Phys 221

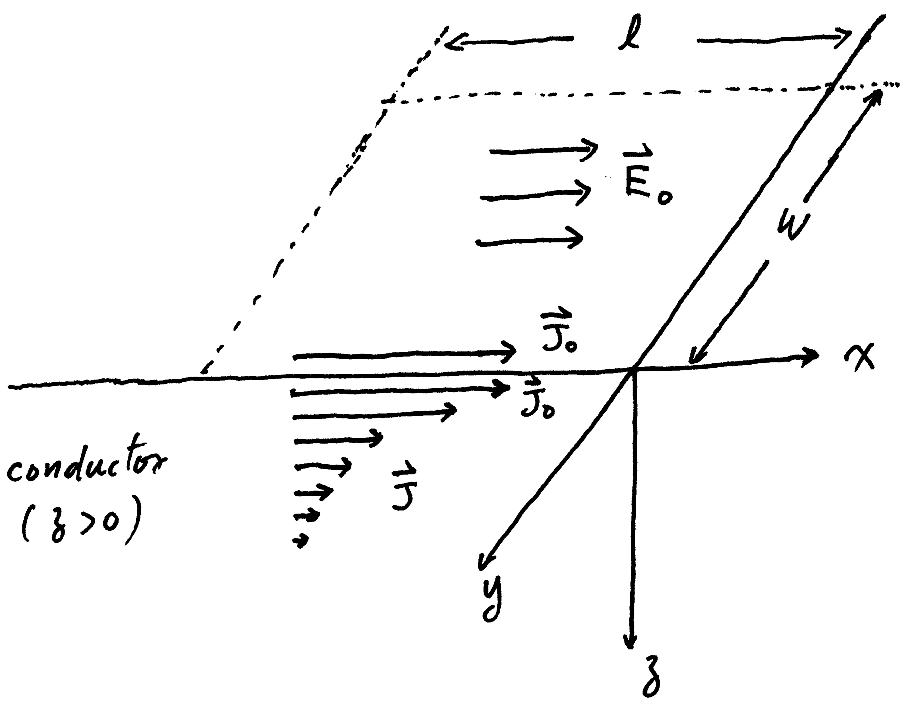
Lecture 26

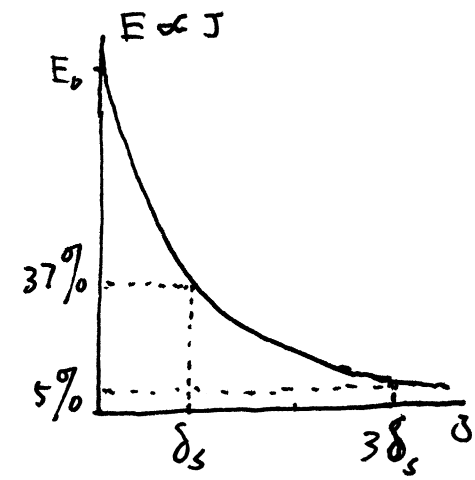
Ref. 7-6. Current flow in a good conductor

$$\alpha = \beta = \sqrt{\pi f \mu \sigma}$$

skin depth $\delta_s = \frac{1}{\alpha} \propto \frac{1}{\sqrt{f}}$

- A conducting solid occupying half-space ($z > 0$)





at high frequencies, the current flows in a thin layer at surface.

When the thickness of the conducting block is greater than $5\delta_s$, it can be considered as infinitely thick. (semi infinite).

$$\tilde{\vec{E}}(z) = \hat{x} E_0 e^{-\alpha z} e^{-j\beta z}$$

$$\vec{J}(z) = \sigma \vec{E}$$

$$= \hat{x} J_0 e^{-\alpha z} e^{-j\beta z} \qquad (J_0 = \sigma E_0)$$

$$= \hat{x} J_0 e^{-(1+j) z/\delta_s} \qquad \left(\alpha = \beta = \frac{1}{\delta_s}\right)$$

- Current (in width W)

$$\tilde{I} = W \cdot \int_0^\infty J_0 e^{-(1+j)z/\delta_s} dz = \frac{J_0 W \delta_s}{1+j}$$

(as if it's a conducting sheet with a thickness δ_s)

- Voltage (accross length l at surface):

$$\tilde{V} = E_0 l = \frac{J_0 l}{\sigma}$$

- Impedance:

$$Z = \frac{\tilde{V}}{\tilde{I}} = \frac{1+j}{\sigma \delta_s} \cdot \frac{l}{W}$$

- Surface impedance (or internal impedance)

$$Z_s = \frac{1+j}{\sigma \delta_s}$$

(when $l = 1$, unit length
$W = 1$, unit width)

since $\mathrm{Im}(Z_s) > 0$. (inductive)

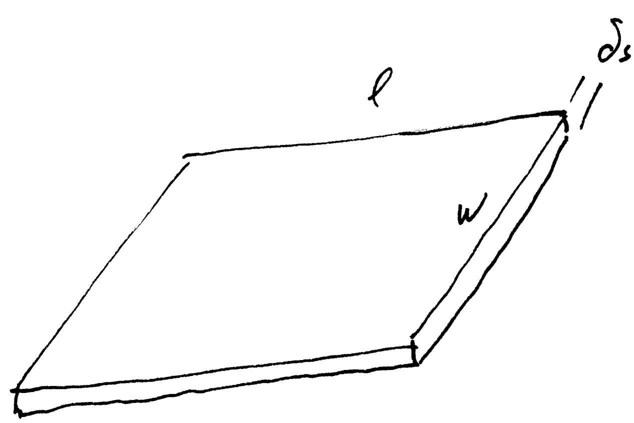
Z_s can be written as $Z_s = R_s + j\omega L_s$

$$R_s = \frac{1}{\sigma \delta_s} = \sqrt{\frac{\pi f \mu}{\sigma}}$$

$$L_s = \frac{1}{\omega \sigma \delta_s} = \frac{1}{2}\sqrt{\frac{\mu}{\pi f \sigma}}$$

- ac resistance of a slab of width W and length l:

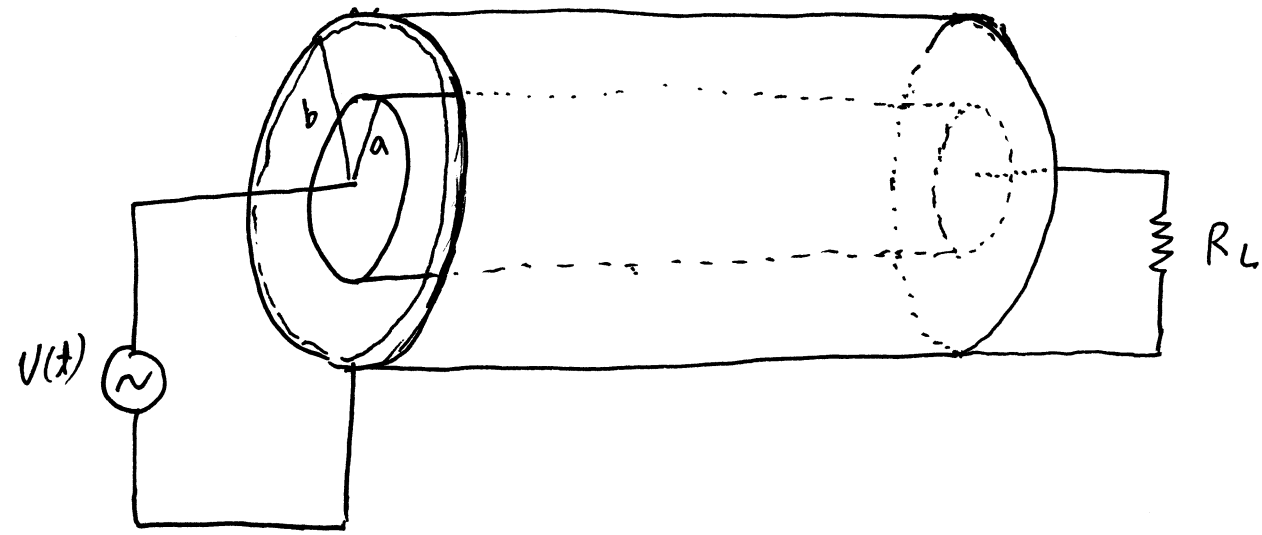
$$R = R_s \cdot \frac{l}{W} = \frac{l}{\sigma \delta_s W}$$



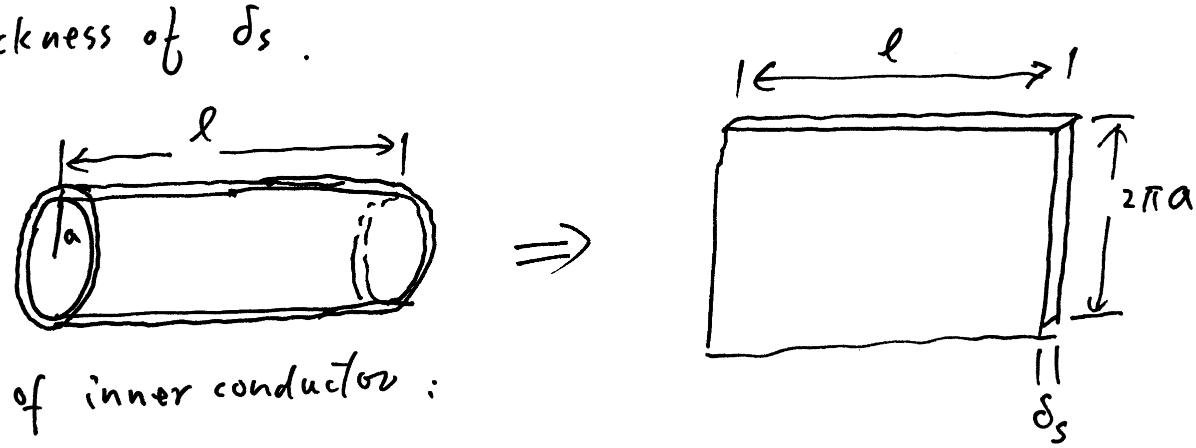
as if it's a conducting sheet with thickness δ_s. (again!)

- Coaxial Cable. Copper: $\sigma = 5.8 \times 10^{7}$ S/m.

$f = 1$ MHz ; $\delta_s = \sqrt{\pi f \mu \sigma} = 0.066$ mm



For the inner conductor, if $a \gg \delta_s$ (e.g: $a > 5\delta_s$), then ~~the~~ it can be considered as a thin sheet with a thickness of δ_s.



Resistance of inner conductor:

$$R_{in} = \frac{l}{\sigma \cdot 2\pi a \delta_s} = R_s \cdot \frac{l}{2\pi a}.$$

Similarly, outer conductor:

$$R_{out} = \frac{l}{\sigma 2\pi b \delta_s} = R_s \frac{l}{2\pi b}.$$

Total resistance per unit length ($l=1$): $R = \frac{R_{in} + R_{out}}{l} = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$