Ref: 5-7, 8, 9.

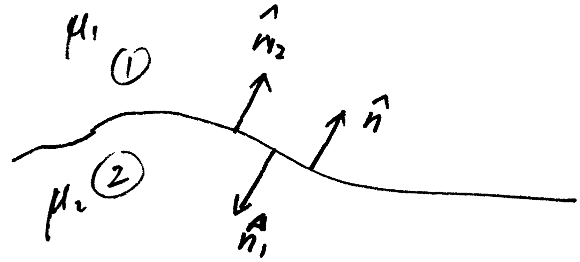
Phys 221

Lecture 17.

Assignment #7.
5.23, 5.25, 5.27, 5.31
5.33, 5.37

$\hat{n} = \hat{n}_2$

- Magnetic Boundary Conditions.



(recall: in electrostatics.

$$\oint_S \vec{D} \cdot d\vec{s} = Q$$

$$\Downarrow$$

$$D_{1n} - D_{2n} = \rho_s \qquad (\hat{n} \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s)$$

Now, magnetostatics, $Q_m = 0$.

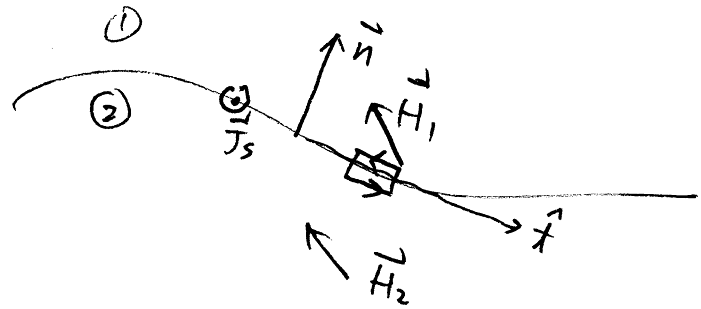
$$\oint_S \vec{B} \cdot d\vec{s} = 0 \quad \Rightarrow \quad \boxed{\hat{n} \cdot (\vec{B}_1 - \vec{B}_2) = 0 \\ \text{i.e.} \quad B_{1n} = B_{2n}.} \qquad (5\text{-}79)$$

For homogeneous and isotropic materials.

$$\boxed{\mu_1 H_{1n} = \mu_2 H_{2n}.} \qquad 5\text{-}80$$

Also, from electrostatics.

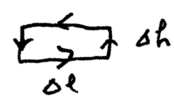
$$\left(\oint_C \vec{E} \cdot d\vec{l} = 0 \quad \Rightarrow \quad E_{1t} = E_{2t} \right)$$



Now $\oint_C \vec{H} \cdot d\vec{l} = I$.

$\Downarrow$

$I = J_s \cdot \Delta l$



$\Delta h \to 0$

Δl ~ small.

$$H_{2t} - H_{1t} = J_s.$$

OR: $\boxed{\hat{n} \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s}$. (5.84).

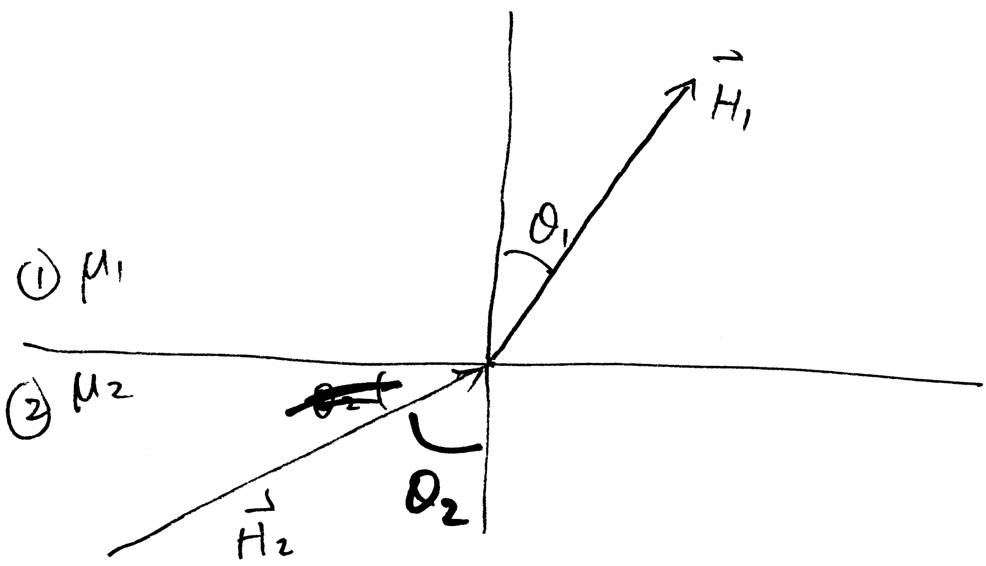
surface current density (free)

if $\vec{J}_s = 0$.

Then $H_{1t} = H_{2t}$.

· when there is no free surface current.

$\vec{J}_s = 0$.



relation $\theta_1 \sim \theta_2$?

$$H_{1t} = H_{2t}. \qquad (J_s = 0)$$

$$H_1 \sin\theta_1 = H_2 \sin\theta_2 \qquad (A)$$

$$B_{1n} = B_{2n}. \qquad (\vec{B} = \mu\vec{H})$$

$$\mu_1 H_1 \cos\theta_1 = \mu_2 H_2 \cos\theta_2 \qquad (B).$$

$$\frac{(A)}{(B)} : \quad \frac{\sin\theta_1}{\mu_1 \cos\theta_1} = \frac{\sin\theta_2}{\mu_2 \cos\theta_2} \qquad \frac{\tan\theta_1}{\mu_1} = \frac{\tan\theta_2}{\mu_2}$$

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}.$$

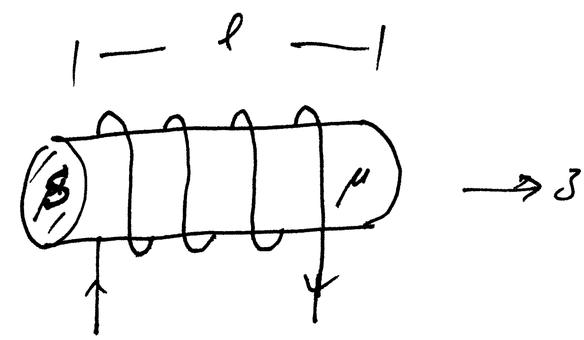
- Inductance

— An inductor stores magnetic energy.

(just like a capacitor stores electric energy. $W_e = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$)

- Self inductance :

e.g. A solenoid



$$\vec{B} = \hat{z}\,\frac{\mu N I}{l}.$$

flux through a loop : $\Phi = \frac{\mu N I}{l} S$ S — area.

flux through N loops (Total flux) :

$$\Lambda = N\Phi = \frac{\mu N^2 S}{l} I.$$

$$\Lambda \propto I. = L I.$$

L — self inductance.

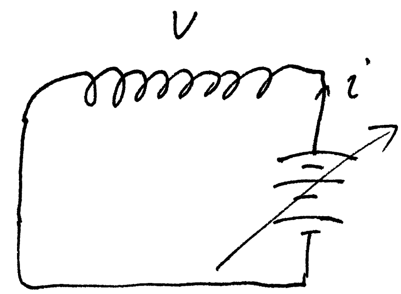
for a solenoid : $L = \mu \frac{N^2}{l} S.$

- Note : Faraday's law :

$$\mathcal{E}_{emf} = -N\frac{d\Phi}{dt} = -\frac{d\Lambda}{dt} = -L\cdot\frac{dI}{dt}.$$

electromotive force.

· Magnetic energy.



$$W_m = \int P \cdot dt$$

$$= \int v\, i\, dt$$

$$= \int L \frac{di}{dt} \cdot i\, dt.$$

$$= \int_0^I L\, i\, di$$

$$= \frac{1}{2} L I^2.$$

$$V = -\mathcal{E}_{emf} = L \cdot \frac{di}{dt}.$$

Solenoid:

$$W_m = \frac{1}{2}\mu \frac{N^2}{\ell} S I^2. \qquad \left(H = \frac{NI}{\ell} \right)$$

$$H^2 = \frac{N^2 I^2}{\ell^2}$$

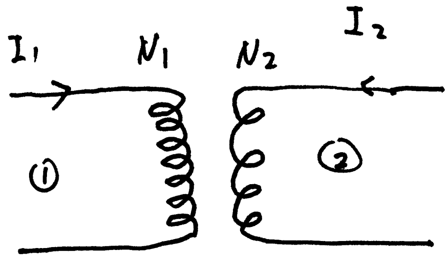
$$= \frac{1}{2}\mu \underbrace{\ell S}_{\text{volume}} \cdot H^2.$$

$$= \frac{1}{2}\mu \underbrace{V}_{\text{volume}} H^2.$$

general: magnetic energy density: $w_m = \frac{1}{2}\mu H^2$.

Total magnetic energy: $W_m = \int \frac{1}{2}\mu H^2 \, d\underbrace{V}_{\text{volume}}$.

• Mutual Inductance



$\vec{B}$-field at ② due to ① : $\vec{B}_1$

Flux through a loop in ② : $\Phi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{s}$

Total magnetic flux linkage through ②:

$$\Lambda_{12} = N_2 \Phi_{12} = N_2 \int_{S_2} \vec{B}_1 \cdot d\vec{s}$$

Mutual Inductance :

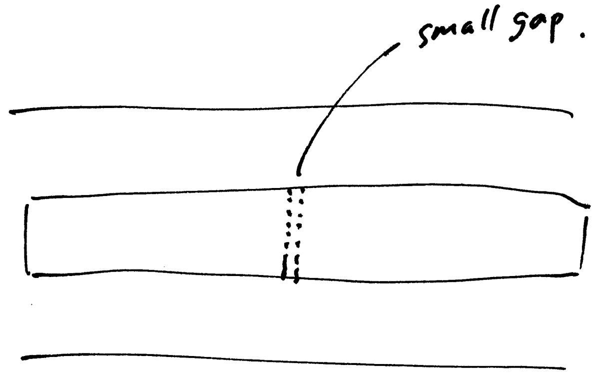
$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2}{I_1} \int_{S_2} \vec{B}_1 \cdot d\vec{s}$$

Can be shown :

$$L_{21} = L_{12}$$

$$\left(L_{21} = \frac{N_1}{I_2} \int_{S_1} \vec{B}_2 \cdot d\vec{s} \right)$$

(b). If there were a small vertical gap in the middle of the rod, what would be $\vec{H}$ and $\vec{B}$ inside the gap. (Assume the opening of the gap does not affect the magnetic field inside the rod).



Inside the gap : $(\hat{n} \parallel \hat{z})$

Boundary conditions : $B_{1n} = B_{2n}$

$\therefore B_{inside} = B_{outside}$.

$$\vec{B} = \hat{z}\, \mu n I .$$

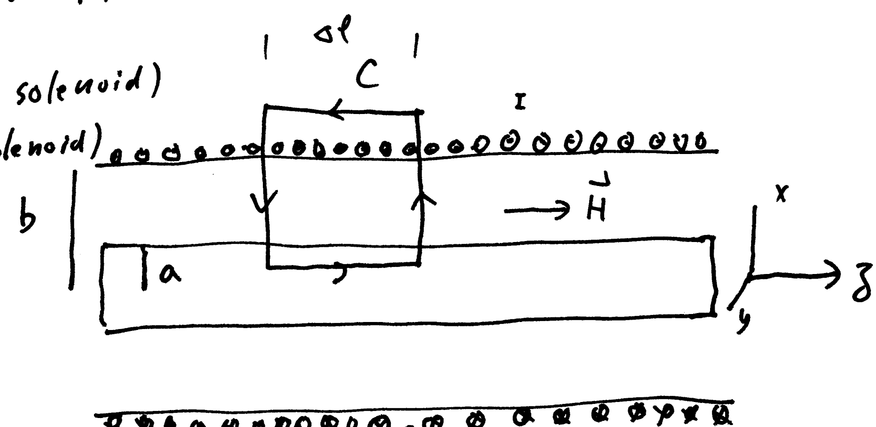
$$\vec{H} = \frac{\vec{B}}{\mu_0} = \hat{z}\, \frac{\mu}{\mu_0} n I .$$

e.g. A circular rod with magnetic permeability μ is inserted in a long solenoid. The radius of the rod is a and the inner radius of the solenoid is b. $b > a$. The solenoid's winding is n turns per meter and carries a current I.

(a). Find $\vec{B}$, $\vec{H}$, and $\vec{M}$ inside the solenoid

RHR: $\vec{H} = \hat{z} H$ (inside solenoid)

$\vec{H} = 0$ (outside solenoid)



Ampere's law:

$$\oint_C \vec{H} \cdot d\vec{l} = I \cdot \delta l .$$

$\vec{H} = \hat{z} \cdot nI$. (for $r < b$, inside and outside the rod).

$\vec{B} = \mu \vec{H}$: for $r < a$ (inside the rod): $\vec{B} = \hat{z} \mu n I$.

for $a < r < b$: $\vec{B} = \hat{z} \mu_0 n I$.

$\vec{M} = \frac{\vec{B}}{\mu_0} - \vec{H}$: for $r < a$: $\vec{M} = \hat{z} \left(\frac{\mu}{\mu_0} nI - nI \right) = \hat{z} nI \left(\frac{\mu}{\mu_0} - 1 \right)$

for $a < r < b$: $\vec{M} = 0$