Phys 221.

Assignment #7. Solutions.

5.23.

$$\nabla \times \vec{H} = \vec{J}. \qquad \vec{H} = \hat{\phi}\,\frac{4}{r}\left[1-(1+2r)\,e^{-2r}\right]$$

$$\vec{J} = \hat{z}\,\frac{1}{r}\left(\frac{\partial (H_\phi \cdot r)}{\partial r}\right)$$

$$= \hat{z}\,\frac{1}{r}\left(r\frac{\partial H_\phi}{\partial r} + H_\phi\right)$$

$$= \hat{z}\left[\frac{\partial H_\phi}{\partial r} + \frac{H_\phi}{r}\right]$$

$$= \hat{z}\left[\cancel{-\frac{4}{r^2}\left[1-(1+2r)e^{-2r}\right]} + \frac{4}{r}\left[-2e^{-2r} + 2(1+2r)e^{-2r}\right] \cancel{+\frac{4}{r^2}[\ ]}\right]$$

$$= \hat{z}\left[\frac{4}{r}\cdot 4r\,e^{-2r}\right]$$

$$= \hat{z}\,16e^{-2r}\ \mathrm{A/m^2}.$$

5.25 . $\vec{A} = \hat{x}\,5\cos\pi y + \hat{z}\,(z + \sin\pi x)$ a7-2.

(a) $\vec{B} = \nabla \times \vec{A}$.

$$= \hat{x}\left(\frac{\partial A_z}{\partial y}\right) + \hat{y}\left(\frac{\partial A_{\cancel{x}}}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(-\frac{\partial A_x}{\partial y}\right)$$

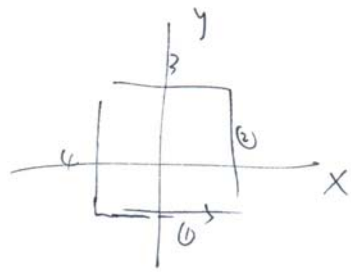
$$= 0 + \hat{y}\left(-\frac{\partial A_z}{\partial x}\right) + \hat{z}\left(-\frac{\partial A_x}{\partial y}\right)$$

$$= -\hat{y}\,\pi\cos\pi x - \hat{z}\,5\pi\,(-\sin\pi y)$$

$$= -\hat{y}\,\pi\cos\pi x + 5\pi\,\hat{z}\,\sin\pi y$$

(b) . $\cancel{\Phi = \oint \vec{B}\cdot d\vec{s}}$ $\Phi = \int \vec{B}\cdot d\vec{s}$

$$d\vec{s} = \hat{z}\,dx\,dy \ .$$



$$\Phi = \int \vec{B}\cdot d\vec{s}$$

$$= 5\pi \int \sin\pi y \cdot dx\,dy_{\;0.125}$$

$$= 5\pi\ 0.25 \int_{-0.125} \sin\pi y\,dy$$

$$= 0$$

(c) . $\oint \vec{A}\cdot d\vec{l} = \int_{(1)} \vec{A}\cdot d\vec{x} + \int_{(2)} \vec{A}\cdot d\vec{y} + \int_{(3)} \vec{A}\cdot d\vec{x} + \int_{4} \vec{A}\cdot d\vec{y}$

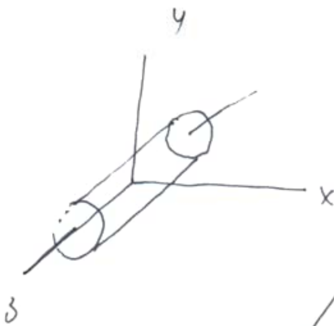
$$= 5\cos\pi(-0.125)\,(0.25) + 0 + 5\cos\pi(0.125)\cdot(-0.25) + 0$$

$$= 0 \ .$$

5.27

(a) $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\; dV'}{R}$

$= \hat{z}\frac{\mu_0 I L}{4\pi R}$

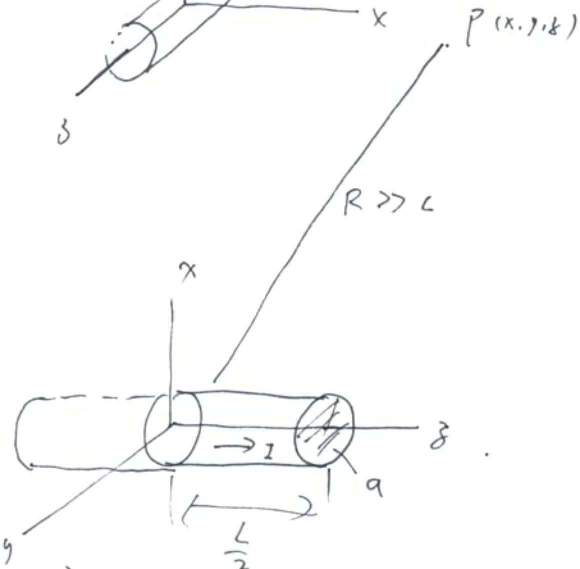


(b) $\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{\nabla \times \vec{A}}{\mu_0}$

$= \frac{IL}{4\pi}\; \nabla \times \frac{\hat{z}}{R}$

$= \hat{x}\frac{\partial A_z}{\partial y} - \hat{y}\frac{\partial A_z}{\partial x}$

$= \frac{IL}{4\pi}\left(\hat{x}\frac{\partial}{\partial y}\frac{1}{R} - \hat{y}\frac{\partial}{\partial x}\left(\frac{1}{R}\right)\right)$



$= \frac{IL}{4\pi}\left(\hat{x}\frac{-y}{(x^2+y^2+z^2)^{3/2}} - \hat{y}\frac{-x}{(\quad)^{3/2}}\right)$ $R = \sqrt{x^2+y^2+z^2}$.

$= \frac{IL}{4\pi R^{3/2}}\left(\hat{y}\,x - y\,\hat{x}\right)$

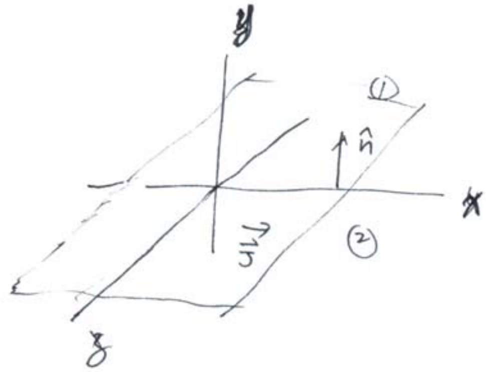
5-31.

$\vec{J} = \hat{x}4.$

$\vec{H}_1 = \hat{z}8.$

$H_{1n} = 0$

$H_{1t} = 0$



B.C.

$\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$ $\qquad \hat{n} = \hat{y}.$

$\hat{n} \times \vec{H}_1 - \hat{n} \times \vec{H}_2 = \vec{J}_s. \qquad \hat{n} \times \hat{H}_1 = \hat{y} \times \hat{z}8 = \hat{x}8.$

$$\hat{n} \times \vec{H}_2 = \hat{n} \times \vec{H}_1 - \vec{J}_s$$
$$= \hat{x}8 - \hat{x}4$$
$$= \hat{x}4.$$

$\hat{x} \perp \vec{H}_2 : H_{2x} = 0.$

$H_{2n} = \frac{\mu_1}{\mu_2} H_{1n} = 0. \therefore H_{2y} = 0.$

$$\therefore \vec{H}_2 = \hat{z}4.$$

OR: use integral form of Maxwell's eq.

(Ampere's law. $\oint_c \vec{H} \cdot d\vec{l} = I.$

5.33.

$\vec{B}_1 = \hat{x}4 - \hat{y}6 + \hat{z}8$.

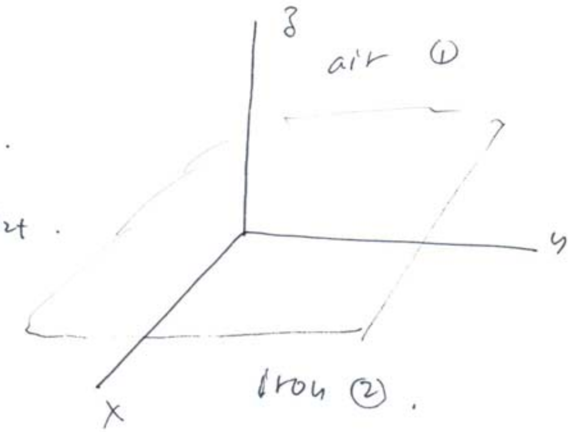
$B_{1n} = \hat{z}8 = B_{2n} = \hat{z}8$.

$H_{1t} = \frac{1}{\mu_0}(\hat{x}4 - \hat{y}6) = H_{2t}$.

$B_{2t} = \mu H_{2t}$

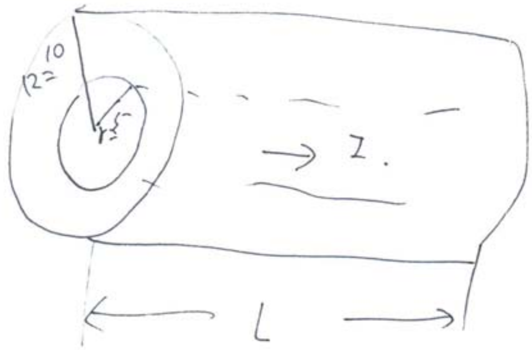
$= \frac{\mu}{\mu_0}(\hat{x}4 - \hat{y}6)$

$= 5000(\hat{x}4 - \hat{y}6)$.



$\vec{B}_2 = \hat{x}20000 - \hat{y}30000 + 8\hat{z}$

5.37. 97-6.



$$W_m = \frac{1}{2} L I^2 = \frac{1}{2} \int_V \mu H^2 dV.$$

$$\vec{B} = \hat{\phi} \frac{\mu_0 I}{2\pi r} \qquad \vec{H} = \hat{\Phi} \frac{I}{2\pi r}.$$

$$W = \frac{\mu_0}{2} \int \frac{I^2}{4\pi^2} \cdot \frac{1}{r^2} dV$$

$$= \frac{\mu_0 I^2}{8\pi^2} \int \frac{1}{r^2} dV.$$

$$= \frac{\mu_0 I^2}{8\pi^2} \cdot 2 \int_a^b \frac{2\pi r \, dr}{r^2}$$

$L=2$

$$= \frac{\mu_0 I^2}{2\pi} \int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 I^2}{2\pi} \ln \frac{b}{a}.$$

$$L = \frac{\mu_0}{\pi} \ln \frac{b}{a}. \quad W_m = \frac{1}{2} L I^2.$$

OR:

$B = LI$

$L = \mu_0$

$$\Phi = \frac{\mu_0 I}{2\pi} \int \frac{1}{r} ds$$

$$= \frac{\mu_0 I}{2\pi} \int_a^b \frac{L \, dr}{r}$$

$$= \frac{\mu_0 I}{\pi} \ln \frac{b}{a}.$$