Phys221 Assignment #1

Due: 10:20am Friday May 8, 2009

1. Textbook page 44, # 2.5.

2. A solid sphere with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ. Figure A below depicts its cross section on the x-y plane. The centre of the sphere is at the origin.
(a) Determine the electric field E at point P(2R/3, 0, 0).
(b) A spherical cavity of radius R/2 is created as shown in figure B. The centre of the cavity is located at (R/2, 0, 0). Determine the electric field inside the cavity at point P(2R/3, 0, 0).

(c) If the cavity inside the sphere has a radius w and the centre of the cavity is located at (a, b, 0), find the electric field at a point (x, y, 0) inside the cavity.

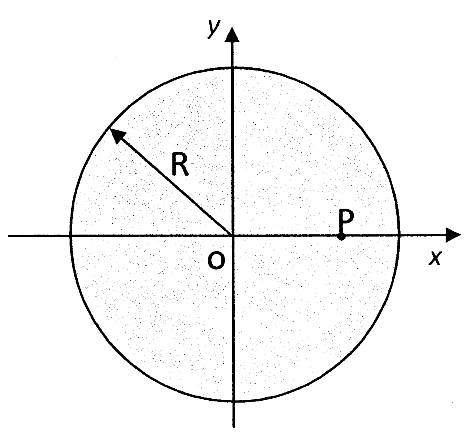


Figure A

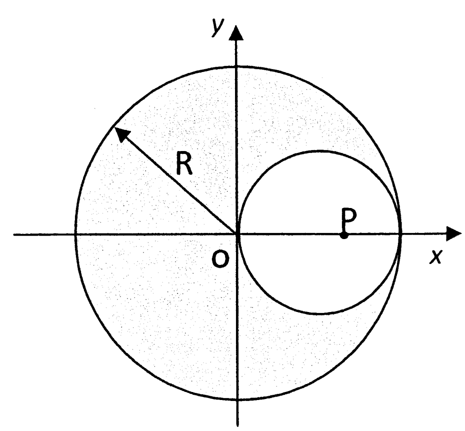


Figure B

Phys 221

Assignment 1. Solutions.

1. Page 44. # 2.5.

Given: $\vec{A} = \hat{x} + \hat{y}2 - \hat{z}3$, $\vec{B} = \hat{x}3 - \hat{y}4$, $\vec{C} = \hat{y}3 - \hat{z}4$.

(a). $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$

$$\hat{a} = \frac{\vec{A}}{A} = \frac{1}{\sqrt{14}}\left(\hat{x} + \hat{y}2 - \hat{z}3\right)$$

(b). $$\vec{B}\cdot\hat{C} = \vec{B}\cdot\frac{\vec{C}}{C} = \frac{B_xC_x + B_yC_y + B_zC_z}{\sqrt{C_x^2 + C_y^2 + C_z^2}} = \frac{(-4)(3)}{\sqrt{3^2+(-4)^2}} = -\frac{12}{5}$$

(c) $\vec{A}\cdot\vec{C} = AC\cos\theta_{AC}$.

$$\cos\theta_{AC} = \frac{\vec{A}\cdot\vec{C}}{AC} = \frac{(2)(3) + (-3)(-4)}{\sqrt{14}\cdot 5} = 0.9621$$

$$\theta_{AC} = \cos^{-1} 0.9621 = 15.8^\circ$$

(d) $$\vec{A}\times\vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 0 & 3 & -4 \end{vmatrix} = \hat{x}(-8+9) + \hat{y}(-4) + \hat{z}(3)$$

$$= \hat{x} + \hat{y}4 + \hat{z}3.$$

(e). $\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} 1 & 2 & -3 \\ 3 & -4 & 0 \\ 0 & 3 & -4 \end{vmatrix}$

$= 16 - 2(-4)(3) + (-3)(3)(3)$

$= 16 + 24 - 27$

$= 13$.

(f). $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

$= (\hat{x}3 - \hat{y}4)\left[2\times 3 + (-3)(-4)\right] - (\hat{y}3 - \hat{z}4)(3-8)$

$= (\hat{x}3 - \hat{y}4)(18) + (\hat{y}3 - \hat{z}4)5$

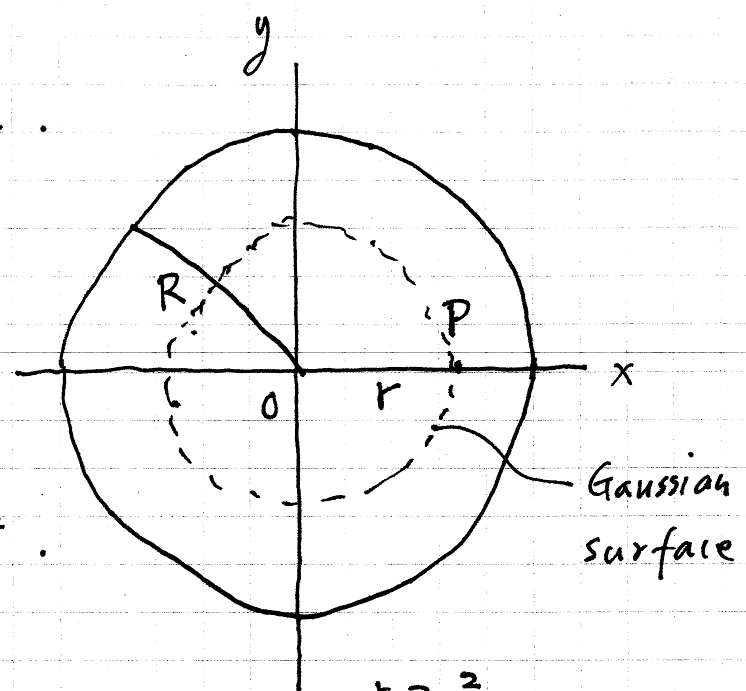
$= \hat{x}54 - \hat{y}57 - \hat{z}20$.

(g). $\hat{x}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ 3 & -4 & 0 \end{vmatrix} = \hat{z}(1)(-4) = -\hat{z}4$.

(h) $(\vec{A}\times\hat{y})\cdot\hat{z} = \hat{z}\cdot(\vec{A}\times\hat{y}) = \begin{vmatrix} 0 & 0 & 1 \\ 1 & -2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = 1$.

(a). Gaussian surface:

spherical surface $r = \frac{2}{3}R$.



$r = \frac{2}{3}R$.

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0}.$$

by symmetry: $\vec{E} = E\hat{R}$.

On Gaussian surface: $E = \text{const}$.

$$\therefore \oint \vec{E} \cdot \vec{ds} = \oint E\, ds$$

$$= E \int ds = E \cdot 4\pi r^2.$$

(surface area of a sphere: $4\pi r^2$)

$$\therefore \quad E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Q — charge inside the Gaussian surface.

$$\therefore \quad Q = \rho \cdot V = \rho \cdot \frac{4}{3}\pi r^3$$

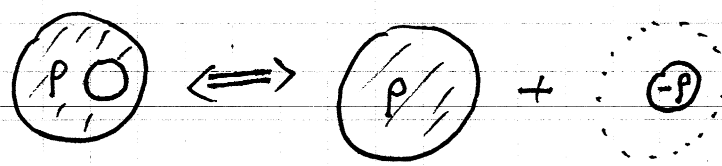
$$\therefore \quad E = \frac{\rho r}{3\varepsilon_0} = \frac{2\rho R}{9\varepsilon_0}$$

at point P: $\vec{E} = \hat{x}\frac{2\rho R}{9\varepsilon_0}$.

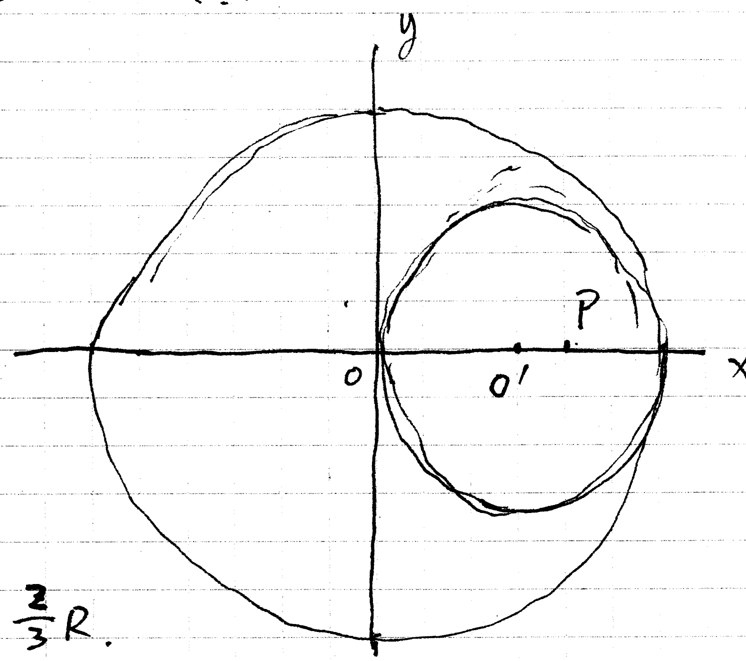
at any point inside sphere: $\vec{E} = \frac{\rho \vec{r}}{3\varepsilon_0}$

(b). Creating a cavity is equivalent to adding a negatively charged object.

i.e.:



$\therefore \quad \vec{E} = E_{\rho} + E_{-\rho}$



at point P: $\quad \vec{E} = E_x \hat{x}$

$$E_x = E_x^{+} + E_x^{-}$$

$$= \frac{\rho r^{+}}{3\varepsilon_0} + \frac{\rho r^{-}}{3\varepsilon_0}$$

r^{+} — from o to P. $r^{+} = \frac{2}{3}R$.

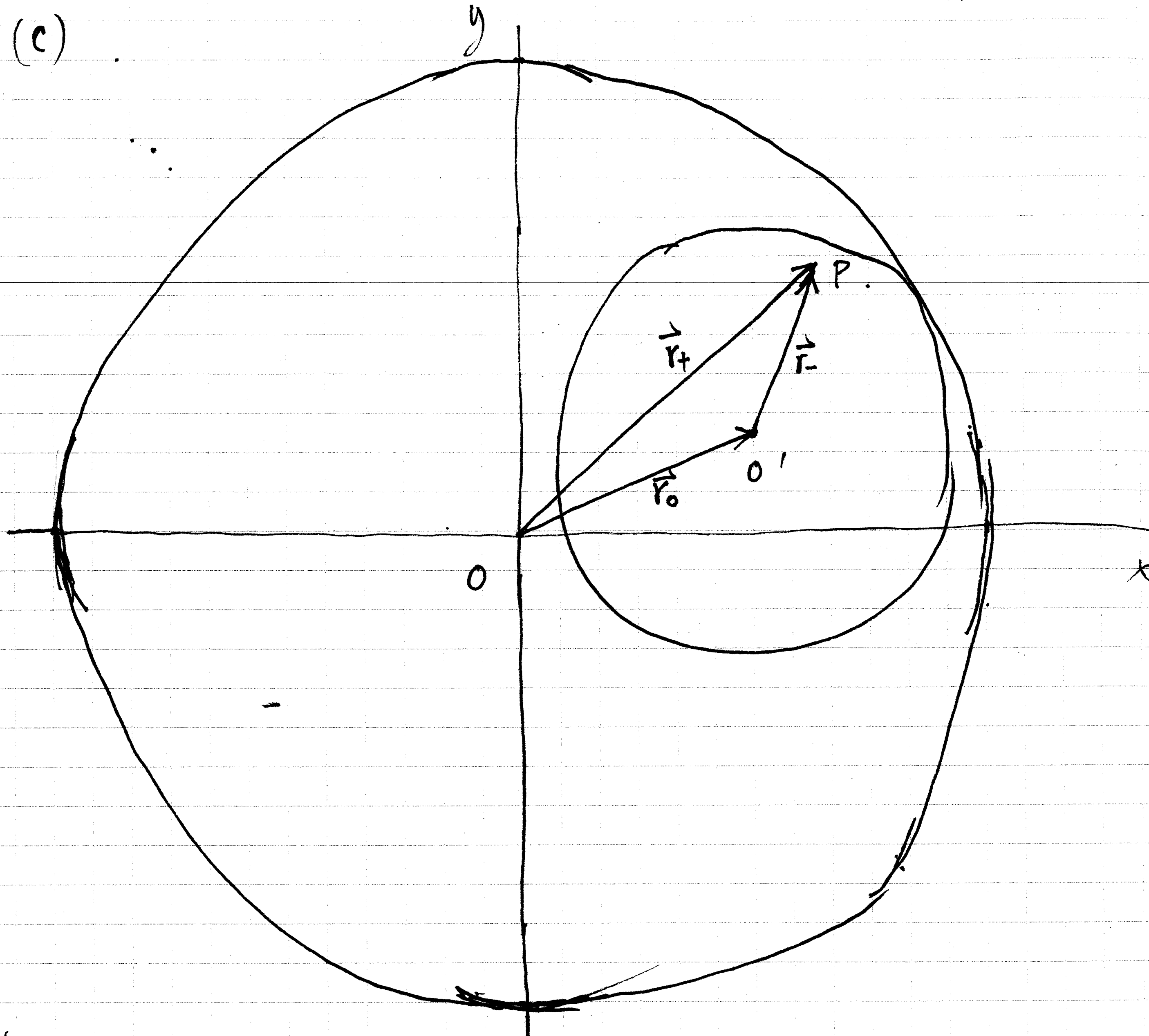
r^{-} — from o' to P: $r^{-} = \left(\frac{2}{3}R - \frac{1}{2}R\right) = \frac{1}{6}R$.

$$\therefore \quad E_x = \frac{\rho}{3\varepsilon_0}\left(\frac{2}{3}R - \frac{1}{6}R\right) = \frac{\rho R}{6\varepsilon_0}$$

at P:

$$\therefore \quad \vec{E} = \hat{x}\,\frac{\rho R}{6\varepsilon_0}.$$

(c)



$\vec{E}$ field at P:

$$\vec{E} = \vec{E}_\rho + \vec{E}_{-\rho} = \frac{\rho}{3\varepsilon_0}\vec{r}_+ - \frac{\rho}{3\varepsilon_0}\vec{r}_- = \frac{\rho}{3\varepsilon_0}(\vec{r}_+ - \vec{r}_-)$$

but: $\vec{r}_+ - \vec{r}_- = \vec{r}_0$ = Position vector of centre of cavity.

$$\therefore \quad \vec{E} = \frac{\rho \vec{r}_0}{3\varepsilon_0}$$