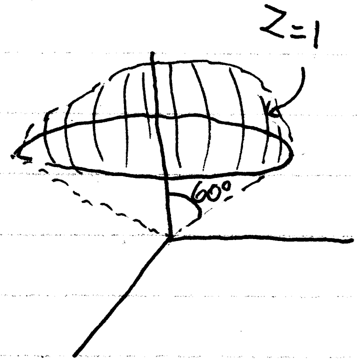
Phys. 221 Assignment 12

10.3) $F(\theta, \phi) = \begin{cases} 1, & 0^\circ \leq \theta \leq 60^\circ, \quad \phi \in \{0, 2\pi\} \\ 0, & \text{elsewhere} \end{cases}$



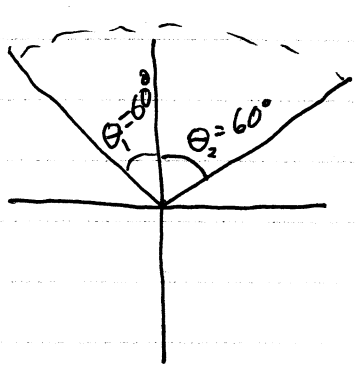
a) direction of maximum propagation:
$F(\theta, \phi)$ is uniform over the cone given by $\theta < 60^\circ$
so the direction is any direction in that cone.

b) $$D \equiv \frac{F_{max}}{F_{av}} = \frac{1}{\frac{1}{4\pi} \iint_{4\pi} F(\theta,\phi)\, d\Omega} = \frac{4\pi}{\int_0^{\pi/3} \int_0^{2\pi} \sin\theta \, d\theta \, d\phi}$$

$$= \frac{4\pi}{(2\pi)\left(\cos(0) - \cos(\pi/3)\right)} = \frac{2}{(1 - 1/2)} = 4$$

c) $$\Omega_p = \iint F(\theta,\phi)\, d\Omega = \int_0^{\pi/3} \sin\theta \, d\theta \int_0^{2\pi} d\phi = (1 - 1/2)(2\pi) = \pi$$

d) Half power beam width in x, z plane:



doesn't drop to 'half', so take point where it goes from 1 to 0,

$\therefore \ \beta = 120^\circ$

10.5) Given: 2 m - length of antenna ℓ Wire is copper

1×10^{-3} m - radius of antenna

$f = 1\,MHz = 10^6\,Hz \Rightarrow \ell/\lambda = \ell f/c = \frac{2}{3}\times10^{-2} < \frac{1}{50}$, so short antenna.

a) $P_{rad} = \frac{1}{2} I_0^2 R_{rad} = \frac{4\pi R^2 S_{max}}{D} \Rightarrow R_{rad} = 80\pi^2(\ell/\lambda)^2 = 0.0351$

look up expression for S_{max} in book and D

$\Rightarrow R_{loss} = \frac{\ell}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = 0.083$

$\Rightarrow \xi = \frac{R_{rad}}{R_{rad}+R_{loss}} = \frac{0.0351}{0.0351+0.083} = 29.7\%$

b) $G = \xi \cdot D = (0.297)\cdot(1.5) = 0.4456$

$G_{dB} = 10\log_{10}(0.4456) = -3.51\,dB$

c) $P_{rad} = 40\pi I_0^2 \left(\frac{\ell}{\lambda}\right)^2 \Rightarrow I_0 = \frac{\lambda}{2\pi\ell}\left(\frac{P_{rad}}{10}\right)^{1/2} = 33.762$

$P_{TOT} = P_{rad} + P_{loss} = 20 + \frac{1}{2} I_0^2 R_{loss} = 20 + \frac{1}{2}(33.762)^2(0.083)$

$= 67.3\,W$

10.(1) 1 m long $\frac{1}{2}\lambda$ dipole, 150 MHz:

a) $P_{rad} = \frac{1}{2} I_o^2 R_{rad} = 36.6 I_o^2$ ⇒ from page 365

$\Rightarrow R_{rad} = 2 \cdot 36.6 = 73.2$

$$R_{loss} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} \xrightarrow{l=\lambda/2} R_{loss} = \frac{c}{4f\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$$

$$= \frac{3 \cdot 10^8}{4 \cdot (150 \cdot 10^6)\pi \cdot (1 \cdot 10^{-3})}\sqrt{\frac{\pi(150 \cdot 10^6)(4\pi \cdot 10^{-7})}{5.8 \cdot 10^7}} = 0.5086$$

$$\xi = \frac{R_{rad}}{R_{rad} + R_{loss}} = 0.9931$$

b) $G = \xi \cdot D = 0.9931 \cdot (1.64)$ ⇒ from pg. 365

$= 1.63$

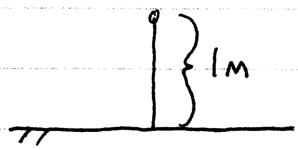
$G_{dB} = 10 \log_{10}(1.63) = 2.1 \text{ dB}$

c) $I_o = \sqrt{\frac{P_{rad}}{36.6}} = \sqrt{\frac{20}{36.6}} = 0.73922 \text{ A}$

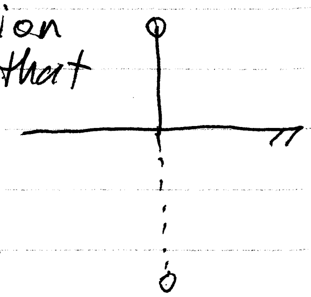
$P_{TOT} = P_{rad} + P_{loss} = 20 + \frac{1}{2} I_o^2 R_{loss} = 20.1 \text{W}$

Very efficient antenna!

10.14)



by Image theorem, radiation Pattern will look like that of a $2\cdot l$ antenna!



For a $2\cdot l$ antenna $\Rightarrow \frac{l}{\lambda} = \frac{lf}{c} = \frac{2m\cdot 10^6 Hz}{3\cdot 10^8}$

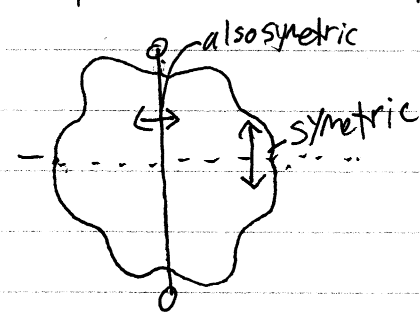
$= \frac{2}{3}\cdot 10^{-2} < \frac{1}{50}$ So we can consider this a <u>short</u> antenna.

a) $P_{rad} = 4\pi R^2 \frac{S_{max}}{D}$ $\Rightarrow$ this is the same as (10.5) except, D changes. (S_{max} is still the same)

$D = \frac{S_{max}}{S_{av}}$ $\Rightarrow$ S_{max} remains the same, but S_{av} will change.

$S_{av} = \cancel{\frac{1}{4\pi R^2}} \cancel{\iint d\theta d\phi}\ \frac{1}{4\pi R^2} \iint_S S(\theta,\phi)$

for the case of a hertzian dipole, as in (10.5) the radiation pattern is symetric about the center of the antenna:



So we can split up the integral for S_{av} into two parts; ~~which are~~

$$S_{av} = \frac{1}{4\pi R^2}\left(\iint_{S\ \text{upper } 1/2 \text{ sphere}} S(\theta,\phi) + \iint_{S\ \text{lower } 1/2 \text{ sphere}} S(\theta,\phi)\right)$$

but since it is symetric, both $\iint_S$ give the same value:

$$\therefore\ S_{av} = \frac{1}{4\pi R^2}\, 2\cdot \iint_{S\ 1/2 \text{ sphere}} S(\theta,\phi)$$

Now for our antenna, we write

$$S_{av} = \frac{1}{4\pi R^2}\left(\iint_{upper} S(\theta,\phi) + \iint_{lower} S(\theta,\phi)\right)$$ but for our antenna there is no radiation in the conductor so $\iint_{lower} S(\theta,\phi) = 0$

$$= \frac{1}{4\pi R^2} \iint_{1/2\,sphere} S(\theta,\phi)$$

$\therefore$ S_{av}(1m long antena over conductor) $= \frac{1}{2} S_{av}$(2m long Hertian dipole)

$\therefore$ Since $D \propto \frac{1}{S_{av}}$, $D_{1m,\,conductor} = 2 \cdot D_{2m\ long\ Hz\ dipole}$

$\therefore$ $D = 3.$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad} = \frac{4\pi R^2 S_{max}}{D} = \frac{4\pi R^2}{D} \frac{15\pi I_o^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 40\pi^2\left(\frac{l}{\lambda}\right)^2 = 0.017546$$

$$R_{loss} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{\cancel{2}\,1}{2\pi(0.5\cdot 10^{-3})}\sqrt{\frac{\pi\, 10^{+6}\, 4\pi\cdot 10^{-7}}{3.5\cdot 10^{7}}} = \frac{0.02138}{2} = 0.01069$$

$$\xi = \frac{R_{rad}}{R_{rad}+R_{loss}} = \cancel{0.45107}\ 0.6214$$

b) $G = \xi \cdot D = 0.6214 \cdot 3 = 1.8642 = 2.7049\ dB$

c) $P_{Rad} = \frac{1}{2} I_o^2 R_{rad} \Rightarrow I_o = \sqrt{\frac{2P_{rad}}{R_{rad}}} = 47.746\ A$

$P_{tot} = 20w + \frac{1}{2} I_o^2 R_{loss} = 32.3W$