Assignment 10 Solutions Phys. 221

7.31) A RHC-polarized wave travelling in the $-z$ direction is equivalent to a LHC polarized wave in the $+z$ direction, with its propagation direction reversed:

$$\tilde{E}(z) = a(\hat{x} + \hat{y}e^{j\pi/2})e^{jkz}$$

$$\Rightarrow E(z,t) = \Re e\left[\tilde{E}(z)e^{j\omega t}\right]$$

$$= \hat{x}\, a\cos(\omega t + kz) + \hat{y}\, a\cos(\omega t + kz + \pi/2)$$

$$= \hat{x}\, a\cos(\omega t + kz) - \hat{y}\, a\sin(\omega t + kz)$$

Now $|E(z,t)| = 2 = \left(a^2\cos^2(\cdots) + a^2\sin^2(\cdots)\right)^{1/2} = a$

$$\& \quad k = \frac{2\pi}{\lambda} = \frac{2\pi}{.06} = \frac{100\pi}{3}$$

$$\omega = k\cdot c = \frac{100\pi}{3}\cdot 3\times10^8 = 10^{10}\pi$$

$$\Rightarrow E(z,t) = \hat{x}\,2\cos\left(10^{10}\pi t + \frac{100\pi}{3}z\right) - \hat{y}\,2\sin\left(10^{10}\pi t + \frac{100\pi}{3}z\right)$$

7.43)

nonmagnetic conducting material

$\Rightarrow$ For Good conductor: (From table 7-2)

$$u_p = \sqrt{\frac{4\pi f}{\mu \sigma}} \qquad \alpha = \sqrt{\pi f \mu \sigma} \Rightarrow \sigma = \frac{\alpha^2}{\pi f \mu}$$

$$\Rightarrow u_p = \frac{2\pi f}{\alpha} = 2\pi f \delta = 2\pi(5 \times 10^9)(2 \times 10^{-6})$$

$$= 2\pi \times 10^4 \ \frac{m}{s}$$

7.47) For copper $\sigma = 5.8 \times 10^7$ S/m

$\mu \approx \mu_0$

$$R_{DC} = \frac{1}{\sigma}\frac{l}{A} = \frac{1}{\sigma}\frac{l}{w \cdot h}$$ (For current in x direction)

Now first lets find skin depth. Since Cu is good Conductor:

$$\delta = 1/\sqrt{\pi \sigma \mu f} = 1/\sqrt{\pi(5.8\times10^7)(4\pi\times10^{-7})(1000)}$$
$$= 0.002089 \text{m}$$

Since the height of the block is 0.3 m, we can see that its thickness is about 150 times the skin depth.
We can therefore use the expression derived in the book for an infinite block in the z-direction (pg. 230-231)

$$R_{AC} = \frac{1}{\sigma \delta_s}\frac{l}{w}$$

$$\Rightarrow \quad \frac{R_{AC}}{R_{dc}} = \frac{1}{\sigma\delta_s}\frac{l}{w}\left(\frac{\sigma w h}{l}\right) = \frac{h}{\delta_s} = \frac{0.3}{0.002089} = 143.55$$

7.51) a) $\tilde{E} = \hat{x}\, 10 e^{-0.2z} e^{-j0.2(z)}$ $\sigma = 4$

We can see from $\tilde{E}$ that $\alpha = \beta$, so we are dealing with a good conductor.
(Note: since ε is so small, water satisfies our "good conductor criteria: $\sigma/\varepsilon\omega \gg 100$, even at pretty high frequencies)

$\Rightarrow \alpha = \beta = 0.2$ $\eta_c = (1+j)\dfrac{\alpha}{\sigma} = \dfrac{5}{100}(1+j) = \sqrt{\dfrac{5}{1000}}\, e^{j\pi/4}$

$$\Rightarrow S_{av} = \hat{z}\frac{|E_0|^2}{2|\eta_c|}\cos\theta_\eta\, e^{-2\alpha z} = \hat{z}\frac{10^2}{2\sqrt{5/1000}}\cos(\pi/4)\, e^{-2(0.2)z}$$

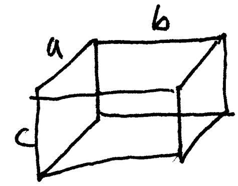
$$= \hat{z}\, 500 e^{-0.4z} \text{ W/m}^2$$

b) $$A = 10\log_{10}\left(\frac{S_{av}(z)}{S_{av}(0)}\right) = 10\log_{10}\left(\frac{500e^{-0.4z}}{500e^{0}}\right) = -4z\log_{10}(e)$$

$$= -1.7372\, z$$

c) From (b), $\alpha_{dB} = 1.7372$ dB/m

$$\therefore \quad d = \frac{40 \text{ dB}}{1.7372 \text{ dB/m}} = 23.026 \text{ m}$$



7.55) $E = \hat{x} E_0 \cos(\omega t - ky)$

The wave is propagating in y direction, ∴ flux has 2 components, The wave entering the box on the left face & leaving the box on the right face.

We are in air, so $\eta = \eta_0$

$$\Rightarrow \quad H = -\hat{z}\frac{E_0}{\eta_0}\cos(\omega t - ky)$$

$$S(y,t) = E \times H = \hat{y}\frac{E_0^2}{\eta_0}\cos^2(\omega t - ky)$$ is the power density at time t & $y_0 = y$

So the power entering the left is

$$P_L(t) = S(y=0, t)\cdot a \cdot c = \hat{y}\frac{acE_0^2}{\eta_0}\cos^2(\omega t)$$

and from the right: $P_R = \hat{y} S(y=b,t)\, a\cdot c = \hat{y}\frac{acE_0^2}{\eta_0}\cos^2(\omega t - kb)$

So the total instantaneous flux is:

$$P(t) = P_L - P_R = \hat{y}\frac{acE_0^2}{\eta_0}\left(\cos^2(\omega t) - \cos^2(\omega t - kb)\right)$$

b) $$P_{av} = \frac{1}{T}\int_0^T P(t)\,dt = \hat{y}\frac{acE_0^2}{\eta_0 T}\underbrace{\int_0^T \cos^2(\omega t) - \cos^2(\omega t - kb)\,dt}_{} = 0$$

Since we are integrating over a whole period, both terms give the same magnitude, but oposite sign