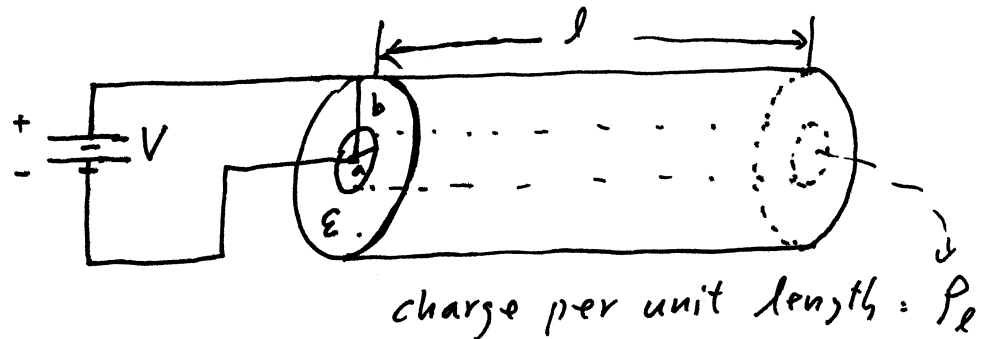


Lecture 11 .

Ref: 4-10, 4-11, 4-12; 5-1.

e.g. Capacitance of coaxial line



in ϵ : $\vec{E} = -\hat{r} \frac{\lambda}{2\pi\epsilon r}$ (Using Gauss' law)

$$= -\hat{r} \frac{Q}{2\pi\epsilon r \cdot l}$$

$$V = - \int_a^b \vec{E} \cdot d\vec{r} = \frac{Q}{2\pi\epsilon l} \int_a^b \frac{dr}{r} = \frac{Q}{2\pi\epsilon l} \ln \frac{b}{a}.$$

$$C = \frac{Q}{V} = \frac{2\pi\epsilon l}{\ln(b/a)}, \quad \text{per unit length: } C' = \frac{2\pi\epsilon}{\ln(b/a)}$$

Resistance: $R = \frac{1}{\sigma} \int_a^b \frac{dr}{2\pi r l} = \frac{1}{2\pi\sigma l} \ln \frac{b}{a}.$

$$RC = \frac{\epsilon}{\sigma}. \quad \text{--- same as parallel-plate capacitor.}$$

Image Method

11-2

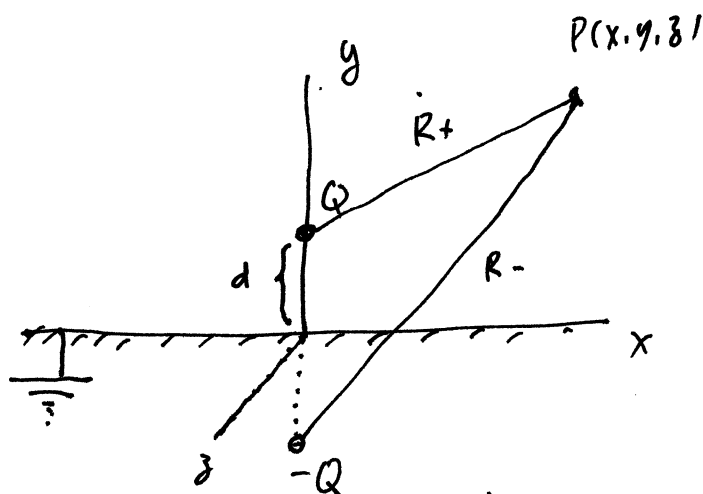
— A very clever way to solve certain electrostatic problems.

e.g. A point charge Q and a grounded plane conductor

find $V(x, y, z)$ for $y > 0$

Difficulty: unknown charge distribution on the conductor surface.

$$\rho_s \neq 0.$$



Idea: Use an equivalent charge ("image charge") to replace the conductor.

This "equivalent" problem has the same equations

$$\nabla^2 V = \rho / \epsilon_0 \text{ in } y > 0.$$

and the same boundary condition $V=0$ at $y=0$.

We don't care if it's equivalent in $y < 0$, because we know that $V=0$ in $y < 0$.

The image charges do not change the charge distributions in the area of interest, they ensure the boundary condition is satisfied on the conductor surfaces.

$$\therefore V(x, y, z) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R_+} + \frac{-Q}{R_-} \right)$$

$$= \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{x^2 + (y-d)^2 + z^2}} - \frac{1}{\sqrt{x^2 + (y+d)^2 + z^2}} \right)$$

To find $\vec{E}$: $\vec{E} = -\nabla V$. or use the textbook approach. (p.106).

To find ρ_s on conductor: exercise 4.19. p.107.

- Magnetostatics

- t -independent Maxwell's equations on magnetism:

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} \quad \left(\frac{\partial \vec{D}}{\partial t} = 0 \right)$$

$$\left(\vec{J} \neq 0, \text{ but } \vec{J} - \text{steady} \right) \quad \left(\frac{\partial \vec{J}}{\partial t} = 0 \right)$$

charge can move!

- Material: μ (magnetic permeability)

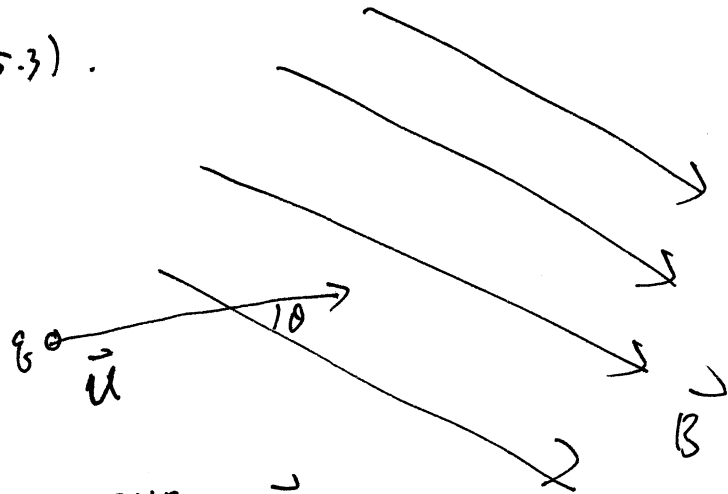
$$\vec{B} = \mu \vec{H} \quad (\text{except ferromagnetic materials})$$

- Magnetic force on a moving charge:

$$\vec{F}_m = q \vec{u} \times \vec{B}$$

$\vec{u}$ - velocity

(5.3)



(5.3) can be considered as the definition of $\vec{B}$.

RHR: $\vec{F}_m$: $\otimes$
(into page)

$$F_m = q u B \sin \theta$$

$$= q u_{\perp} B$$

$$= q u B_{\perp}$$

• Lorentz force

electromagnetic force on a moving charge q .

$$\vec{F} = q (\underbrace{\vec{E}} + \underbrace{\vec{u} \times \vec{B}})$$

electric
force

magnetic force

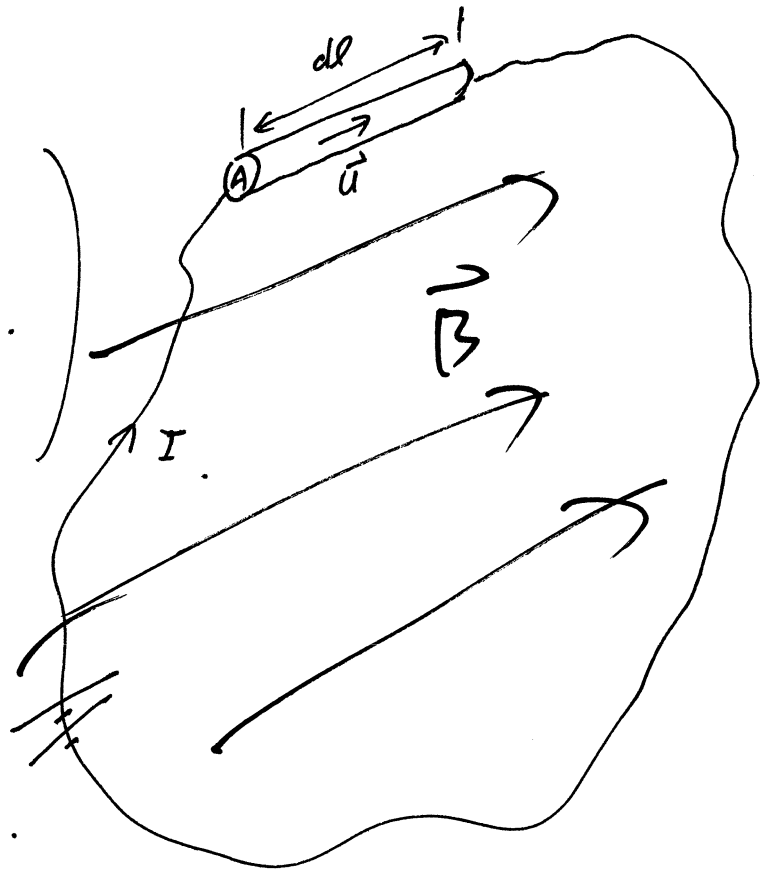
• Magnetic force on a current element:

$$d\vec{F}_m = I d\vec{\ell} \times \vec{B}$$

$$\left(\begin{array}{l} \text{Note: } I d\vec{\ell} = Nq \vec{u} \\ \quad \quad \quad = Nq \frac{d\vec{\ell}}{dt} \\ \quad \quad \quad \frac{Nq}{dt} = I. \end{array} \right)$$

Note: $I d\vec{\ell}$ is an element
of a current I .

It can not exist
without a ~~the~~ closed current.



Total magnetic force on the closed circuit:

$$\vec{F}_m = I \oint_C d\vec{\ell} \times \vec{B}$$

5.10 .