Phys 221

Lecture 27.

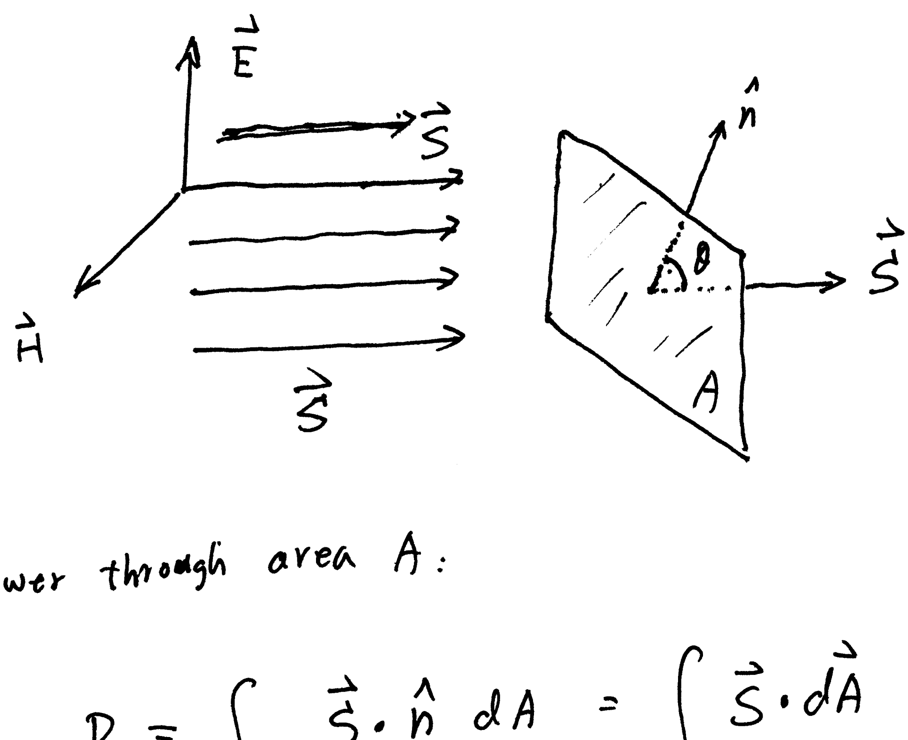
Ref. 7-7. Electromagnetic Power density.

- Poynting vector

$$\vec{S} = \vec{E} \times \vec{H}$$

Meaning: Direction: propagation direction ($\hat{S} = \hat{k}$)

Magnitude: Power through per unit area.



Power through area A:

$$P = \int_A \vec{S} \cdot \hat{n} \, dA = \int_A \vec{S} \cdot d\vec{A}$$

- In a lossless medium, e.g. plane wave travelling in $\hat{z}$ direction

— Time domain: $\vec{S}(z,t) = \vec{E}(z,t) \times \vec{H}(z,t)$

(Instantaneous relation) $= \hat{z} \dfrac{|E_{x0}|^2}{\eta} \cos^2(\omega t - kz)$ $\quad (k=\beta)$

time-average (over a period T):

$$\vec{S}_{av} = \frac{1}{T}\int_0^T \vec{S}(z,t)\,dt = \hat{z}\frac{|E_{x0}|^2}{2\eta} \qquad (7.157)$$

$$\left(\because \frac{1}{2\pi}\int_0^{2\pi} \cos^2 x\,dx = \frac{1}{2}\right)$$

— phasor domain: $\tilde{\vec{S}} \neq \tilde{\vec{E}} \times \tilde{\vec{H}}$ (because it's not a linear operation. $\times$)

It turns out:

(because of 7.157) $\boxed{\vec{S}_{av} = \frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right]}$ $\quad (7.158)$

$\tilde{\vec{H}}^*$ — complex conjugate of $\tilde{\vec{H}}$

(replace j with $-j$.

Verify 7.158: $\frac{1}{2}\mathrm{Re}\left[\tilde{\vec{E}} \times \tilde{\vec{H}}^*\right] = \frac{1}{2}\mathrm{Re}\left[\hat{x}\,E_{x0}\,e^{-jkz} \times \hat{y}\,\frac{E_{x0}^*}{\eta}\,e^{jkz}\right]$

$$= \frac{1}{2}\mathrm{Re}\left[\hat{z}\frac{|E_{x0}|^2}{\eta}\right] = \frac{1}{2}\hat{z}\frac{|E_{x0}|^2}{\eta}.$$

- In a lossy medium.

$$\eta_c = |\eta_c| e^{j\theta_\eta}$$

Then, can be shown:

$$\vec{S}_{av}(z) = \hat{z} \frac{|E_0|^2}{2 \cdot |\eta_c|} e^{-2\alpha z} \cos(\theta_\eta)$$

at $z = \delta_s$ (skin depth).

$$E = E_0 e^{-1} = 37\% \, E_0$$

$$S_{av} = S_{av}(0) \cdot e^{-2} = 14\% \, S(0).$$

- Decibel Scale.

  - For power ratio $G = \frac{P_1}{P_2}$.

$$G\,[dB] = 10 \log \frac{P_1}{P_2}.$$

  - For voltage, current, field $\vec{E}$, $\vec{H}$, $g = \frac{V_1}{V_2}$

$$g\,[dB] = 20 \log \frac{V_1}{V_2}.$$

(why? $\because$ $P = \frac{V^2}{R}$, so G and g in [dB] $P \propto I^2, E^2, H^2$, etc. have the same meaning)