Phys 221

Lecture 4.

Ref: 3-3, 4-4, 5-4, 5-5;

- Curl of a vector field $\vec{B}$ (rotation)

Definition.

$$\nabla \times \vec{B} \equiv \lim_{\Delta S \to 0} \frac{1}{\Delta S} \left[\hat{n} \oint_C \vec{B} \cdot d\vec{l} \right]_{max} \qquad (3.5).$$

ΔS — area enclosed by contour C

$\hat{n}$ — unit vector normal to ΔS

$\oint_C \vec{B} \cdot d\vec{l}$ — circulation of $\vec{B}$

why max? — the circulation depends on orientation of C.

(need to choose C to maximize the circulation).

- Meaning of $\nabla \times \vec{B}$:

  - it's a vector
  - Max circulation of $\vec{B}$ per unit area.
  - points to the normal direction (right-hand-rule)
  - represents the strength of a vortex source that causes the circulation of $\vec{B}$.
  - when $\nabla \times \vec{B} = 0$, $\vec{B}$ is called an irrotational field. OR conservative field.

- Calculation of $\nabla \times \vec{B}$. (see back cover of Textbook)

e.g. In Cartesian coordinates.

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix}$$

(please go over the cylindrical and spherical cases yourself).

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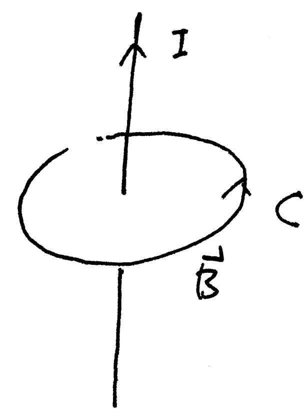
e.g. Magnetostatics



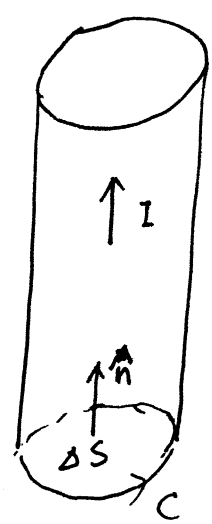
Ampere's law: $\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$

current I is the source of $\vec{B}$.

For a current distribution with current density $\vec{J}$.

$\vec{J}$: direction: $\hat{n}$.

magnitude: $|\vec{J}| = \frac{I}{\Delta S}$.



i.e: $I = \int_{\Delta S} \vec{J} \cdot d\vec{s}$ (4.12).

Then:
$$\nabla \times \vec{B} = \lim_{\Delta S \to 0} \frac{\hat{n} \oint_C \vec{B} \cdot d\vec{l}}{\Delta S}$$

$$= \lim_{\Delta S \to 0} \frac{\hat{n} \mu_0 I}{\Delta S}$$

$$= \mu_0 \vec{J}$$

$$\therefore \quad \nabla \times \vec{B} = \mu_0 \vec{J}$$

Current density $\vec{J}$ represents the strength of vortex source which causes the circulation of $\vec{B}$.

- Stoke's Theorem.

$$\int_S (\nabla \times \vec{B}) \cdot d\vec{S} = \oint_C \vec{B} \cdot d\vec{l}$$

It comes from the definition of $\nabla \times \vec{B}$. (3.5)

When ΔS is not small (becomes S).

Note: Stoke's theorem converts a line integral to a surface integral, and vice versu.

• Vector identities involving curl.

① . $\nabla \times \nabla V = 0$ (for any scalar).

(The curl of a gradient is zero).

e.g.: electrostatics : Since $\vec{E} = -\nabla V$ V - scalar potential of $\vec{E}$-field.

$$\nabla \times \vec{E} = 0 .$$

i.e, $\vec{E}$ is a conservative field.

i.e. $\oint \vec{E} \cdot d\vec{l} = 0$.

(That's true when everything is t-independent).

(but when t-dependence is introduced, $\oint \vec{E} \cdot d\vec{l} \neq 0$. $\vec{E}$-field is no longer conservative)

② . $\nabla \cdot (\nabla \times \vec{A}) = 0$. (for any vector $\vec{A}$)

e.g: magnetostatics : Because $\nabla \cdot \vec{B} = 0$ (no monopole)

∴ $\vec{B}$ can be written as the curl of a vector:

$$\vec{B} = \nabla \times \vec{A} . \qquad (5\text{-}53)$$

$\vec{A}$ — vector potential of $\vec{B}$-field.

Phys221 Assignment #1

Due: 10:20am Friday May 8, 2009

1. Textbook page 44, # 2.5.

2. A solid sphere with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ. Figure A below depicts its cross section on the x-y plane. The centre of the sphere is at the origin.
(a) Determine the electric field E at point P(2R/3, 0, 0).
(b) A spherical cavity of radius R/2 is created as shown in figure B. The centre of the cavity is located at (R/2, 0, 0). Determine the electric field inside the cavity at point P(2R/3, 0, 0).

(c) If the cavity inside the sphere has a radius w and the centre of the cavity is located at (a, b, 0), find the electric field at a point (x, y, 0) inside the cavity.

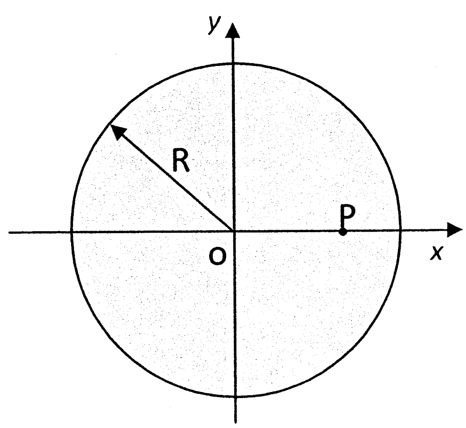


Figure A

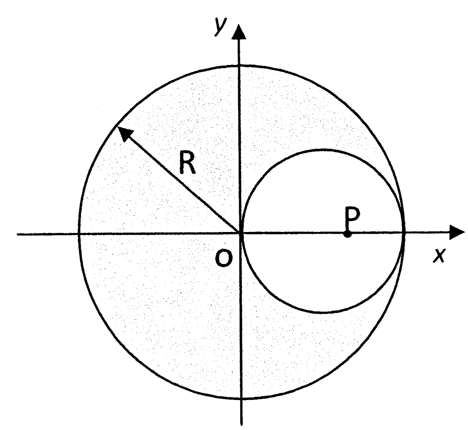


Figure B