Ref: 3-1, 3-2, 4-3, 4-4, 4-5.

Physics 221.

Lecture 3.

Assignment #2: 3.1, 3.12, 3.17. Due next friday.

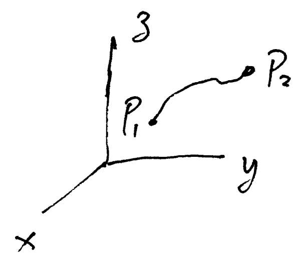
• Differential of V: $V = V(x, y, z)$

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} \cdot dy + \frac{\partial V}{\partial z} dz$$

$$= \nabla V \cdot d\vec{l}$$

Difference of V between locations P_1 and P_2.

$$\int_{P_1}^{P_2} dV = \int_{P_1}^{P_2} \nabla V \cdot d\vec{l}$$



$$V_2 - V_1 = \int_{P_1}^{P_2} (-\vec{E}) \cdot d\vec{l}$$

i.e.

$$\boxed{V_2 - V_1 = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}} \qquad (4.39)$$

$\vec{E}$ — electric field. $\vec{E} = \vec{F}/q$.

= electric force per unit charge.

V — electric potential.

= potential energy per unit charge.

(4-39) can be rewritten as

$$\int_{P_1}^{P_2} \vec{E}\cdot d\vec{\ell} = -(V_2 - V_1) = -\Delta V$$

Meaning:

work done by $\vec{E}$-field = Loss of potential energy.

($\vec{E}$ - field is conservative).

Along a closed path:

$$\oint \vec{E}\cdot d\vec{\ell} = 0 \qquad (\because V_P - V_P = 0) \qquad (4\text{-}40)$$

(circulation of $\vec{E} = 0$).

Please read 3-1.1 and 3-1.2.

(Gradient operator in cylindrical and spherical coordinates)

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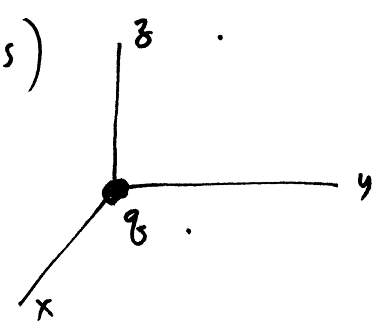
e.g.: electric potential of a point charge q located at the origin.

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{R}.$$

(spherical coordinates)

ind. of θ, ϕ.



Electric field:

$$\vec{E} = -\nabla V = -\hat{R}\frac{\partial V}{\partial R} = \hat{R}\frac{q}{4\pi\varepsilon_0}\frac{1}{R^2}$$

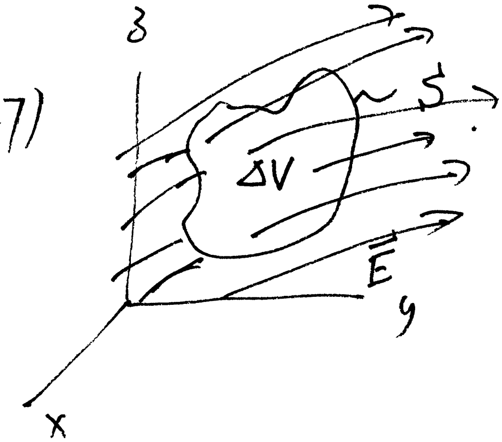
- Divergence of a vector field.

Given a vector field $\vec{E}(x, y, z)$, its divergence is.

$$\nabla \cdot \vec{E} \equiv \text{div}\, \vec{E} \equiv \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \qquad (3\text{-}28)$$

OR: (equivalent definition). (p.53-54).

$$\nabla \cdot \vec{E} = \lim_{\Delta V \to 0} \frac{\oint \vec{E} \cdot d\vec{s}}{\Delta V}. \qquad (3\text{-}27)$$



Meaning.

① $\nabla \cdot \vec{E}$ is a scalar

②. It's the total outward flux of $\vec{E}$ per unit volume.

③. when $\nabla \cdot \vec{E} > 0$, it's a source

$\nabla \cdot \vec{E} < 0$ it's a sink.

④ The magnitude of $\nabla \cdot \vec{E}$ represents the strength of the source (or sink).

e.g. For an electric field $\vec{E}$.

$$\nabla \cdot \vec{E} = \lim_{\Delta V \to 0} \frac{\frac{Q}{\varepsilon_0}}{\Delta V} = \frac{1}{\varepsilon_0} \frac{Q}{\Delta V}\Big|_{\Delta V \to 0} = \frac{\rho}{\varepsilon_0}.$$

ρ — charge per unit volume. (charge density — strength of source)

Meaning: $+q$ — source of $\vec{E}$.

$-q$ — sink of $\vec{E}$.

e.g: For a magnetic field. $\vec{B}$.

$$\nabla \cdot \vec{B} = 0.$$

why? because there is no "magnetic charges".
(monopoles).

- When $\nabla \cdot \vec{B} = 0$ everywhere,

We say that $\vec{B}$ is divergenceless (sourceless)
or solenoidal.

Gauss' Theorem.

- When the volume in (3.27) is not small:

$$\boxed{\int_V \nabla \cdot \vec{E} \cdot dV = \oint_S \vec{E} \cdot d\vec{s}} \qquad 3-30$$

called the divergence theorem.

(V is the volume enclosed by S).

- Calculation of divergence in different coordinate systems:

plug in the formulas !

e.g. 3-3. p. 55.

Read: 4-1, 4-2, 4-3, 4-4, 4-5. (Review)

(during the weekend).

- $\vec{E}$ - field due to a point charge $\vec{q}$ at $(0,0,0)$:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\hat{R}}{R^2}$$

$$\nabla\cdot\vec{E} = \frac{1}{R^2}\frac{d}{dR}\left(R^2\frac{1}{4\pi\varepsilon_0}\frac{1}{R^2}\right) = 0 \quad \text{when } R \neq 0.$$

$$= \infty \quad \text{when } R = 0.$$

$$\left.\begin{aligned} \rho &= 0 \quad \text{when } R = 0 \\ &= \infty \quad \text{when } R = 0 \end{aligned}\right\} \text{ - for a point charge at } R = 0.$$