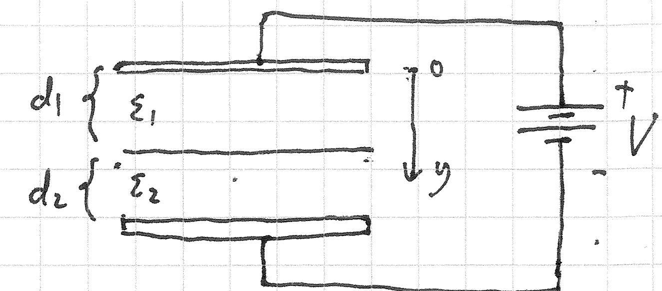
Phys 221

Assignment #5 solutions.

4.54.




a). $\vec{E}_1 = \hat{y}\, E_1$, $\quad \vec{E}_2 = \hat{y}\, E_2$.

$V_1 = E_1 d_1$, $\quad E_1 = \frac{V_1}{d_1}$

$$E_2 = \frac{V_2}{d_2}$$

Boundary condition: $\quad D_1 = D_2 \qquad (\vec{D}_1 = \hat{y}\, D_1,\ \vec{D}_2 = \hat{y}\, D_2)$

i.e. $\quad E_1 \varepsilon_1 = \varepsilon_2 E_2$.

$$V = V_1 + V_2 = E_1 d_1 + E_2 d_2$$

$$V = E_1 d_1 + \frac{E_1 \varepsilon_1}{\varepsilon_2} d_2 . = E_1 \left(d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2\right)$$

$$\therefore \quad E_1 = \frac{V}{d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2} = \frac{\varepsilon_2 V}{\varepsilon_2 d_1 + \varepsilon_1 d_2}$$

$$E_2 = \frac{V}{\frac{\varepsilon_2}{\varepsilon_1} d_1 + d_2} = \frac{\varepsilon_1 V}{\varepsilon_2 d_1 + \varepsilon_1 d_2}.$$

b). $$W_{e1} = \frac{1}{2} \varepsilon_1 E_1^2 \cdot A \cdot d_1 = \frac{1}{2} \varepsilon_1 A d_1 \cdot \frac{\varepsilon_2^2 V^2}{(\varepsilon_2 d_1 + \varepsilon_1 d_2)^2}$$

$$W_{e2} = \frac{1}{2} \varepsilon_2 E_2^2 \cdot A \cdot d_2 = \frac{1}{2} \varepsilon_2 A d_2 \frac{\varepsilon_1^2 V^2}{(\varepsilon_2 d_1 + \varepsilon_1 d_2)^2}$$

$$W_e = W_{e1} + W_{e2}$$

$$= \frac{1}{2} \frac{A V^2}{(\varepsilon_2 d_1 + \varepsilon_1 d_2)^2} \cdot \left( \varepsilon_1 d_1 \varepsilon_2^2 + \varepsilon_2 d_2 \varepsilon_1^2 \right)$$

$$= \frac{1}{2} A V^2 \varepsilon_1 \varepsilon_2 \frac{\cancel{d_1 \varepsilon_2 + d_2 \varepsilon_1}}{\cancel{(\varepsilon_2 d_1 + \varepsilon_1 d_2)}^2}$$

$$= \frac{1}{2} A V^2 \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_2 d_1 + \varepsilon_1 d_2} .$$

$$= \frac{1}{2} C V^2 .$$

$$\therefore \quad C = \frac{A \varepsilon_1 \varepsilon_2}{\varepsilon_2 d_1 + \varepsilon_1 d_2} .$$

(c). $C_1 = \varepsilon_1 \frac{A}{d_1}$ , $C_2 = \varepsilon_2 \frac{A}{d_2}$

$$\frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_1 \frac{A}{d_1} \cdot \varepsilon_2 \frac{A}{d_2}}{\varepsilon_1 \frac{A}{d_1} + \varepsilon_2 \frac{A}{d_2}} \cdot \left( \frac{d_1 d_2}{d_1 d_2} \right)$$

$$= \frac{\varepsilon_1 \varepsilon_2 A}{\varepsilon_1 d_2 + \varepsilon_2 d_1}$$

$$= C .$$

4.55. $\varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = 4\varepsilon_0$.

$d_1 = 1\times10^{-2}\,m$, $d_2 = 3\times10^{-2}\,m$.

$A = 5\times10^{-2}\times2\times10^{-2} = 1\times10^{-3}\,m^2$.

$$C = \frac{A\varepsilon_1\varepsilon_2}{\varepsilon_2 d_1 + \varepsilon_1 d_2} = \frac{(2\varepsilon_0)(4\varepsilon_0)\times10^{-3}}{4\varepsilon_0\times1\times10^{-2} + 2\varepsilon_0\times3\times10^{-2}} = \frac{8\times10^{-3}\varepsilon_0}{1.0\times10^{-1}}$$

$$= 8\times10^{-2}\varepsilon_0$$

$$= 8\times10^{-2}\times 8.85\times10^{-12}$$

$$= 70.8\times10^{-14}\ F$$

$$\approx 7.1\times10^{-13}\ F.$$

• 4.56

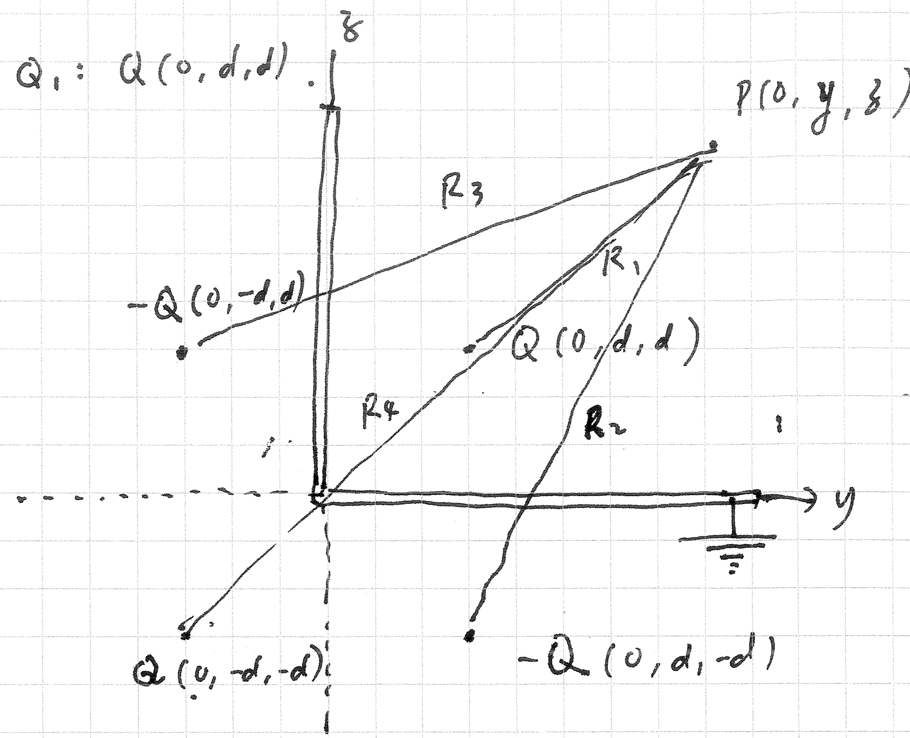
(a) given charge: $Q_1$: $Q(0,d,d)$.

Image charges

$Q_2$: $-Q\ (0, d, -d)$

$Q_3$: $-Q\ (0, -d, d)$

$Q_4$: $Q\ (0, d, d)$.




(b)

$$V = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2} + \frac{Q_3}{4\pi\varepsilon_0 R_3} + \frac{Q_4}{4\pi\varepsilon_0 R_4}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{(y-d)^2+(z-d)^2}} - \frac{1}{\sqrt{(y-d)^2+(z+d)^2}} + \frac{1}{\sqrt{(y+d)^2+(z+d)^2}} - \frac{1}{\sqrt{(y+d)^2+(z-d)^2}}\right)$$