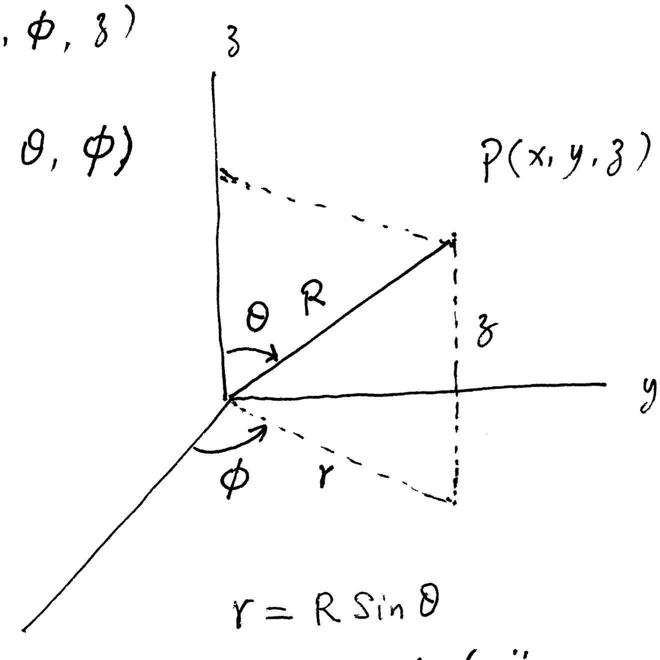
Physics 221. Lecture 2.

- Orthogonal Coordinate Systems.
  - coordinates are perpendicular to each other.

The position of a point in 3-D space is defined by 3 ind. variables.

- Cartesian (x, y, z)
- Cylindrical (r, ϕ, z)
- Spherical. (R, θ, ϕ)



$r = R \sin\theta$

zenith: θ — "latitude"

azimuth: ϕ — "longitude".

— Why do we need different coordinate systems?

Simplify the description of problems ~~when~~ by choosing a coordinate system that best fits the geometry of the problem.

e.g: Coaxil cable : use cylindrical system.

- Table 2-1 Summary of vector relations.
  (Properties of Cartesian, Cylindrical and Spherical systems).

Note 1. Base vectors.

$\hat{x}$, $\hat{y}$, $\hat{z}$

$\hat{r}$, $\hat{\phi}$, $\hat{z}$

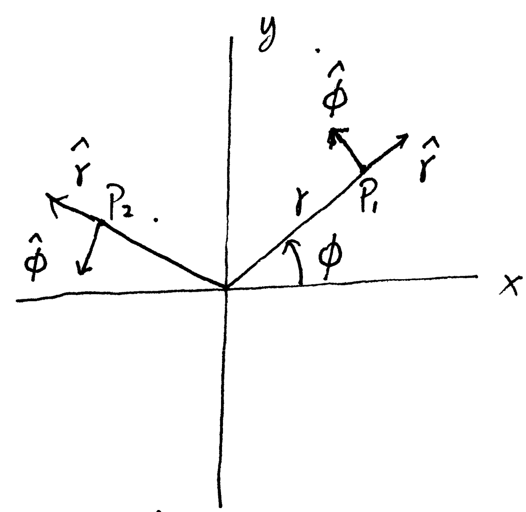
$\hat{R}$, $\hat{\theta}$, $\hat{\phi}$

[redacted].

$\hat{r}$, $\hat{\phi}$, $\hat{R}$, $\hat{\theta}$ — not constant!

Direction varies with location.

e.g: $\hat{r}$, $\hat{\phi}$, vewed on x-y plane. (Polar coord.).



$\hat{r}$ (at P_1) $\neq$ $\hat{r}$ (at P_2)

$\hat{\phi}$ (at P_1) $\neq$ $\hat{\phi}$ (at P_2).

Note 2. Differential length $d\vec{l}$

$\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$. — Cartesian

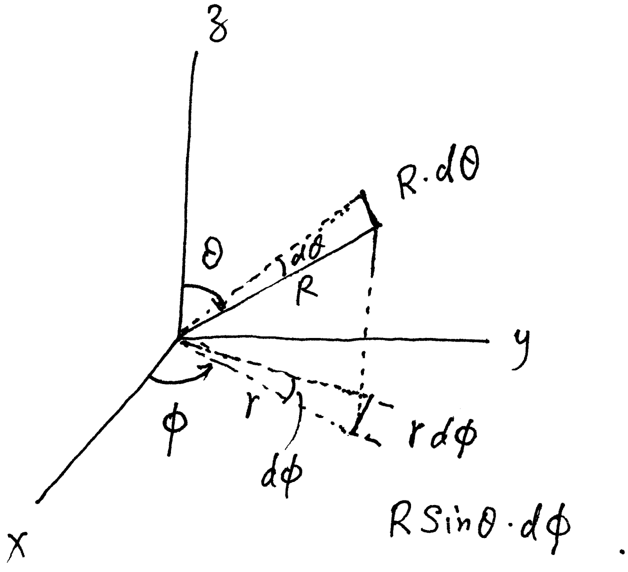
$\hat{r}\,dr + \hat{\phi}\,r\,d\phi + \hat{z}\,dz$. — Cylindrical

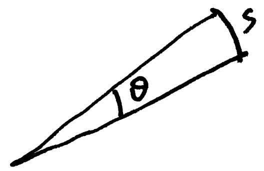
$\hat{R}\,dR + \hat{\theta}\cdot R\,d\theta + \hat{\phi}\,R\sin\theta\cdot d\phi$ — spherical.

watch out : ϕ, θ are angles.

$\therefore$ $d\phi$, $d\theta$ are not lengths!

but, $R\,d\theta$, is, and so are $r\,d\phi$, $R\sin\theta\,d\phi$.





arc length : $s = r\cdot\theta$ if θ is in radians.

$ds = r\,d\theta$.

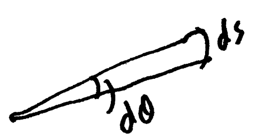


Table 2-2. Coordinate Transformation Relations.

Matrix Form. (optional)

e.g. Cartesian to Cylindrical. given (x, y, z), want (r, ϕ, z) } for a vector $\vec{A}$

$$\begin{pmatrix} A_r \\ A_\phi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

$$A_{cyl.} = \left(T_{cart-cyl.} \right) A_{cart}.$$

Cylindrical to Cartesian.

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_r \\ A_\phi \\ A_z \end{pmatrix}$$

$$A_{cart.} = \left(T_{cyl.-cart} \right) A_{cyl.}$$

$$T_{cart-cyl} = T^{-1}_{cyl.-car.} \quad \text{(inverse matrix)}$$

- Gradient of a scalar field. V

$$\text{grad } V \triangleq \nabla V$$

$$\triangleq \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z}.$$

— It's a vector.

— Represents the magnitude and direction of the max space rate of change of V.

∇ — "del". is a vector operator.

$$\nabla \triangleq \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}.$$

e.g. Electrostatics. V — electric potential.

$\vec{E}$ — electric field.

$$\boxed{\vec{E} = -\nabla V}$$

└ "$-$" sign — $\vec{E}$ direction is from higher V to lower V.

("max" decreasing direction)

e.g.: $E_x = -\dfrac{\partial V}{\partial x}$

$E_y = -\dfrac{\partial V}{\partial y}$