

physics 221. Lecture 2.

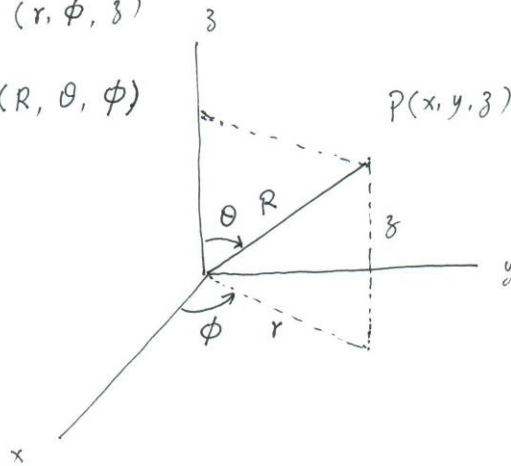
- Orthogonal Coordinate Systems.

- coordinates are perpendicular to each other.

- Cartesian (x, y, z)

- Cylindrical (r, ϕ, z)

- Spherical (R, θ, ϕ)



- Why do we need different coordinate systems?

Simplify the description of problems ~~when~~
by choosing a coordinate system that best
fits the geometry of the problem.

e.g: coaxial cable : use cylindrical system.

• Table 2. Summary of vector relations.

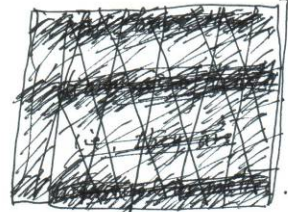
(Properties of Cartesian, Cylindrical and Spherical systems).

Note 1. Base vectors.

$\hat{x}, \hat{y}, \hat{z}$

$\hat{r}, \hat{\phi}, \hat{z}$

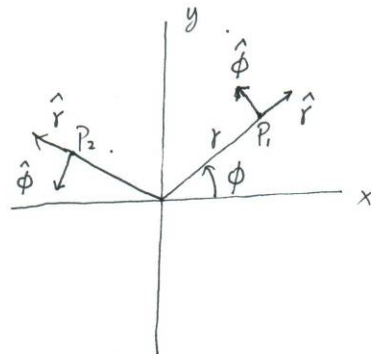
$\hat{R}, \hat{\theta}, \hat{\phi}$



$\hat{r}, \hat{\phi}, \hat{R}, \hat{\theta}$ — not constant!

Direction varies with location.

e.g. $\hat{r}, \hat{\phi}$, viewed on x - y plane. (Polar coord.)



$$\hat{r}(\text{at } P_1) \neq \hat{r}(\text{at } P_2)$$

$$\hat{\phi}(\text{at } P_1) \neq \hat{\phi}(\text{at } P_2)$$

Note 2. Differential length $d\vec{l}$

$$\hat{x} dx + \hat{y} dy + \hat{z} dz \quad . \quad - \text{Cartesian}$$

$$\hat{r} dr + \hat{\phi} r d\phi + \hat{z} dz \quad . \quad - \text{Cylindrical}$$

$$\hat{R} dR + \hat{\theta} \cdot R d\theta + \hat{\phi} R \sin\theta \cdot d\phi \quad - \text{spherical}.$$

watch out: ϕ, θ are angles.

$\therefore d\phi, d\theta$ are not lengths!

but, $R d\theta$, is, and so are $r d\phi$, $R \sin\theta d\phi$.

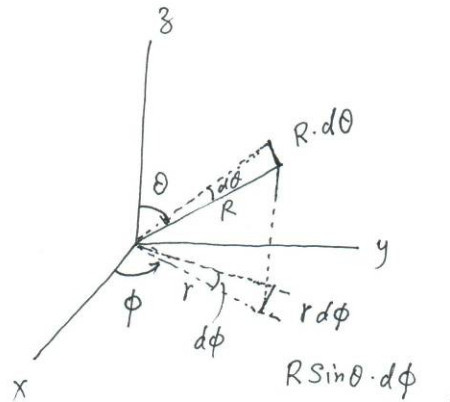


Table 2-3. Coordinate Transformation Relations.

Matrix Form.

e.g. Cartesian to Cylindrical. given (x, y, z) } for a vector $\vec{A}$
 want (r, ϕ, z) .

$$\begin{pmatrix} A_r \\ A_\phi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$A_{\text{cyl.}} = \cancel{A_{\text{cyl.}}} (T_{\text{cart-cyl.}}) A_{\text{cart.}}$$

Cylindrical to Cartesian.

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_r \\ A_\phi \\ A_z \end{pmatrix}$$

$$A_{\text{cart.}} = (T_{\text{cyl.-cart}}) A_{\text{cyl.}}$$

$$T_{\text{cart-cyl}} = T_{\text{cyl.-car}}^{-1} \quad (\text{inverse matrix})$$

- Gradient of a scalar field V

$$\begin{aligned}\text{grad } V &\triangleq \nabla V \\ &\triangleq \hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z}.\end{aligned}$$

— It's a vector.

— Represents the magnitude and direction of the max space rate of change of V .

∇ — "del". is a vector operator.

$$\nabla \triangleq \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}.$$

e.g. Electrostatics. V — electric potential.

$\vec{E}$ — electric field.

$$\boxed{\vec{E} = -\nabla V}$$

↳ "-" sign — $\vec{E}$ direction is from higher V to lower V .

("max" decreasing direction)