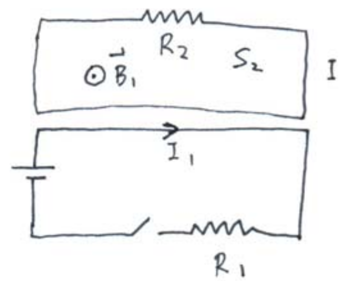
Phys. 221

Assignment #8 solutions.

6.1




$\vec{B}_1$ — Magnetic field due to $I_1$.

at $t = 0$

$$\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{s}$$

is increasing.

The induced current I should reduce the total $\vec{B}$-field in loop 2.

i.e. against $\vec{B}_1$.

Therefore I should be clockwise.

at $t = t_1$, $I_1$ is reduced to zero.

$\Phi_{12}$ is decreasing.

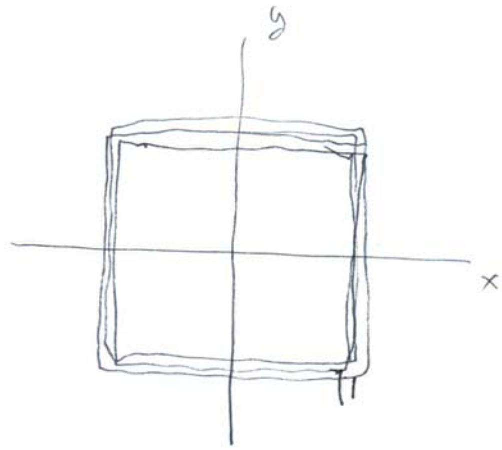
The induced current I shoud increase the $\vec{B}$-field in 2.

i.e., same direction as $\vec{B}_1$.

Therefore, I should be counterclockwise.

6.3.

(a). $\vec{B} = \hat{z}\, 10 \cdot e^{-2t}$ (T)




$$V_{emf} = -N\frac{d\Phi}{dt} = -NA\frac{dB}{dt}$$

$$= 10NA \cdot 2 \cdot e^{-2t}$$

$$= 20\, NA \cdot e^{-2t}$$

$$= 125\, e^{-2t} \text{ V}$$

$N = 100$

$A = 0.25^2 = 0.0625 \text{ m}^2$

(b). $\vec{B} = \hat{z}\, 10 \cos x \cos 10^3 t$

$$\Phi = \int \vec{B} \cdot d\vec{s} = 10 \cos^3 t \cdot \int_{-0.125}^{0.125} \cos x \, dx \cdot 0.5 = 5(\sin 0.125) \cdot 2 \cdot \cos^3 t$$

$$= 10 \sin 0.125 \cdot \cos^3 t$$

$$V_{emf} = -N\frac{d\Phi}{dt} = -(100)(10)(\sin 0.125) \cdot 3 \cdot \cos^2 t \cdot (-\sin t)$$

$$= 3000 \sin 0.125 \cdot \cos^2 10^3 t \cdot \cancel{\sin t} \cdot 1000$$

$$= 3$$

$$\Phi = \int \vec{B} \cdot d\vec{s} = 10 \cdot \cos 10^3 t \cdot 2 \int_0^{0.125} \cos x \cdot dx \cdot 0.25$$

$$= -5 \cos 10^3 t \cdot \sin 0.125$$

$$V_{emf} = -N\frac{d\Phi}{dt} = -100 \cdot (-5) \sin 0.125 \cdot 10^3 \cdot (-\sin 10^3 t)$$

$$= 5 \times 10^5 \sin 0.125 \cdot \sin 10^3 t$$

$$= 62.5 \sin 10^3 t \cdot \text{KV}$$

(c) 0.

6.5 $V_{emf} = -\frac{d\Phi}{dt} = -A\cdot\frac{dB}{dt}$.

$B = B_0 \cos(2\pi f t)$

$\frac{dB}{dt} = -2\pi f\ B_0 \sin(2\pi f t)$

$V_{emf} = 2\pi f A B_0 \sin(2\pi f t)$

$V_{emf\,(max)} = 2\pi f A B_0$

$$B_0 = \frac{20\ mV}{2\pi\cdot 3\times10^8\times 0.01}$$

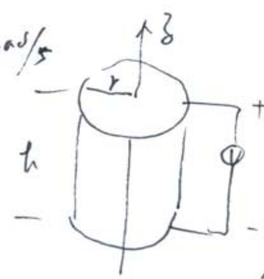
$B_0 = 1\times10^{-9}\ T$

6.11. $1200\ RPM = \omega = \frac{2\pi\cdot1200}{60}$ rad/s

$\vec{B} = \hat{r}\,6\ (T)$

$r = 0.05$ m.

$h = 0.1$ m.




$u = r\omega = (0.05)\,40\pi$ m/s $= 2\pi$ m/s. $\vec{u} = 2\pi\hat{\phi}$

$V_{emf} = \int(\vec{u}\times\vec{B})\cdot d\vec{l} = -uB\cdot h = -(2\pi)(6)(\cancel{0.05}\ 0.1)$

$= -1.2\pi\ (V)$

$= -3.77\ V$.

6.18

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}.$$

$$\nabla \cdot \vec{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}.$$

$$= 0 - 6y \cos \omega t.$$

i.e. $$\frac{\partial \rho_v}{\partial t} = 6y \cos \omega t.$$

$$\rho_v = \int 6y \cos \omega t \, dt$$

$$= \frac{6y}{\omega} \sin \omega t + C.$$