Phys 221

Lecture 6.

Room change for Tue Tutorials. to: AQ 5018

Ref: 4-3, 4-4, 4-5.

Assignment. #3. on course webpage.

- Electrostatics. (t-independent)

3 Methods to calculate the $\vec{E}$-field of given charges.

① Coulomb's law.

② Gauss' law.

③ potential.

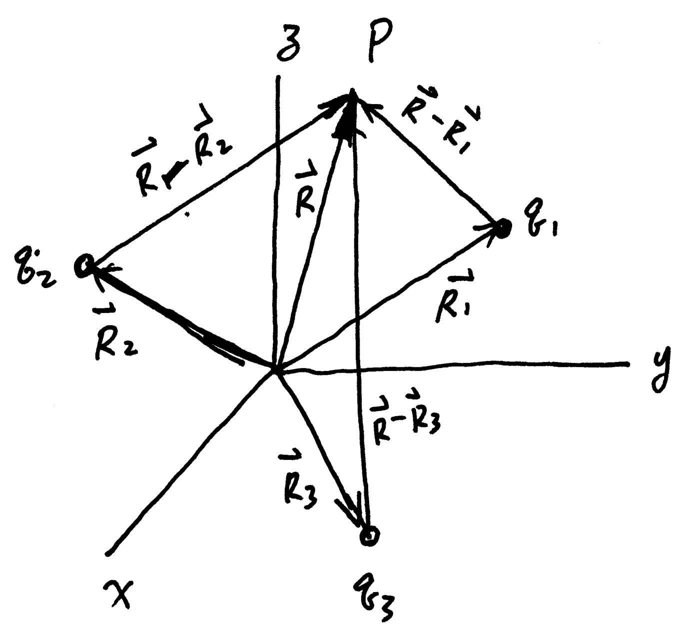
It can be shown: Gauss' law is equivalent to Coulomb's law.

- Coulomb's law

When many charges $q_1, q_2, q_3, \ldots\ldots$ are located at $\vec{R}_1, \vec{R}_2, \vec{R}_3 \cdots$

The $\vec{E}$-field at $\vec{R}$ is:

$$\vec{E}(\vec{R}) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i(\vec{R}-\vec{R}_i)}{\left(|\vec{R}-\vec{R}_i|\right)^3}$$



$(\vec{R}-\vec{R}_i)$ — vector from $\vec{R}_i$ to $\vec{R}$

$\vec{R}_i$ ↑ source point

$\vec{R}$ ↑ field point.

- For continuous charge distribution in volume V' with charge density ρ_v

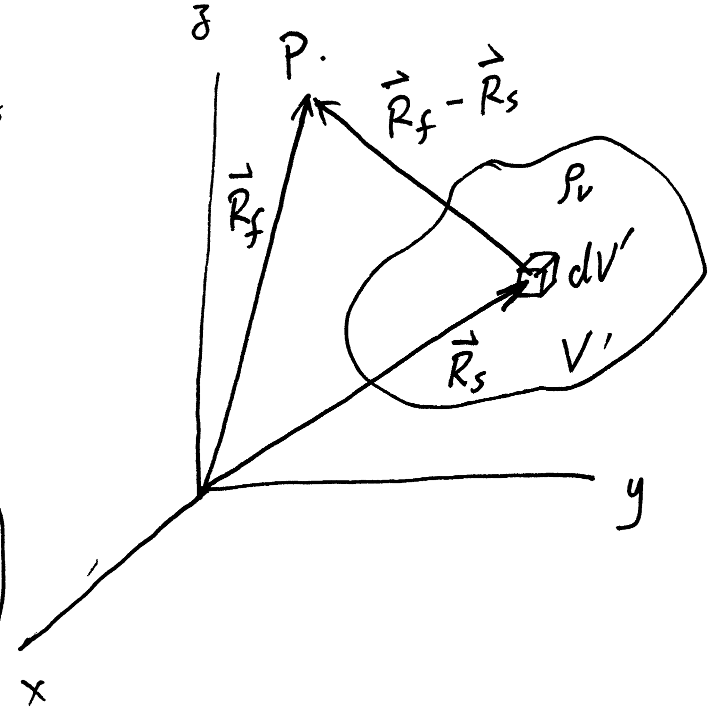
— charge element dq

$$dq = \rho_v \, dV'$$

located at $\vec{R}_s$

— Field point : $\vec{R}_f$

(We want to find $\vec{E}$ at $\vec{R}_f$)



— distance vector from source dq to field point $\vec{R}_f$:

$$(\vec{R}_f - \vec{R}_s) \triangleq \vec{R}'$$

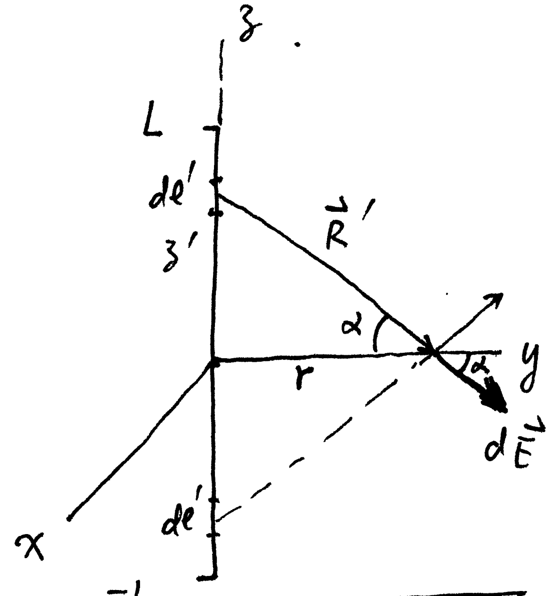
Some books use $(\vec{R} \to \vec{R}')$ ~~We used it~~ briefly ~~last time~~

Total $\vec{E}$-field at $\vec{R}_f$:

$$\vec{E} = \frac{1}{4\pi\varepsilon} \int \frac{\hat{\vec{R}}'}{R'^2} \, dq = \frac{1}{4\pi\varepsilon} \int_{V'} \frac{\hat{\vec{R}}' \rho_v \, dV'}{R'^2} \qquad (4.21a)$$

for a charged wire: $dq = \rho_\ell \, d\ell'$ (4.21c)

for a charged sheet: $dq = \rho_s \, ds'$. (4.21b) e.g. A straight wire of length $2L$ carries a uniform linear charge density ρ_ℓ. Determine the electric field $\vec{E}$ at Point P, a distance r from the wire along the perpendicular bisector.



$R' = \sqrt{r^2 + z'^2}$.

$\cos\alpha = \dfrac{r}{R'}$

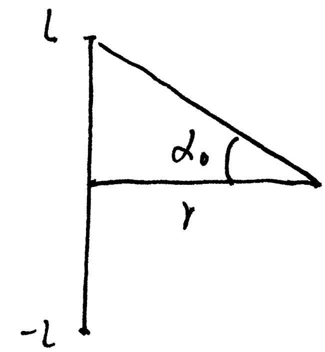
by symmetry, $\vec{E} = \hat{r}\, E_r$.

Coulomb's law:

$$E_r = \frac{1}{4\pi\varepsilon_0}\int \frac{dq \cdot \cos\alpha}{R'^2} = \frac{\rho_\ell}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{r\,dz'}{R'^3}$$

$$= \frac{2\rho_\ell}{4\pi\varepsilon_0}\int_0^L \frac{r\,dz'}{(r^2+z'^2)^{3/2}}$$

$$= \frac{\rho_\ell}{2\pi\varepsilon_0}\left[\frac{z'}{r\sqrt{r^2+z'^2}}\right]_0^L$$



$$= \frac{\rho_\ell L}{2\pi\varepsilon_0 r\sqrt{r^2+L^2}}$$

$$= \frac{\rho_\ell \sin\alpha_0}{2\pi\varepsilon_0 r}.$$

$\dfrac{L}{\sqrt{r^2+L^2}} = \sin\alpha_0$

$$\left(= \frac{Q}{4\pi\varepsilon_0 r\sqrt{r^2+L^2}} \right)$$

When $L \gg r$ (very long wire)

$$\vec{E} = \hat{r} \frac{\rho_\ell}{2\pi \varepsilon_0 r}$$

when $r \gg L$ (very far away)

$$\vec{E} = \hat{r} \frac{\rho_\ell L}{2\pi \varepsilon_0 r^2} = \hat{r} \frac{Q}{4\pi \varepsilon_0 r^2}$$ (just like a point charge).

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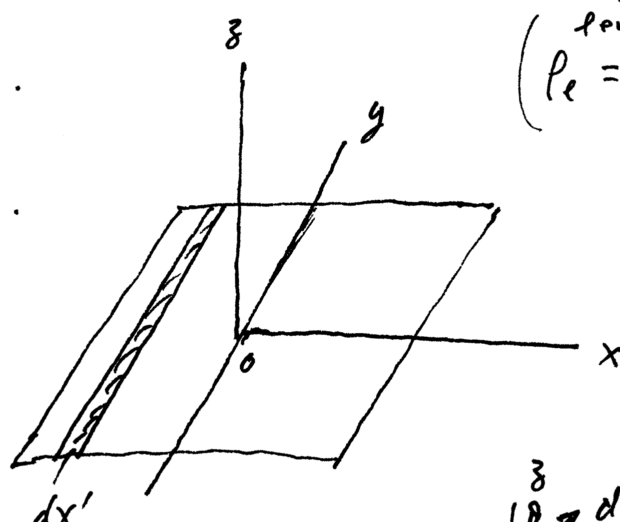
e.g. For a large flat sheet of charge with a uniform area charge density ρ_s (charge per unit area).

Find $\vec{E}$-field at a point on z-axis.

O ~ centre of the area.

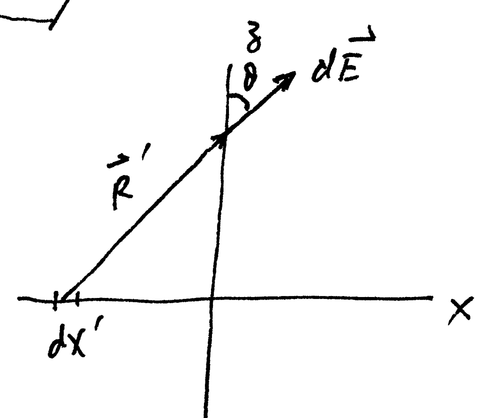
charge per unit length in y
$\left(\rho_\ell = \rho_s \, dx' \right)$



by symmetry, $\vec{E} = \hat{z} E_z$.

$$E_z = \int_{-W}^{W} \frac{\rho_s \, dx' \cos\theta}{2\pi \varepsilon_0 R'}$$

$$= \cancel{\hat{z}} \frac{\rho_s}{2 \varepsilon_0} \quad (\text{as } W \to \infty).$$



Note: $\frac{x'}{z} = \tan\theta$, $dx' = \frac{z \, d\theta}{\cos^2\theta}$, $\frac{z}{R'} = \cos\theta$

$$E_z = 2 \int_0^{\theta_0 \to \frac{\pi}{2}} \frac{\rho_s \, z \cdot \cos\theta \, d\theta}{2\pi \varepsilon_0 R' \cdot \cos^2\theta} = \frac{\rho_s}{\pi \varepsilon_0} \int_0^{\frac{\pi}{2}} d\theta$$

$$= \frac{\rho_s}{2 \varepsilon_0}$$

(R' is the r in the wire case)

- Gauss' Law.

$$\nabla \cdot \vec{D} = \rho_v \qquad (4.26)$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q \qquad (4.29)$$

$\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$ for istropic and uniform medium

ρ_v — free charge per unit volume.

e.g.: A Long straight wire with a uniform linear charge density ρ_ℓ.

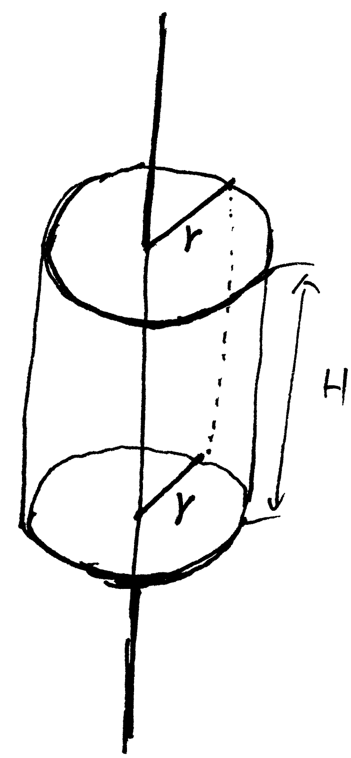
Find $\vec{E}$ field.

Gaussian surface.

by symmetry: $\vec{E} = \hat{r} E_r$.

$\vec{D} \cdot d\vec{s} = 0$ on top and bottom.

E_r = const on lateral surface $\doteq E$



$$Q \doteq \oint_S \vec{D} \cdot d\vec{s} = \oint_S \varepsilon_0 \vec{E} \cdot d\vec{s}$$

use cylindrical coord.

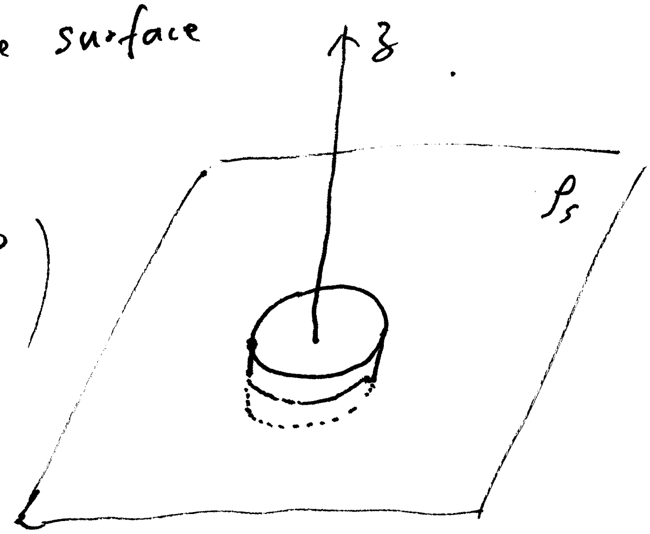
$$= \varepsilon_0 E_r \cdot 2\pi r \cdot H.$$

$$E_r = \frac{Q}{2\pi \varepsilon_0 H} = \frac{\rho_\ell}{2\pi \varepsilon_0}.$$

(same as previous example)

e.g. For a large flat sheet of charge with a uniform area density of charge ρ_s.

Find $\vec{E}$-field neer the surface.

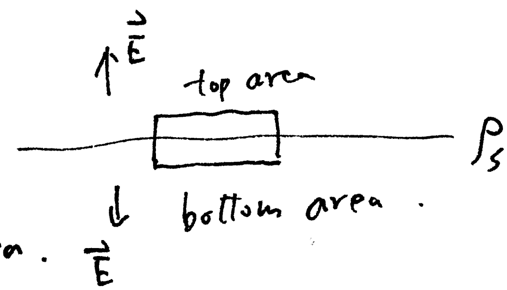


by symmetry.

$E_x = 0$. $E_y = 0$. $\begin{pmatrix} E_r = 0 \\ E_\phi = 0 \end{pmatrix}$

Thin drum-shaped Gaussian surface.

$$\oint_S \vec{D} \cdot \vec{ds} = \varepsilon_0 \oint_S \vec{E} \cdot \vec{ds} = Q.$$



Since $E_r = 0$

$\vec{E} \cdot \vec{ds} = 0$ on lateral area.

on top and bottom : $\vec{E} \parallel \vec{ds}$. also, $E = \text{const}$.

$$\therefore Q = \varepsilon_0 \oint_S \vec{E} \cdot \vec{ds} = \varepsilon_0 \left(E A_{Top} + E A_{bottom} \right)$$

$$= 2 \varepsilon_0 E A.$$

$$E = \frac{Q}{2 \varepsilon_0 A} = \frac{\rho_s}{2 \varepsilon_0}$$ (same as previous example)