

Physics 221

Lecture 8

Ref. 4-5, 4-6, 4-7, 4-8, 4-9, 4-10, 4-11.

Assignment #4.

p. 114, 4.45, 4.52, 4.53.

- Electric scalar potential

from Coulomb's law:

$$V(\vec{R}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{R'} + C$$

(4.48)

- $V(\vec{R})$ can be ~~solved~~ determined by solving its differential equation.

In electrostatics, $\vec{E} = -\nabla V$.

since $\nabla \cdot \vec{E} = \frac{\rho_v}{\epsilon}$

(in medium, ϵ)
 ρ_v - free charge density.

$$\nabla \cdot (\nabla V) = -\frac{\rho_v}{\epsilon}$$

i.e. $\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$

Poisson's equation.
 (4.60)

When there is no free charge, $\rho_v = 0$.

$$\boxed{\nabla^2 V = 0}$$

(Laplace's equation)
 (4.61)

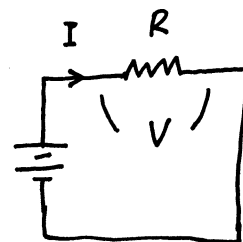
Note: In general,

(4.60) and (4.61) can be solved along with boundary conditions.

• Ohm's law.

In a circuit: $R = \frac{V}{I}$

R - resistance



$$I = \frac{1}{R} \cdot V$$

OR: $I = G V$ G - conductance
(in S). (Siemens)

In a field: $\boxed{\vec{J} = \sigma \vec{E}} \quad (4.67) \quad \text{point form of ohm's law.}$

$\vec{J}$ - current density

σ - conductivity

$\vec{E}$ - electric field

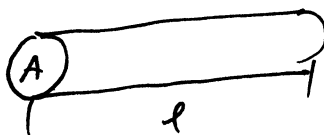
{ Very useful law
in addition to Maxwell's eqs.
for conductors: $\sigma \neq 0$
can be poor conductors! }

- σ - material dependent. (in S/m): Silver 6.2×10^7
copper 5.8×10^7
pure Silicon 4.4×10^{-4}
glass 10^{-12}

$$\sigma = \frac{1}{\rho}$$

ρ - resistivity (Ω/m)

$$R = \rho \frac{l}{A} = \frac{l}{\sigma A}$$



• Perfect conductor: $\sigma = \infty$, $\rho = 0$

$$\therefore \vec{E} = 0$$

• Perfect dielectric: $\sigma = 0$, $\vec{J} = 0$ (perfect insulator)

• Two kinds of charge carriers: $+$ and $-$.
(hole) (electron)

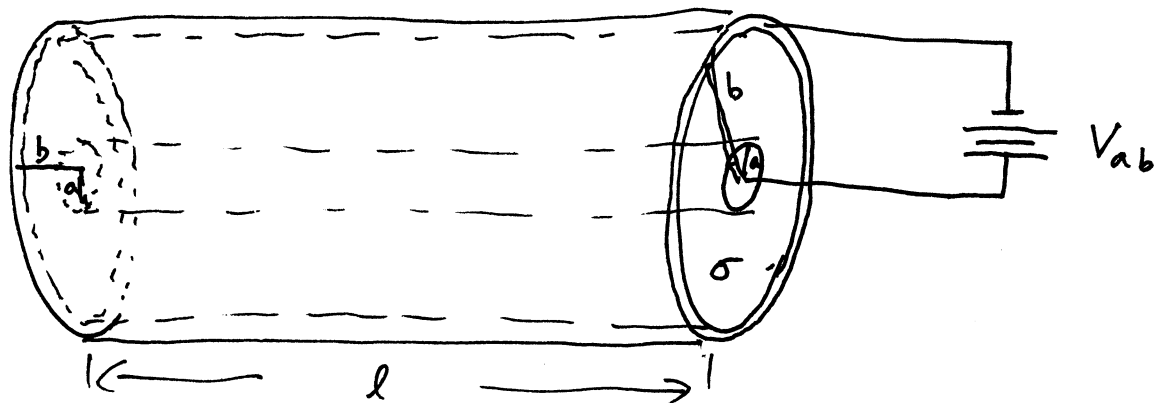
average drift velocity: $\sim 10 \mu\text{m/s}$.

very slow!

but the electrical force (field) transmits much faster!
(speed of light).

• e.g. 4-9. p. 87.

conductance of Coaxial Cable.



Find G' (conductance per unit length).

current can "leak" through the insulating material ($\sigma \neq 0$)
 $\hat{I}$.

Current density: $\vec{J} = \hat{r} \frac{I}{A} = \hat{r} \frac{I}{2\pi r l}$

Since $\vec{J} = \sigma \vec{E}$:

$$\vec{E} = \hat{r} \frac{I}{2\pi r \sigma l}$$

$$V_{ab} = V_a - V_b = - \int_b^a \vec{E} \cdot d\vec{l}$$

$$= - \int_b^a \frac{I dr}{2\pi \sigma l \cdot r}$$

$$(d\vec{l} = d\vec{r})$$

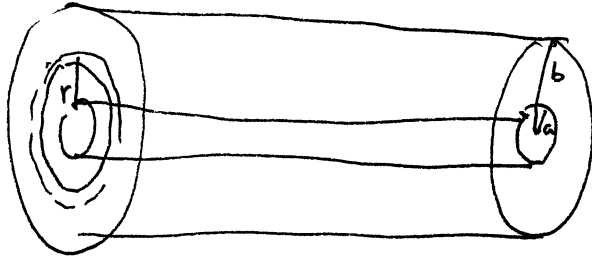
$$= \frac{I}{2\pi \sigma l} \ln \frac{b}{a}$$

Conductance: $G = \frac{I}{V_{ab}} = \frac{2\pi \sigma l}{\ln \frac{b}{a}}$

conductance per unit length:

$$G' = \frac{G}{l} = \frac{2\pi \sigma}{\ln \frac{b}{a}}$$

$$\text{OR: } dR = \rho \frac{dr}{2\pi r l} = \frac{dr}{2\pi \sigma l r} \quad \left(= \rho \frac{\text{length}}{\text{cross section area}} \right)$$



$$R = \int_a^b \frac{dr}{2\pi \sigma l r} = \frac{1}{2\pi \sigma l} \ln \frac{b}{a}$$

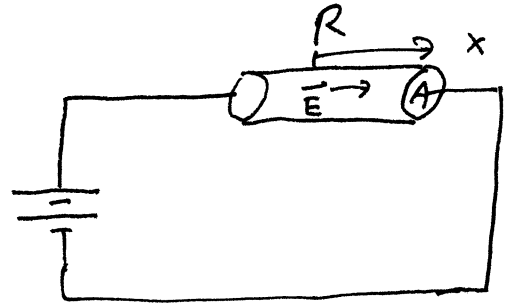
$$G = \frac{1}{R} = \frac{2\pi \sigma l}{\ln \frac{b}{a}}$$

$$G' = \frac{G}{l} = \frac{2\pi \sigma}{\ln \frac{b}{a}}$$

• Joule's law.

Power dissipated in R :

circuit: $P = IV = I^2 R$.



field: $P = \int_V \vec{E} \cdot \vec{J} dV = \int_V \sigma |\vec{E}|^2 dV$

verify: $\int \sigma |\vec{E}|^2 dV$

$$= \sigma \int_A E_x dA \cdot \int_0^l E_x dx$$

$$= \sigma E_x A E_x l$$

$$= J \cdot A \cdot E \cdot l$$

$$= I \cdot V$$

(Assuming uniform $\vec{E}$
along x-direction.)