vec{s}$$

2 (1/20 marks). Write down the boundary conditions for **E**, **D**, **H** and **B**.

$$\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$$

$$\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0$$

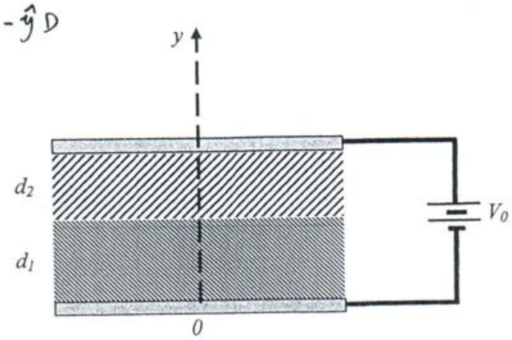
$$\hat{n} \times (\vec{E}_1 - \vec{E}_2) = 0$$

$$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$$

(a, b) .

3(6/20 marks). The space between two parallel conducting plates each having an area $S$ is filled with two parallel dielectric layers of thickness $d_1$ and $d_2$. The permittivity is $\varepsilon_1$ and $\varepsilon_2$ respectively. A d-c voltage $V_0$ is applied across the plates.
(a) Determine the electric field intensity between the plates.
(b) Determine the capacitance.
(c) If the conductivity of the dielectric layers is $\sigma_1$ and $\sigma_2$ respectively (i.e., they are not perfect insulators), determine the distribution of free changes.

(a) . $\oint_S \vec{D}\cdot d\vec{S} = Q \qquad \vec{D} = -\hat{y} D$




$DS = \rho_s \cdot S$

$\therefore D = \rho_s$ .

$\vec{D} = -\hat{y}\rho_s$

$\vec{E}_1 = \dfrac{\vec{D}}{\varepsilon_1} = -\hat{y}\dfrac{\rho_s}{\varepsilon_1}$

$\vec{E}_2 = \dfrac{\vec{D}}{\varepsilon_2} = -\hat{y}\dfrac{\rho_s}{\varepsilon_2}$ .

$$V_0 = -\int \vec{E}\cdot d\vec{l} = \frac{\rho_s d_1}{\varepsilon_1} + \frac{\rho_s d_2}{\varepsilon_2} \Rightarrow \rho_s = \frac{V_0}{d_1/\varepsilon_1 + d_2/\varepsilon_2} .$$

$$\therefore \vec{E}_1 = -\hat{y}\frac{V_0/\varepsilon_1}{d_1/\varepsilon_1 + d_2/\varepsilon_2} = -\hat{y}\frac{V_0\varepsilon_2}{d_1\varepsilon_2 + d_2\varepsilon_1}$$

$$\vec{E}_2 = -\hat{y}\frac{V_0/\varepsilon_2}{d_1/\varepsilon_1 + d_2/\varepsilon_2} = -\hat{y}\frac{V_0\varepsilon_1}{d_1\varepsilon_2 + d_2\varepsilon_1}$$

(b) . $$C = \frac{Q}{V_0} = \frac{\rho_s S}{\frac{\rho_s d_1}{\varepsilon_1} + \frac{\rho_s d_2}{\varepsilon_2}} = \frac{S}{d_1/\varepsilon_1 + d_2/\varepsilon_2} = \frac{S\varepsilon_1\varepsilon_2}{d_1\varepsilon_2 + d_2\varepsilon_1}$$

(OR: $C = \dfrac{C_1C_2}{C_1+C_2}$ . $\quad C_1 = \dfrac{S\varepsilon_1}{d_1}$ , $\quad C_2 = \dfrac{S\varepsilon_2}{d_2}$

$$= \frac{\frac{S\varepsilon_1}{d_1}\frac{S\varepsilon_2}{d_2}}{\frac{S\varepsilon_1}{d_1} + \frac{S\varepsilon_2}{d_2}} = \frac{S\varepsilon_1\varepsilon_2}{\varepsilon_1 d_2 + \varepsilon_2 d_1}$$ )

(c)

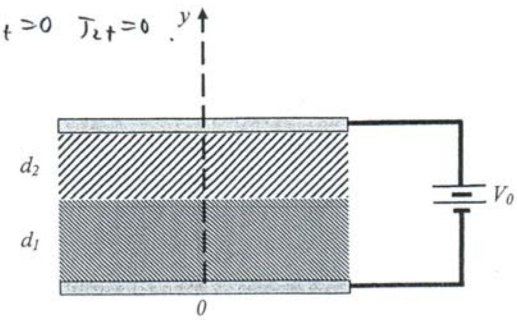
3(6/20 marks). The space between two parallel conducting plates each having an area $S$ is filled with two parallel dielectric layers of thickness $d_1$ and $d_2$. The permittivity is $\varepsilon_1$ and $\varepsilon_2$ respectively. A d-c voltage $V_0$ is applied across the plates.

(a) Determine the electric field intensity between the plates.

(b) Determine the capacitance.

(c) If the conductivity of the dielectric layers is $\sigma_1$ and $\sigma_2$ respectively (i.e., they are not perfect insulators), determine the distribution of free changes.

(c). DC : $J_{1n} = J_{2n}$. $J_{1t} = 0$ $J_{2t} = 0$.




$$\vec{J}_1 = \vec{J}_2 = -\hat{y} J .$$

$$\vec{J} = \sigma \vec{E}$$

$$\vec{E}_1 = \frac{\vec{J}_1}{\sigma_1} = -\hat{y}\frac{J}{\sigma_1}$$

$$\vec{E}_2 = \frac{\vec{J}_2}{\sigma_2} = -\hat{y}\frac{J}{\sigma_2}$$

$$\vec{D}_1 = \varepsilon_1 \vec{E}_1 = -\hat{y}\frac{J\varepsilon_1}{\sigma_1}, \qquad \vec{D}_2 = \varepsilon_2 \vec{E}_2 = -\hat{y}\frac{J\varepsilon_2}{\sigma_2}$$

$J$ :

$$E_1 d_1 + E_2 d_2 = V_0 \Rightarrow \frac{J d_1}{\sigma_1} + J\frac{d_2}{\sigma_2} = V_0$$

$$J = \frac{V_0}{d_1/\sigma_1 + d_2/\sigma_2} = \frac{V_0 \sigma_1 \sigma_2}{d_1\sigma_2 + d_2\sigma_1}$$

Surface charge density on upper plate:

$$\rho_{su} : \quad \rho_{su} = \vec{D}_2 \cdot (-\hat{y}) = \frac{J\varepsilon_2}{\sigma_2} = \frac{V_0 \varepsilon_2 \sigma_1}{d_1\sigma_2 + d_2\sigma_1}$$

lower plate:

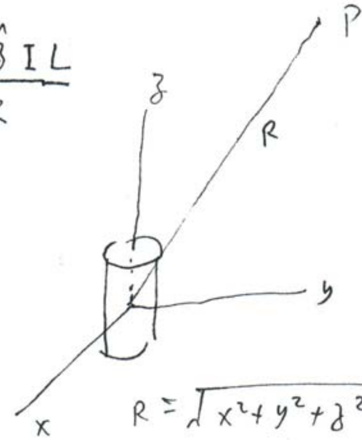
$$\rho_{sL} = \vec{D}_1 \cdot (\hat{y}) = -\frac{J\varepsilon_1}{\sigma_1} = \frac{-V_0 \sigma_2 \varepsilon_1}{d_1\sigma_2 + d_2\sigma_2}$$

Interface between two media:

$$\rho_{si} = -\hat{y}\cdot(\vec{D}_1 - \vec{D}_2) = J\left(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2}\right) = \frac{V_0(\varepsilon_1\sigma_2 - \varepsilon_2\sigma_1)}{d_1\sigma_2 + d_2\sigma_1}$$

4$_{\text{(6/20 marks)}}$. A thin current element extending between z = −L/2 and z = L/2 carries a current I along the z-direction through a circular cross-section of radius $a$.
(a) Find the magnetic vector potential **A** at a point P located very far from the origin (assume that the distance from P to every point in the current element is the same).
(b) Determine the corresponding **H**.

(a). $\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J} \cdot dV'}{R} = \frac{\mu_0 \hat{z} I L}{4\pi R}$




$R = \sqrt{x^2+y^2+z^2}$

(b) $\vec{H} = \nabla \times \frac{\vec{A}}{\mu_0}$

$= \nabla \times \frac{IL}{4\pi} \frac{\hat{z}}{R}$.

$= \frac{IL}{4\pi} \nabla \times \frac{\hat{z}}{R} = \frac{IL}{4\pi} \nabla \times \frac{\hat{z}}{\sqrt{x^2+y^2+z^2}}$

$= \frac{IL}{4\pi} \left\{ \hat{x} \frac{\partial A_z}{\partial y} - \hat{y} \frac{\partial A_z}{\partial x} \right\}$

$= \frac{IL}{4\pi} \left\{ \hat{x} \frac{-y}{(x^2+y^2+z^2)^{3/2}} - \hat{y} \frac{-x}{(x^2+y^2+z^2)^{3/2}} \right\}$

$= \frac{IL}{4\pi \ R^{3/2}} \left( -\hat{x} y + \hat{y} x \right)$

5(6/20 marks). The circular disk shown in the figure below lies in the $x$–$y$ plane and rotates with uniform angular velocity $\omega$ about the $z$-axis. The disk is of radius $a$ and is present in a uniform magnetic flux density $\boldsymbol{B}$ in the z-direction. Obtain an expression for the emf induced at the rim relative to the center of the disk.

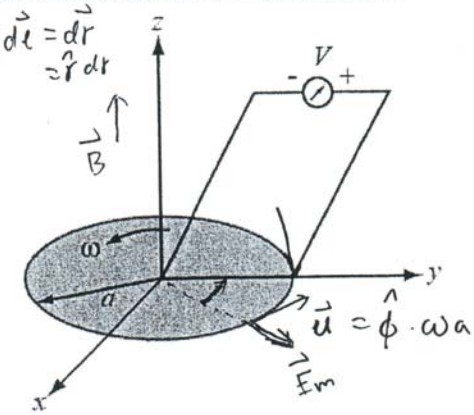
$$V_{emf} = \int_0^a \vec{u} \times \vec{B} \cdot \vec{dl} \qquad (\vec{dl} = \vec{dr} = \hat{r}\, dr$$

$$= \int_0^a (\vec{u} \times \vec{B}) \cdot \hat{r}\, dr$$

$$= \int_0^a \omega \cdot r B\, dr$$

$$= \frac{1}{2} \omega B r^2 \Big|_0^a$$

$$= \frac{1}{2} \omega B a^2 .$$




$$|\vec{u} \times \vec{B}| = \omega \cancel{r} \cdot B .$$