Assignment #11 Solutions

8.1) Condition is that if $l/\lambda \gtrsim 0.01$, we should include the line in our analysis

$$\frac{l}{\lambda} = \frac{lf}{c}$$

a) $\frac{lf}{c} = 6.667 \times 10^{-6}$ $\Rightarrow$ can confidently ignore T.L. effects

b) $\frac{lf}{c} = 0.01$ $\Rightarrow$ This is borderline, so decision will depend on how accurate a calculation we want

c) $\frac{lf}{c} = 0.2$ $\Rightarrow$ Should include T.L. effects

d) $\frac{lf}{c} = \frac{1}{3}$ $\Rightarrow$ include T.L. effects

8.2) Coaxial line given: $b = 1.0\,cm/2$ $a = 0.5\,cm/2$

insulator: $\mu = 1$ $\varepsilon_r = 2.25$ $\sigma = 10^{-3}\,S/m$

conductor: $\mu_c = \mu_0$ $\sigma_c = 5.8 \times 10^7\,S/m$

$f_0 = 1 \times 10^9\,Hz$

$$\ln(b/a) = \ln((0.01\,m/2)/(0.005\,m/2)) = \ln(2)$$

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = \sqrt{\frac{f\mu_c}{4\pi\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right) = \sqrt{\frac{10^9 \cdot (1.2566\times10^{-6})}{4\cdot\pi\cdot(5.8\times10^7)}}\cdot\left(\frac{\Omega}{0.005}+\frac{\Omega}{0.01}\right)$$

$$= 0.7878\ \Omega/m$$

$$L' = \frac{\mu}{2\pi}\ln(b/a) = \frac{1.2566\times10^{-6}}{2\pi}\cdot\ln(2) = 1.3863\times10^{-7}\ H/m$$

$$G' = \frac{2\pi\sigma}{\ln(b/a)} = \frac{2\pi(10^{-3})}{\ln(2)} = 9.065\times10^{-3}\ S/m$$

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} \quad \varepsilon = \varepsilon_r\cdot\varepsilon_0 = \frac{2\pi(2.25)(8.854\times10^{-12})}{\ln(2)} = 1.806\times10^{-10}\ F/m$$

8.3) Parralell Plate: $f_o = 10^9$ Hz

$W_c = 1.5\,cm$ $\quad d_i = 0.2\,cm$

copper: $\mu_c = \mu_o$ $\quad \sigma_c = 5.8 \times 10^7$

insulator: $\varepsilon_r = 2.6$, $\mu = \mu_o$, $\sigma \simeq 0$

$$R_s' = \frac{2R_s}{W_c} = \frac{2}{W_c}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2}{(1.5\times10^{-2})}\sqrt{\frac{\pi(10^9)(4\pi\cdot10^{-7})}{5.8\times10^7}} = 1.091\ \Omega/m$$

$$L' = \frac{\mu d}{W} = \frac{(4\pi\times10^{-7})(0.2\times10^{-2})}{(1.5\times10^{-2})} = 1.6755\times10^{-7}\ H/m$$

$$G' = \frac{\sigma W}{d} = \frac{(0)(1.5\times10^{-2})}{(0.2\times10^{-2})} = 0$$

$$C' = \frac{\varepsilon W}{d} = \frac{2.6(8.854\times10^{-12})(1.5\times10^{-2})}{0.2\times10^{-2}} = 1.7265\times10^{-10}\ F/m$$

8.5) From 8.2) $f_o = 1 \times 10^9$ Hz

$R' = 0.7878\ \Omega/m$ $\quad L' = 1.3863 \times 10^{-7}$ H/m

$G' = 9.065 \times 10^{-3}$ S/m $\quad C' = 1.806 \times 10^{-10}$ F/m

---

$$R' + j\omega L' = 0.7878 + j(2\pi \cdot 10^9)(1.3863 \times 10^{-7}) = 0.7878 + j871.04$$

$$= 871.04\, e^{j1.5696}$$

$$G' + j\omega C' = 9.065 \times 10^{-3} + j2\pi \cdot 10^9 \cdot 1.806 \times 10^{-10} = 9.065 \times 10^{-3} + j1.1347$$

$$= 1.1348\, e^{j1.5628}$$

$$\sqrt{(R' + j\omega L')(G' + j\omega C')} = \sqrt{(988.46)\, e^{j(1.5696 + 1.5628)}} = \sqrt{9.8846 \times 10^2}\, e^{j\frac{3.1324}{2}}$$

$$= 31.44\, e^{j1.5662} = 0.14451 + j31.439$$

$\therefore \quad \alpha = 0.1445$ 1/m $\quad \beta = 31.439$ rad/m

$$Z_o = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{871.04}{1.1348}}\, e^{j(1.5696 - 1.5628)} = 27.705\, e^{j0.0068}$$

$\simeq 27.7 + j0.1$ - the imaginary part is sensitive to how many digits you carry through the problem.

$$v_p = \frac{\omega}{\beta} = \frac{2\pi f_o}{\beta} = \frac{2\pi(10^9)}{31.439} \simeq 2 \times 10^8 \text{ m/s}$$