Phy 221.

Lecture 15-16. Ref: 5-5, 5-6.

• Vector magnetic potential $\vec{A}$

Since $\nabla \cdot \vec{B} = 0$ (No magnetic monopole)

and $\nabla \cdot (\nabla \times \vec{A}) = 0$ (identity for any vector $\vec{A}$).

$\therefore$ We define the magnetic vector potential $\vec{A}$

such that
$$\vec{B} = \nabla \times \vec{A} \qquad (5.53)$$

(similar to electrostatics. Since $\nabla \times \vec{E} = 0$
and. $\nabla \times (\nabla V) = 0$
$\therefore$ define. $\vec{E} = -\nabla V$.)

magnetic potential $\vec{A}$ is a vector function whose curl is $\vec{B}$.

Note: (5.53) can not define $\vec{A}$ uniquely.

註. $\because$ if $\vec{A}$ satisfies $\vec{B} = \nabla \times \vec{A}$.

Then, $\vec{W} = \vec{A} + \nabla f$ also:

$$\nabla \times \vec{W} = \nabla \times \vec{A} + \nabla \times \nabla f = \nabla \times \vec{A}$$

$(\because \nabla \times \nabla f = 0)$.

- Source of magnetic field : $\vec{J}$.

$$\nabla \times \vec{B} = \mu \vec{J}$$

Then: $$\nabla \times (\nabla \times \vec{A}) = \mu \vec{J}$$

$$\nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J}.$$

Since. (5.53) can not define $\vec{A}$ uniquely,

we can add more conditions to define $\vec{A}$ better.

Coulomb's choice : $\nabla \cdot \vec{A} = 0$.
(gauge).

Then. $$\boxed{\nabla^2 \vec{A} = -\mu \vec{J}}$$ (use $\nabla \cdot \vec{A} = 0$.)

(5.60).

This is a vetor poisson's eq.

i.e, we have 3 components :

$$\left.\begin{aligned} \nabla^2 A_x &= -\mu J_x \\ \nabla^2 A_y &= -\mu J_y \\ \underbrace{\nabla^2 A_z}_{\text{potential}} &= \underbrace{-\mu J_z}_{\text{source}}. \end{aligned}\right\}$$

ALL scalar equations!

similar to

$$\nabla^2 V = -\rho/\varepsilon$$

Since.
$$\nabla^2 V = -\rho/\varepsilon$$

has a solution.
$$V = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho_v\, dv'}{R'}.$$

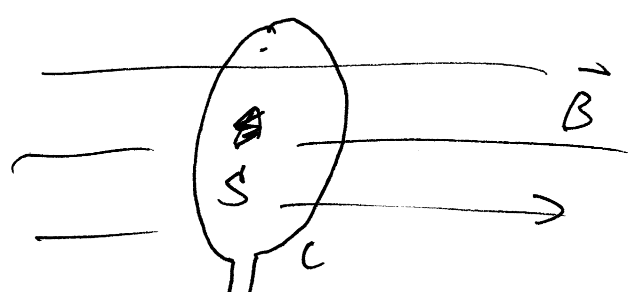
$$\nabla^2 A_x = -\mu J_x$$ should have a solution:

$$A_x = \frac{\mu}{4\pi}\int_{V'} \frac{J_x\, dv'}{R'}$$

$$\therefore \quad \vec{A} = \frac{\mu}{4\pi}\int_{V'} \frac{\vec{J}\, dv'}{R'}$$

} can be computed one component at a time.

• Magnetic flux:



$$\phi = \int \vec{B}\cdot\vec{ds}$$

$$= \int_S (\nabla \times \vec{A})\cdot\vec{ds} \qquad \text{(stoke's Thm)}$$

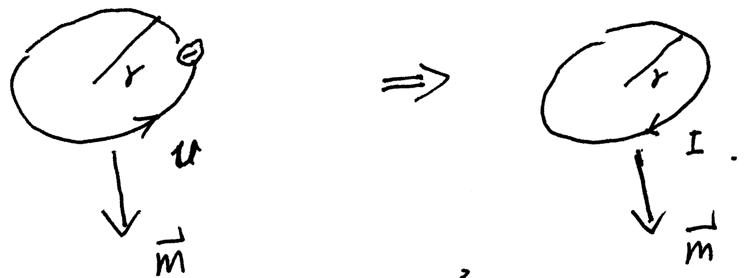
$$= \oint_C \vec{A}\cdot\vec{dl} \qquad \text{(circulation of } \vec{A}\text{)}$$

• Magnetic Properties of Materials

magnetic dipole moment of an electron:

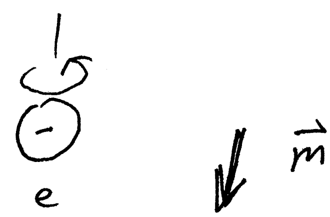
a). orbital.

$$I = -\frac{e}{T} = -\frac{eu}{2\pi r}.$$



$$m = \pi r^2 I.$$

b). spin:



• Magnetization vector:

$$\vec{M} = \frac{\sum \vec{m}_i}{V}.$$

$$\vec{M} = \chi_m \vec{H}$$

χ_m ↑ dep. on material.

$\vec{H}$ ↑ depends on applied magnetic field.

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 (\vec{H} + \vec{M})$$
$$= \mu_0 \vec{H} + \mu_0 \chi_m \vec{H}$$
$$= \mu_0 (1 + \chi_m) \vec{H}$$

$$\vec{B} = \mu \vec{H} \qquad \mu = \mu_0 \mu_r$$

$$\mu = \mu_0 (1 + \chi_m). \qquad \mu_r = 1 + \chi_m$$

μ — magnetic permeability.

χ_m — magnetic susceptibility.

$$\begin{cases} \chi_m < 0 \\ \chi_m > 0 \text{ and } \chi_m \ll 1 \\ \chi_m \gg 1. \end{cases}$$

- Diamagnetism. $\chi_m < 0$.

  i.e, $\vec{M}$ is in opposite direction as $\vec{H}$.

  (because of the precession of orbital electrons under the influence of an applied $\vec{H}$ field)

  $$\chi_m \sim -10^{-5}.$$

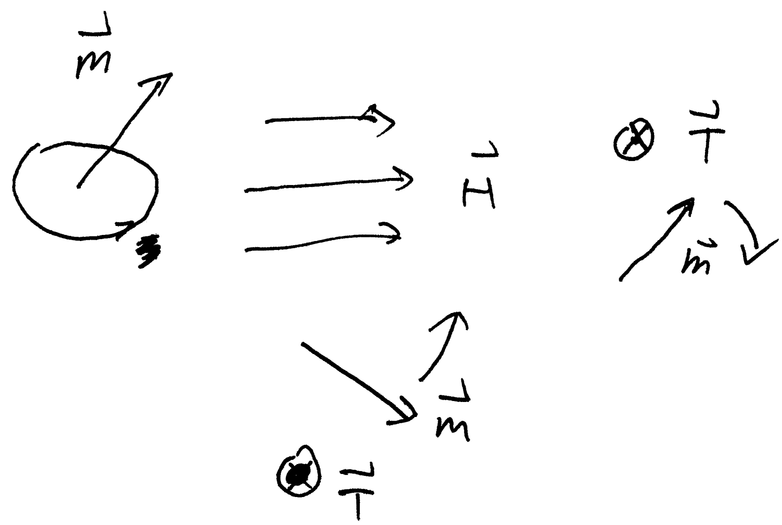
  $\mu \approx \mu_0$. — χ_m has a very small effect.

  (~~sometimes called "non-magnetic material"~~)

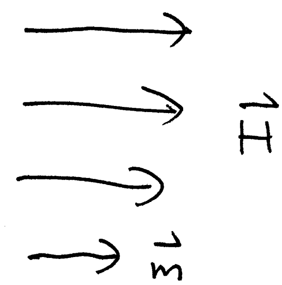
- paramagnetism : $\chi_m > 0$

  but $\chi_m \ll 1$

electron spins can be aligned with the applied $\vec{H}$-field.



~~Be~~ Happiest:



but weak. $\chi_m \sim 10^{-5}$.

$\mu \approx \mu_0$.

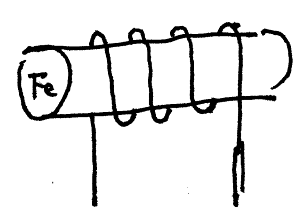
- Ferromagnetism.

— materials with <u>Magnetized domains</u>, e.g. (iron, nickel, cobalt).

↑ many $\vec{m}$ are aligned in the same direction.

$\chi_m \gg 1$.

How to increase magnetic field $\vec{B}$ in a solenoid : put iron in it.



- magnetic Hysteresis of <u>ferromagnetic materials</u>.

(There are magnetized domains
$\vec{m}$'s are aligned within a domain)

when ~~$\vec{B}$~~ $\vec{H}=0$, The domains are randomly oriented.

Net $\vec{M}=0$.

when $\vec{H}\uparrow$, The domains are more aligned.

Net $\vec{M}\neq 0$.

($\vec{M}$ can be very strong*!)
i.e. $M>H$.

$$\vec{B}=\mu_0(\vec{H}+\vec{M}).$$

When $\vec{H}=0$.
$\vec{M}\neq 0$.

(spontaneous magnitization).

(permanent magnet!)