

PHYS 221 Midterm examination #1

May 30, 2008

Name Key

Time: 50 minutes

Student No. _____

Please show complete solutions and explain your reasoning, stating any principles that you have used.

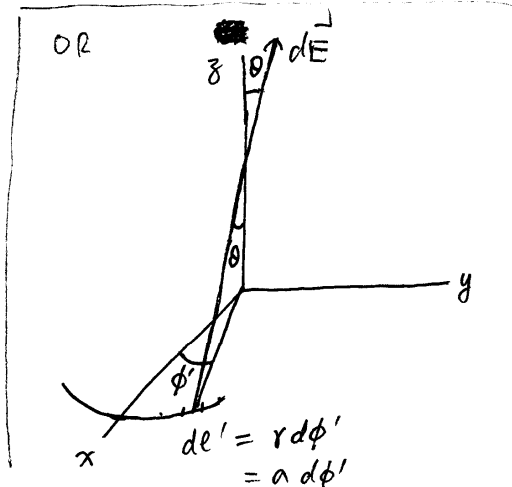
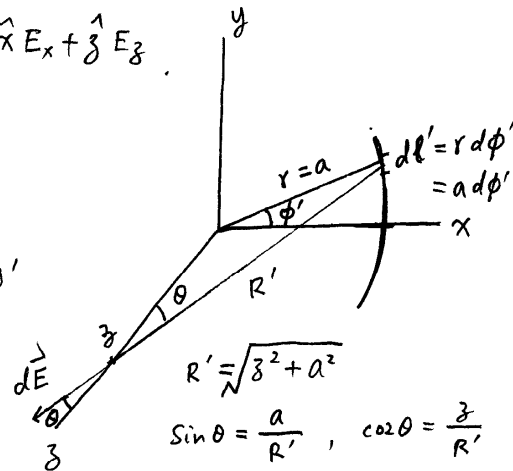
1(5/15 marks). Electric charge is uniformly distributed along an arc located in the x - y plane and defined by $r = a$ and $-\frac{\pi}{8} \leq \phi \leq \frac{\pi}{8}$. The linear charge density is ρ_l .

Determine the electric field $\vec{E}$ at $(0, 0, z)$.

By symmetry, $E_y = 0$. i.e., $\vec{E} = \hat{x} E_x + \hat{z} E_z$.

$$\begin{aligned}
 E_x &= \frac{-1}{4\pi\epsilon_0} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{\rho_l a d\phi' \sin\theta \cdot \cos\phi'}{(z^2 + a^2)} \\
 &= \frac{-\rho_l a^2}{4\pi\epsilon_0 (z^2 + a^2)^{3/2}} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \cos\phi' d\phi' \\
 &= \frac{-\rho_l a^2 \sin\frac{\pi}{8}}{2\pi\epsilon_0 (z^2 + a^2)^{3/2}}
 \end{aligned}$$

$$\begin{aligned}
 E_z &= \frac{1}{4\pi\epsilon_0} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{\rho_l a d\phi' \cdot \cos\theta}{z^2 + a^2} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{\rho_l a z}{(z^2 + a^2)^{3/2}} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} d\phi' \\
 &= \frac{\rho_l a z}{16\epsilon_0 (z^2 + a^2)^{3/2}}
 \end{aligned}$$



2(5/15 marks). An electric dipole consists of two charges q and $-q$ located at $(0, 0, d/2)$ and $(0, 0, -d/2)$ respectively.

(a) Determine the electric potential V at any point $P(x, y, z)$ in free space, given that P is far away from the dipole.

(b) Determine the electric field $\mathbf{E}$ (both magnitude and direction) at point P .

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{R_1} - \frac{q}{R_2} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \frac{R_2 - R_1}{R_1 R_2} \quad (V(\infty) = 0)$$

a). when $R \gg d$.

$$V \approx \frac{q}{4\pi\epsilon_0} \frac{d \cos \theta}{R^2}$$

OR:
$$V = \frac{q d \cdot z}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{3/2}}$$

b). $\vec{E} = -\nabla V$

$$= - \left[\hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi} \right]$$

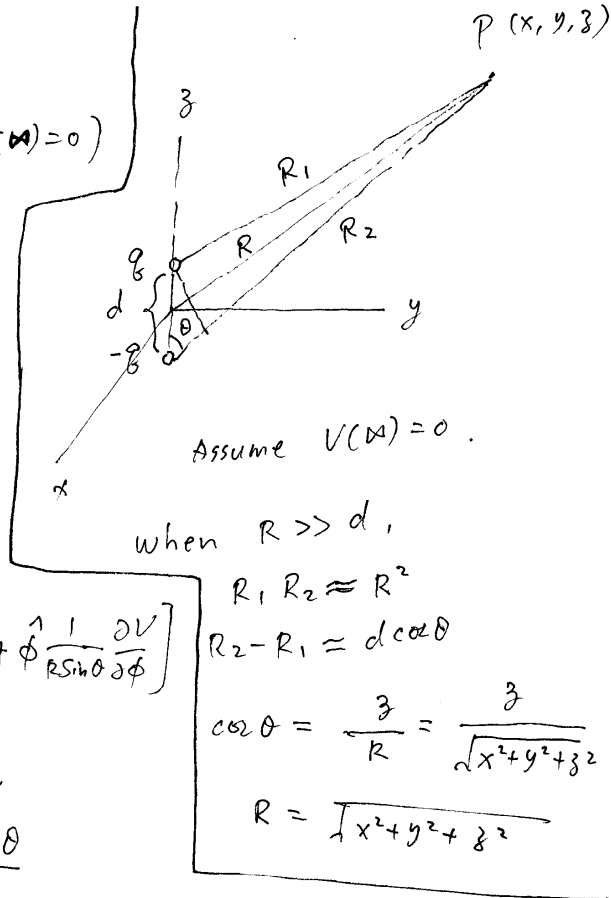
$$= -\hat{R} \frac{\partial V}{\partial R} - \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta}$$

$$E_R = -\frac{\partial V}{\partial R} = \frac{q}{2\pi\epsilon_0} \frac{d \cos \theta}{R^3}$$

$$E_\theta = -\frac{1}{R} \frac{\partial V}{\partial \theta} = \frac{1}{R} \frac{q \cdot d \cdot \sin \theta}{4\pi\epsilon_0 R^2} = \frac{q d \cdot \sin \theta}{4\pi\epsilon_0 R^3}$$

$$E_\phi = 0$$

$$\therefore \vec{E} = \hat{R} \frac{q d \cos \theta}{2\pi\epsilon_0 R^3} + \hat{\theta} \frac{q d \sin \theta}{4\pi\epsilon_0 R^3} \quad (R \gg d)$$



3(5/15 marks). A solid sphere with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ . Figure A below depicts its cross section on the x - y plane. The centre of the sphere is at the origin.

(a) Determine the electric field E at point $P(2R/3, 0, 0)$.

(b) A spherical cavity of radius $R/2$ is created as shown in figure B. The centre of the cavity is located at $(R/2, 0, 0)$. Determine the electric field inside the cavity at point $P(2R/3, 0, 0)$.

(c) If the cavity inside the sphere has a radius w and the centre of the cavity is located at $(a, b, 0)$, find the electric field at a point $(x, y, 0)$ inside the cavity.

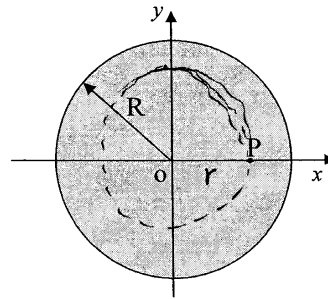


Figure A

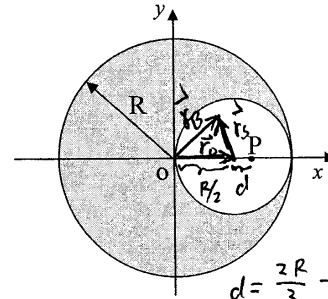


Figure B

$$(a) \quad \vec{E} = \hat{R} E_R$$

spherical Gaussian surface.

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\epsilon_0}$$

$$E_R \cdot 4\pi r^2 = \frac{\frac{4}{3}\pi r^3 \cdot \rho}{\epsilon_0}$$

$$E_R = \frac{\rho r}{3\epsilon_0}$$

$$\text{i.e., } \vec{E} = \hat{R} \frac{\rho r}{3\epsilon_0} = \frac{\rho \vec{r}}{3\epsilon_0}$$

at point P:

$$\vec{E} = \hat{x} \cdot \frac{\rho}{3\epsilon_0} \cdot \frac{2R}{3} = \hat{x} \frac{2\rho R}{9\epsilon_0}$$

$$(b): \quad \vec{E} = \hat{x} \left(\frac{2\rho R}{9\epsilon_0} - \frac{\rho R}{18\epsilon_0} \right) = \hat{x} \frac{\rho R}{6\epsilon_0}$$

$$\left[\text{OR: } E_x = E_x^+ + E_x^- = \frac{\rho}{3\epsilon_0} \cdot \frac{2R}{3} - \frac{\rho}{3\epsilon_0} \cdot \frac{R}{6} = \frac{\rho R}{6\epsilon_0} \right]$$

$$E_y = E_z = 0$$

$$(c) \quad \vec{E} = \vec{E}_\rho + \vec{E}_{-\rho}$$

$$= \frac{\rho}{3\epsilon_0} (\vec{r}_B - \vec{r}_S) \quad r_S = w$$

$$= \frac{\rho}{3\epsilon_0} \vec{r}_0 \quad \left(\vec{r}_0 = \text{position of center of cavity} \right)$$

$$\left(\begin{array}{c} \vec{r}_B \\ \vec{r}_S \\ \vec{r}_0 \end{array} \right) \quad \vec{r}_B - \vec{r}_S = \vec{r}_0$$