Physics 221.

Lecture 7.

Electrostatics

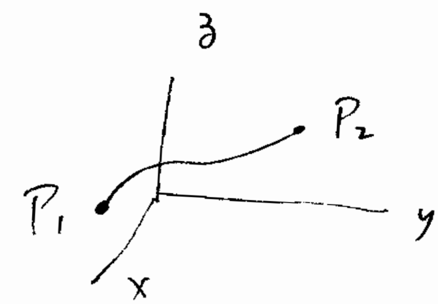
- Electric scalar potential (4-5) p.79.

— A nice way to get arround the vector sum.

$$\vec{E} = -\nabla V$$

and: $$V_{21} = V_2 - V_1 = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{l} \qquad (4.39)$$

OR: $$\underbrace{-\Delta V}_{\text{P.E. loss per unit charge}} = \underbrace{\int_{P_1}^{P_2} \vec{E}\cdot d\vec{l}}_{\text{work done by } \vec{E}\text{-field}}$$



(Since $V_1 - V_1 = 0$.

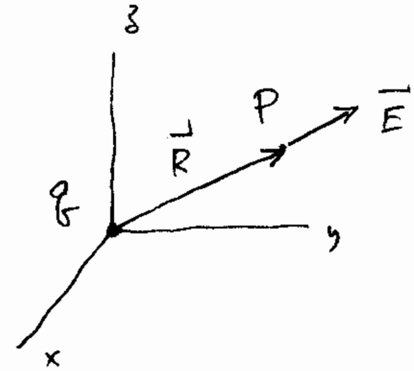
$\oint_C \vec{E}\cdot d\vec{l} = 0$ for any closed C)

i.e: $\nabla \times \vec{E} = 0$ everywhere.

i.e: $\vec{E}$ — conservative.

- For a point charge q at the origin
  - We can define the potential at ∞ as 0 $(V(\infty) = 0)$ $\left\{ \text{NOT always possible} \right.$

Then, the potential at point P is.



$$V(\vec{R}) = V(\vec{R}) - V(\infty)$$

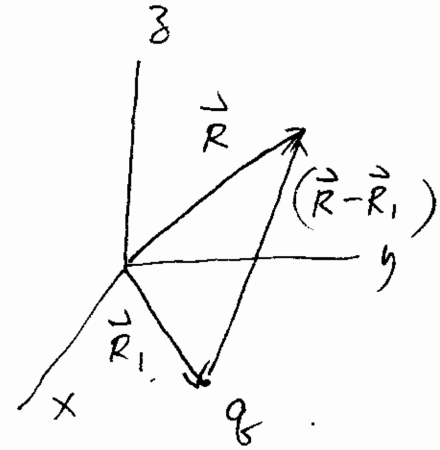
$$= - \int_{\infty}^{P} \vec{E} \cdot \vec{dl}$$

$$= - \int_{\infty}^{R} \frac{q}{4\pi \varepsilon_0 R^2} \cdot dR \qquad \varepsilon_0 \text{ — in vacuum}$$

$$V(\vec{R}) = \frac{q}{4\pi \varepsilon_0 R} \qquad \left[q \text{ at } (0,0,0) \right]$$

- If the charge is at $\vec{R}_1$

$$V(\vec{R}) = \frac{q}{4\pi \varepsilon_0 \left| (\vec{R} - \vec{R}_1) \right|}.$$



$|\vec{R} - \vec{R}_1|$ — distance from source to field point.

V — Scalar !

- For many point charges, $q_1, q_2, \cdots q_N$ at $\vec{R}_1, \vec{R}_2, \cdots \vec{R}_N$

$$V(\vec{R}) = \frac{1}{4\pi\varepsilon_0} \underbrace{\sum_{k=1}^{N}}_{} \frac{q_k}{|\vec{R} - \vec{R}_k|}$$

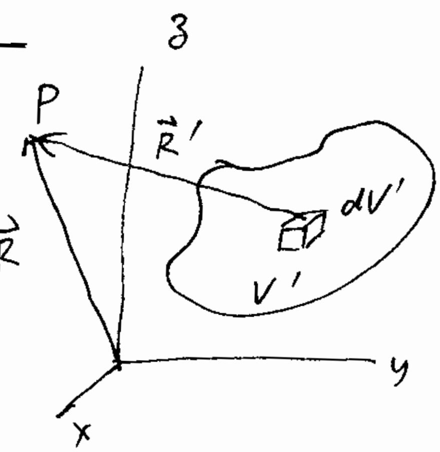
ε — in medium

electric potential at $\vec{R}$.

scalar sum! (algebraic)

- For continuous charge distribution.

$$V(\vec{R}) = \frac{1}{4\pi\varepsilon} \int_{V'} \frac{\rho_v \, dV'}{R'}$$



R' — distance from dV' to the field point P.

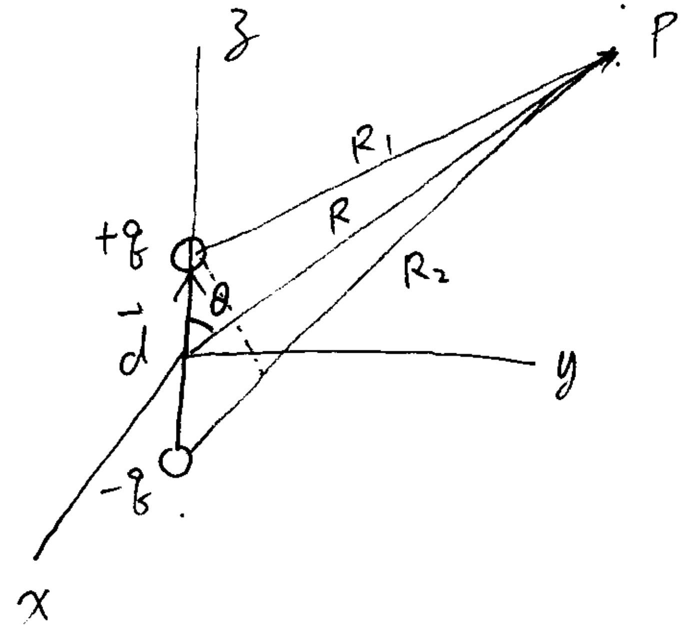
$\rho_v \, dv' = dq$.

for surface charges: $dq = \rho_s \, ds$

for line charges: $dq = \rho_\ell \, d\ell$.

e.g. Electric dipole.

$$\vec{p} = q\vec{d}$$



① Find its potential at $\vec{R}$ for $R \gg d$.

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1} + \frac{-q}{R_2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0} \frac{R_2 - R_1}{R_1 R_2}$$

for $R \gg d$. $R_2 - R_1 \approx d\cos\theta$, $R_1 R_2 \approx R^2$

$$\therefore \quad V = \frac{qd\cos\theta}{4\pi\varepsilon_0 R^2}.$$

② Find its $\vec{E}$-field. (p. 82)

Phys. 221

Lecture 8

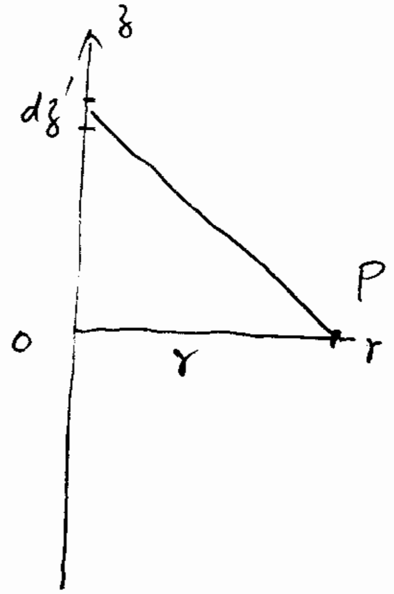
Electric potential. V.

$$V(\vec{R}) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho_v \, dV'}{R'}$$

R' — distance from surce (dV') to field Point

e.g.: Infinitely long straight line charge of a uniform line charge density ρ_ℓ in air.

Find the potential at P



Try: $$V = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty} \frac{\rho_\ell \, dz'}{\sqrt{r^2+z'^2}}$$

$$= \frac{\rho_\ell}{4\pi\varepsilon_0} \ln\left(z' + \sqrt{z'^2+r^2}\right)\Bigg|_{z'\to-\infty}^{z'\to\infty}$$

$$= \lim_{L\to\infty} \frac{\rho_\ell}{4\pi\varepsilon_0} \ln\left(\frac{L+\sqrt{L^2+r^2}}{-L+\sqrt{L^2+r^2}}\right)$$

$$= \lim_{L\to\infty} \frac{\rho_\ell}{4\pi\varepsilon_0} \ln \frac{\left(L+\sqrt{L^2+r^2}\right)^2}{r^2} \Rightarrow \infty$$

problem: when size is infinte, we can't assume the zero potential point at infinity.

How to do it properly :

(Only the potential difference ΔV is uniquely defined)

$\therefore$ potential difference between r and r_0 :

$$\Delta V = V(r) - V(r_0) \qquad \left[V(r) = \frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{\rho_v \, dv'}{R'} + C \right.$$

$$= \frac{\rho}{4\pi\varepsilon_0} \int_{-\infty}^{+\infty} \frac{\rho_\ell \, dz'}{\sqrt{r^2+z'^2}} - \frac{\rho}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{\rho_\ell \, dz'}{\sqrt{r_0^2+z^2}} .$$

$$= \frac{\rho_\ell}{4\pi\varepsilon_0} \ln \frac{z' + \sqrt{z'+r^2}}{z' + \sqrt{z'^2+r_0^2}} \Bigg|_{z' \to -\infty}^{z' \to \infty}$$

r^2

$$= \frac{\rho_\ell}{4\pi\varepsilon_0} \ln \left(\frac{L + \sqrt{L^2+r^2}}{L + \sqrt{L^2+r_0^2}} \cdot \frac{-L + \sqrt{L^2+r_0^2}}{-L + \sqrt{L^2+r^2}} \right)_{L \to \infty}$$

$$= \frac{\rho_\ell}{4\pi\varepsilon_0} \ln \left(\frac{1 + \sqrt{1+\frac{r^2}{L^2}}}{1 + \sqrt{1+\frac{r_0^2}{L^2}}} \cdot \frac{-1 + \sqrt{1+(r_0/L)^2}}{-1 + \sqrt{1+(r/L)^2}} \right)$$

$$= \frac{\rho_\ell}{4\pi\varepsilon_0} \ln \left(\frac{\left(1+\sqrt{1+r^2/L^2}\right)^2}{\left(1+\sqrt{1+\left(\frac{r_0}{L}\right)^2}\right)^2} \; \frac{\left(\frac{r_0}{L}\right)^2}{\left(\frac{r}{L}\right)^2} \right)_{L \to \infty}$$

$$V(r) - V(r_0) = \frac{\rho_0}{4\pi\varepsilon_0} \ln \frac{r_0^2}{r^2} . = -\frac{\rho_\ell}{2\pi\varepsilon_0} \ln \frac{r}{r_0} .$$

choose $\boxed{V(r_0) = 0 : \quad V(r) = -\frac{\rho_\ell}{2\pi\varepsilon_0} \ln \frac{r}{r_0}}$

$\vec{E}$-field: $\vec{E} = -\nabla V$.

$$= -\hat{r}\frac{\partial V}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{z}\frac{\partial V}{\partial z}.$$

$$= -\hat{r}\frac{\partial V}{\partial r}$$

$$= -\hat{r}\left(\frac{-\rho_\ell}{2\pi\varepsilon_0}\right)\frac{1}{r}$$

$$= \hat{r}\frac{\rho_\ell}{2\pi\varepsilon_0 r}$$ — same as before.

Compare 3 Methods:

① . Coulomb's law — intuitive.
but hard to deal with vector sum

②. Gauss' law — Easier math.
but requires high symmetry.

③ potential function — No more vector (sum) integral
Even works for finite length!
but concept and math could be challenging.

- Poisson's Equation:

Since $\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}$ and $\vec{E} = -\nabla V$

$$\nabla \cdot (\nabla V) = -\frac{\rho_v}{\varepsilon}$$

i.e.: $$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}.} \qquad 4.60 \quad \text{(Poisson's Eq)}.$$

When there is no free charge.

$$\nabla^2 V = 0. \quad \text{— Laplace's eq.}$$

Very important (later) and useful.

- Ohm's law. (point form)

$$\vec{J} = \sigma \vec{E}.$$

(circuit form: $R = \frac{V}{I}$
$I = \frac{1}{R} V$)

$\vec{J}$ — current density.
(I per unit area)

$\vec{E}$ — electric field.

σ — electric conductivity.

perfect dielectric: $\vec{J} = 0$. $\boxed{(\because \sigma = 0)}$

perfect conductor: $\vec{E} = 0$. $\boxed{(\because \sigma = \infty)}$

- Conductivity:

charge carriers: electrons. $q_e = -e$. (−)

holes: $q_h = e$. (+)

$e = 1.6 \times 10^{-19}$ (Coulomb).

drift velocity: (average drift velocity. — NOT. newton's law!)

electron: $\vec{u}_e = -\mu_e \vec{E}$ μ_e — electron mobility.

hole $\vec{u}_h = \mu_h \vec{E}$. μ_h — hole mobility.

current density.

$$\vec{J} = \vec{J}_e + \vec{J}_h = \rho_{ve}\vec{u}_e + \rho_{vh}\vec{u}_h$$

$$= \left(-\rho_{ve}\mu_e + \rho_{vh}\mu_h\right)\vec{E}.$$

conductivity: $\vec{J} = \sigma\vec{E}$.

$$\sigma = \left(-\rho_{ve}\mu_e + \rho_{vh}\mu_h\right)$$

$$= \left(N_e\mu_e + N_h\mu_h\right)e.$$

for a good conductor ($\mu_e N_e \gg N_h\mu_h$)

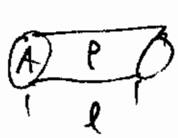
$$\sigma = -\rho_{ve}\mu_e = N_e\mu_e e.$$

units: resistance. R: Ω (ohm)

conductance: $G = \frac{1}{R}$: S (Siemens)

conductivity: σ: S/m.

resistivity. ρ Ω/m. $\rho = \frac{1}{\sigma}$.



$$R = \rho\cdot\frac{\ell}{A} = \frac{\ell}{\sigma A}$$

e.g: 4-8, 4-9.

current density.

$$\vec{J} = \vec{J}_e + \vec{J}_h = \rho_{ve}\vec{u}_e + \rho_{vh}\vec{u}_h$$

$$= (-\rho_{ve}\mu_e + \rho_{vh}\mu_h)\vec{E}.$$

conductivity: $\vec{J} = \sigma\vec{E}$.

$$\sigma = (-\rho_{ve}\mu_e + \rho_{vh}\mu_h)$$

$$= (N_e\mu_e + N_h\mu_h)e.$$

for a good conductor ($\mu_e N_e \gg N_h\mu_h$)

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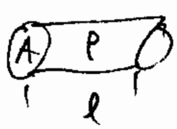

$$R = \rho \cdot \frac{l}{A} = \frac{l}{\sigma A}$$

e.g: 4-8, 4-9.