Phys 221

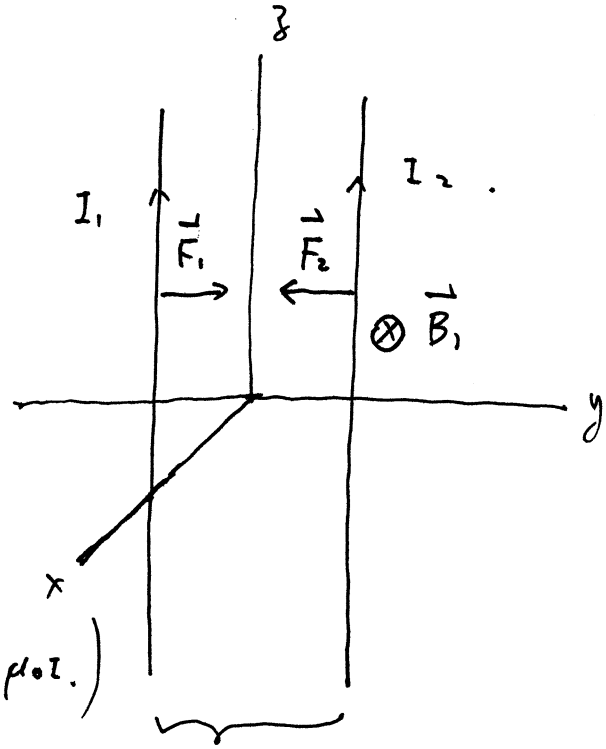
Lecture 14 .

Ref: 5-3, 5-4, 5-6

e.g. Magnetic force between 2 parallel currents

Long wires I_1, I_2.

distance: d .



Force on I_2 due to I_1:

I_1 produces a $\vec{B}$-field at I_2:

$$\vec{B}_1 = -\hat{x}\,\frac{\mu_0 I_1}{2\pi d}$$

(Ampere's law) $\left(\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I .\right)$

The magnetic field $\vec{B}_1$ exerts a force on I_2: $\vec{F}_2 = I_2 \int d\vec{\ell} \times \vec{B}_1$

$$\vec{F}_2 = I_2 \ell\, \hat{z} \times \vec{B}_1 = -\hat{y}\,\frac{\mu_0 I_1 I_2 \ell}{2\pi d} .$$

Force per unit length: $\vec{F}_2{}' = -\hat{y}\,\frac{\mu_0 I_1 I_2}{2\pi d}$. $\left(\vec{F}_2{}' = \frac{\vec{F}_2}{\ell} . \text{ ok: } \ell = 1\right)$

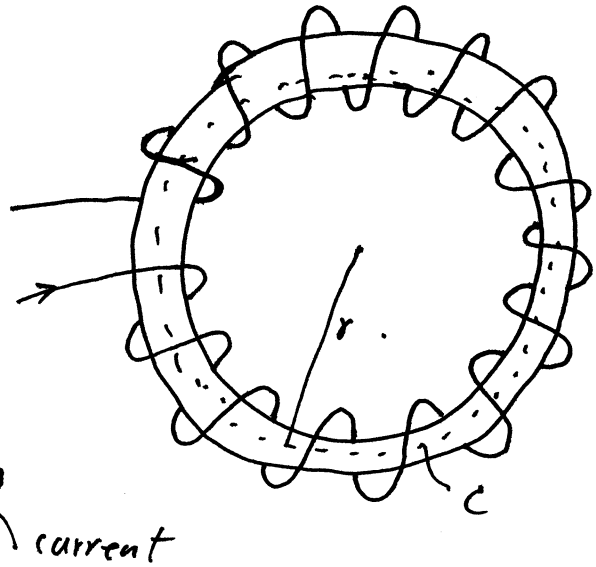
Force on I_1 due to I_2: $\vec{F}_1 = \hat{y}\,\frac{\mu_0 I_1 I_2 \ell}{2\pi d}$ (Newton's 3rd law)

Question: what if the two currents are in opposite directions .? (repulsive) (demo).

• e.g. Magnetic field inside a toroidal coil.

Use Ampere's law:

$$\oint_C \vec{H}\cdot d\vec{\ell} = NI.$$



Symmetry:

$$H \cdot 2\pi r = NI.$$

$$\vec{H} = \hat{\phi} H.$$

$$\vec{H} = \hat{\phi}\frac{NI}{2\pi r} = \hat{\phi} nI.$$

direction of $\hat{\phi}$: RHR:

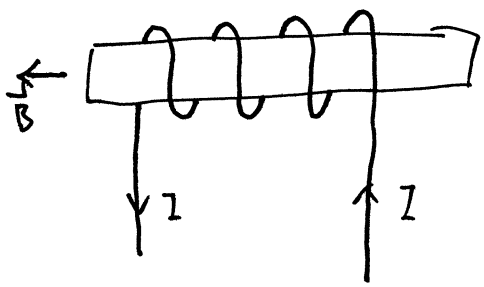
n — Number of turns per unit length.

e.g.: Magnetic field inside a solenoid:

A Toroid with $r \to \infty$.

$$|\vec{H}| = nI.$$

$$|\vec{B}| = \mu nI.$$



direction: right-hand-rule.