Ch. 6. Maxwell's Equs.

for time-varying fields

Ref: 6-1, 6-2,
6-3, 6-4, 6-5, 6-6

- Faraday's law.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$V_{emf} = -N\frac{d\Phi}{dt}. \qquad \Phi = \int_S \vec{B} \cdot d\vec{s}$$

- change in Φ:

① . $\vec{B}$ changes with time. (S does not change)

② . $\vec{S}$ change with time. (moving loop)
(but const. $\vec{B}$).

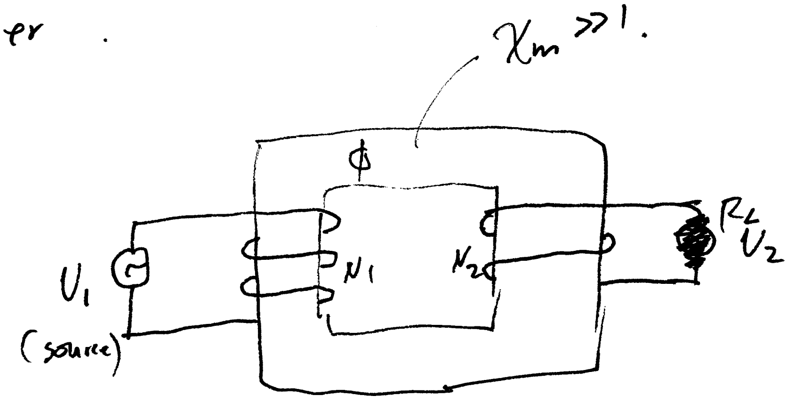
③ . Both $\vec{B}$ and $\vec{S}$ change with time
($\frac{d\Phi}{dt}$ — total differentiation).

- negative sign.

Lenz's Law — the induced current should produce a $\vec{B}$-field that is against the change in Φ: (Effect against cause).

① . $\vec{B}$ changes with time but S does not.

e.g. — Transformer.



$$V_1 = -N_1 \frac{d\Phi}{dt}.$$

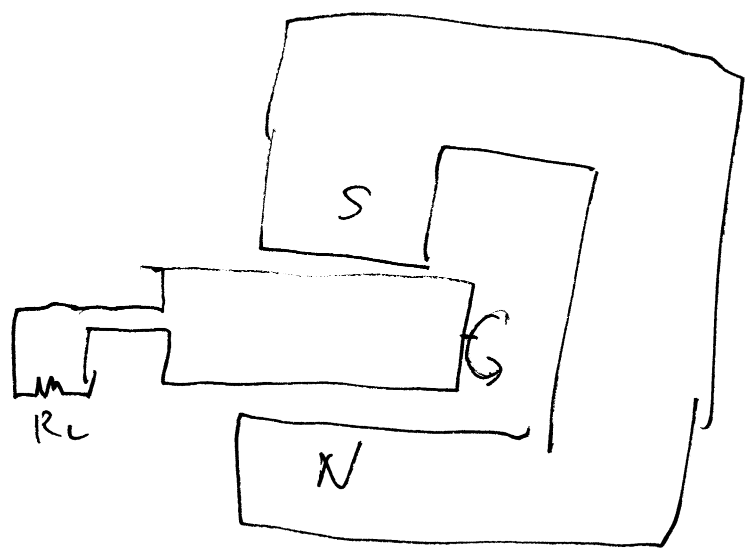
$$V_2 = -N_2 \frac{d\Phi}{dt}.$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}.$$

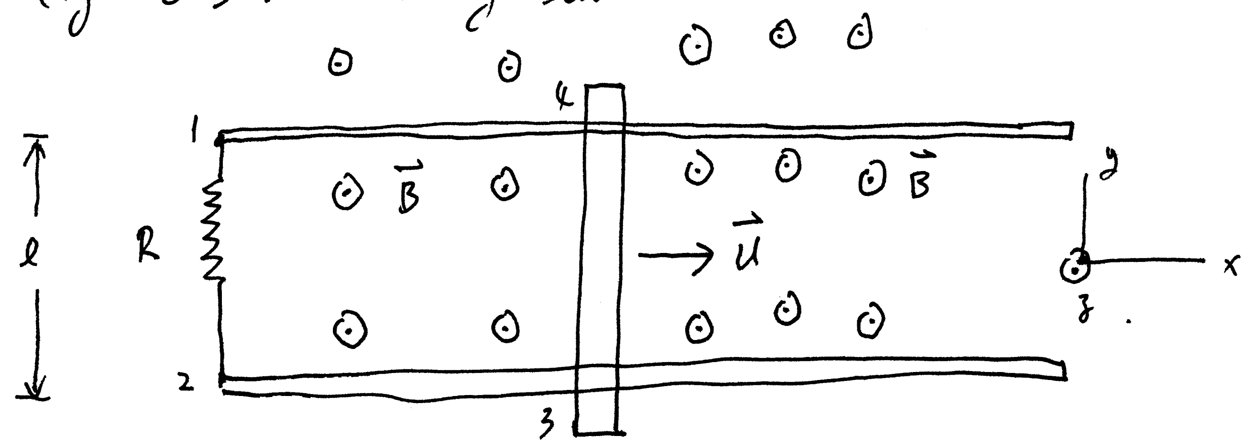
Ideally. $P_1 = P_2$. (No loss): $\frac{I_1}{I_2} = \frac{N_2}{N_1}$, $R_{in} = \frac{V_1}{I_1} = \frac{V_2}{I_2}\left(\frac{N_1}{N_2}\right)^2 = \left(\frac{N_1}{N_2}\right)^2 R_L$.

② moving conductor in a static magnetic field.

— Electromagnetic generator.



e.g. 6-3. Sliding Bar.



$\vec{B} = \hat{z} B_0 x$ — Not uniform!

$\vec{u}$ — constant velocity $\vec{u} = \hat{x} u$. (increase flux in closed loop)

Find: emf due to the motion.

$$\Phi = \int_S \vec{B} \cdot d\vec{S} = \int_S \hat{z} B_0 x \cdot \hat{z}\, dx dy$$

$$= B_0 l \int_0^{x_0} x\, dx$$

(at time t, the position of moving bar is at x_0
$x_0 = ut$.)

$$= \frac{1}{2} B_0 l x_0^2 .$$

$$V_{emf} = -\frac{d\Phi}{dt} = -B_0 l \cdot x_0 \cdot \frac{dx_0}{dt} = -B_0 l x_0 u .$$

$$= -B_0 l u^2 t .$$

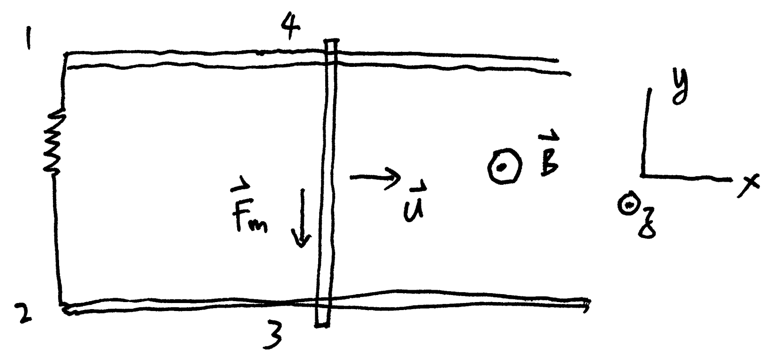
Direction of I: 2143. (4→3, reduce $\vec{B}$-field in loop)

Question: If the bar moves to the left, how do you determine the direction of induced current?
(Key: change in flux. not just flux!)

Another approach. (Lorentz Force)

Magnetic force acting on a moving charge q: $\vec{F}_m = q\vec{u} \times \vec{B}$

"Motional electric field": $\vec{E}_m = \frac{\vec{F}_m}{q} = \vec{u} \times \vec{B}$ $(4 \to 3)$



Direction of current: 4 3 2 1.

Potential: $V_3 > V_4$, (in the generator, ($\mathcal{E}_{emf}$): $\vec{E}_m$: pump charges from lower potential V_4 to higher potential V_3)

$V_2 = V_3$, $V_1 = V_4$

$\therefore$ $V_2 > V_1$. (in the circuit: current flows from higher potential V_2 to lower potential V_1)

In the generator, work by emf = rise in potential energy.

$$|V_{emf}| = \int_4^3 \vec{E}_m \cdot d\vec{l} = V_3 - V_4 = \int_4^3 (\vec{u} \times \vec{B}) \cdot d\vec{l} = \int_4^3 (-\hat{y}\, u B_0 x_0) \cdot (-\hat{y}\, dy)$$

$$= u B_0 x_0 l \qquad (x_0 = ut)$$

$$|V_{emf}| = u^2 B_0 l t = V_3 - V_4 = V_2 - V_1.$$